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Bidder Collusion and the Auction with Target Bids*

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Abstract

This paper studies collusion in one-shot auctions, where a buyer can bribe his competitors into lowering their bids. We modify the single-unit Vickrey auction to incite deviations from the designated-winner scenario and thus undermine collusion. The construction of mechanism does not require the knowledge of colluders' identities or distributions of valuations, in which sense it is entirely detail-free.

Keywords: Bidder collusion, detail-free auctions, Vickrey auction.

JEL codes: D82, D44, C72.

1 Introduction

Bidder collusion in auctions jeopardizes the seller's revenue. In contrast to its numerous virtues, the Vickrey (second-price) auction is extremely susceptible to

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bidder collusion (Ausubel, Milgrom, 2006). In the Vickrey mechanism, bidders are likely to engage in pre-play communication for two reasons. First, information disclosure entails no risk. Since truthful bidding is weakly dominant, identifying a strong competitor never results in more aggressive bidding. Second, the exchange of information often discourages competition and lowers the auction price, in particular if the cartels can share the spoils through transfers.[1]

In this paper, we modify the Vickrey mechanism in order to reverse the incentives for pre-play communication. We change the allocation rules in such a way that disclosing information prior to the auction results in more intense competition. This implies that any bidder who contemplates winning the auction will avoid being identified by the other bidders. Collusive negotiations are then hampered by the incentives to protect private information.

To incite competition upon information disclosure we introduce 'bid targeting' in a mechanism with two bidding rounds. In the first round bidders place a preliminary bid and choose a target bidder; self-targeting is permitted. In the second round the bid is adjusted towards the target. Depending on the bidders' target choices, the seller applies either the standard Vickrey assignment or a novel *gap rule* that favors the second-highest bidder. The Vickrey auction is thus modified in two ways: first, the bidders' message space includes not only the bid, but also the target submission. Second, the new allocation rule incites deviations

[1]Marshall and Marx (2007) show that surplus-sharing cartels are able to eliminate all competition in a one-shot Vickrey auction without controlling their members' actions. The role of side-transfers in first- and second-price auctions has been extensively studied in Graham and Marshall, 1987; Mailath and Zemsky, 1991; McAfee, McMillan, 1992. A different strand of literature studies collusion sustained through repeated interactions between the cartel members: Abreu, Pearce, Stacchetti, 1986; Skrzypacz, Hopenhayn, 2004; Aoyagi 2007; Vergote, 2011. This paper focuses on one-shot interactions.

from the designated-bidder scenario.

In the presence of prior collusion, bid targeting is akin to blowing the whistle on a likely winner. By targeting the cartel leader a non-designated member of the cartel can win the auction by the gap rule. This feature of the mechanism mirrors whistle-blower rewards used by the anti-cartel authorities.[2] The advantage is that, in the auction with target bids, the whistle-blower reward is part of the assignment rule and does not require the bidders to present any proofs of collusion. The seller "cooperates" with the whistle-blower by using the target's information to adjust the whistle-blower's bid. The combination of both information pieces: leader's identity (whistle-blower's information) and his preliminary bid (seller's information), adversely affects the cartel leader. Even if he can credibly commit to reward cooperation, the reward (as long as it does not exceed his own collusion surplus) will be insufficient to preclude deviations. Following the results of Marshall and Marx (2009), we suggest that the seller does not disclose the price at which the good was sold. Facing fewer observables the cartel has less flexibility in the conditional side-transfers to sustain cooperation and thus the non-disclosure of the final price contributes to hindering collusion. At the same time, we can allow for settings where the winner's identity cannot be hidden.

First we find the non-cooperative equilibrium of the modified mechanism and show that when bidders have no information about each other, they will not target bids other than their own. This is true because targeting a bidder at random

[2]For instance, the German *Bundeskartellamt*. US Security and Exchange Commission awards between 10 and 30 percent of money ordered in sanctions. http://www.sec.gov/whistleblower.

may limit one's final bid and prevent him from winning the auction. If bidders are ex ante symmetric and valuations are private and independent, the bidders will self-target and bid their true valuations in equilibrium. This implies that the auction with target bids is outcome and revenue equivalent to the collusion-free Vickrey auction.

Next we study collusion in a model with bribes based on Schummer (2000). Schummer studies allocation mechanisms that are robust to manipulations by two bidders where one of them rewards the other for misrepresenting his type, making both players are better off. We extend the model in two ways. First, we allow for bribing more than one bidder, that is, we allow for coalitions that include two players or more. (The coalition structure is determined by a partition of the player set, such that bidders only observe the composition of coalitions they belong to.) Second, we allow the bribes to be outcome-contingent, and the auction strategies to be chosen in an incentive-compatible way. We model the cartel's side contract as one that specifies the bidding manipulation where one member is designated to win, and the bribes that other members receive if the manipulation succeeds.

We say that a side contract is collusion incentive compatible if two conditions are satisfied. First, no cartel member can profitably deviate from the bidding manipulation and second, the side contract Pareto-improves the cartel members' welfare. We show that, in the auction with target bids, the set of side contracts satisfying collusion incentive compatibility is empty. The proof shows that there always exists a deviation, where a cartel member targets the cartel-designated winner and obtains the exact amount of surplus that his withdrawal from competition would create. The total gains from deviation are at least as large as the surplus from collusion. Thus all surplus that the winner could obtain and redistribute in side-transfers is insufficient to overcome the cartel members' incentives to deviate.

An important assumption here is that cartels do not control their members' actions during the auction or reallocate the object after sale.[3] Cartel power amounts to enforcing rewards conditional on the success of collusion. Collusion where the cartels do not control their members' actions has been documented in a large body of literature on public procurement auctions: Hendricks and Porter, 1989; Porter and Zona, 1993; McMillan, 1991; Levenstein, Suslow, 2006; Pesendorfer, 2000.[4] In this sense, we complement the literature on collusion-robust auction design (Che and Kim, 2009; Pavlov, 2008) that stems from Laffont and Martimort (1997, 2000) who assume full cartel enforcement.[5][6] We show that if the cartels cannot enforce their members' actions, the seller can achieve the revenue of the collusion-free Vickrey auction. Furthermore, he can do so without the knowledge of colluding bidder identities or the distribution of valuations.

The auction with target bids preserves the detail-free property of its prototype Vickrey auction. It thus contributes to the growing body of literature on

[3]For instance, in public procurement auctions the reallocation is ruled out by the terms of contract.

[4]Theoretical studies of non-enforcing cartels include: Robinson, 1985; Graham and Marshall, 1987; von Ungern-Sternberg, 1988; Esö, Schummer, 2004; Marshall and Marx, 2007, 2009

[5]Che and Kim (2009) and Pavlov (2008), independently, obtain robustness to collusion modeled as in Laffont and Martimort (1997, 2000), where the cartels are able fully to enforce their members' actions. Assuming that the bidder identities are known, the auctioneer refuses to sell to any collusive bidder with some positive probability. This serves to tighten the participation constraints of the side mechanism and obtain collusion robustness.

[6]An alternative game-theoretic approach models collusion as part of a cooperative or semi-cooperative game; see Biran and Forges (2011) and references therein.

detail-free auctions,[7] and is, to the best of my knowledge, the first among those to address the problem of bidder collusion. *Detail-free* here means that neither the allocation nor the price depend on the prior distribution of bidders' willingness to pay for the object. Furthermore, the auction with target bids does not require the knowledge of colluding bidder identities, because it provokes a conflict internally within the cartel. Detail-free mechanisms, such as the auction with target bids, alleviate the "garbage in - garbage out"[8] problem pervading one-shot interactions.

2 Motivating Example

Consider a second-price auction of a single indivisible good, where the bidders can communicate before the auction. At the outset, each bidder only knows his valuation, i.e. how much money he is willing to pay for the good. Suppose that two bidders agree that one of them will bid truthfully aiming to win the auction, while the other will place a very low bid. To fix ideas, let bidder l be the designated leader, and let i be the low bidder with valuation $v_i < v_l$. As result of rigging the bids, l can win the auction at a lower price; let $\Delta_i U_l$ denote the respective ex post increase in his surplus. $\Delta_i U_l$ is strictly positive if and only if bidder l wins while i's true valuation is second-highest, such that his withdrawal

[7] Perry and Reny (2002), Caillaud and Robert (2003), Hu, Offerman and Zou (2011); on k^{th}-price auctions: Kagel, Levin (1993), Wolfstetter (2001), and Mezzetti, Tsetlin (2009).

[8] Myerson (1981) borrows the wording from computer science literature. Earlier, Hurwitz (1972) points out that the lack of knowledge of type distributions is a major concern for mechanism design. This problem has particularly strong bite in one-shot interactions where no prior observations are available.

effectively reduces the price (see Fig.1).

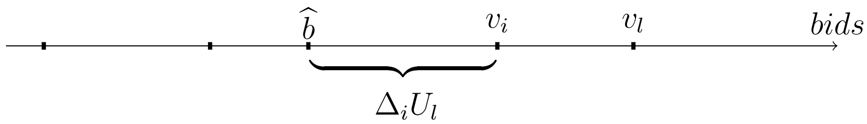


Figure 1: Collusion surplus when bidder i withdraws his bid v_i.

There are mutual benefits to collusion, or a Pareto improvement, for bidders i and l if (1) bidder i receives a positive transfer (bribe) when collusion succeeds, $t_i > 0$, *and* (2) bidder l's surplus is larger than what his payment to bidder i, $\Delta_i U_l > t_i$.

The purpose of modifying the auction is to protect the seller from price reducing manipulations of this kind. We design auction rules to preclude mutual benefits to collusion, and specifically, to introduce the possibility of profitable deviations from bid rigging scenarios. In this example, suppose bidder i could, somehow, receive $\Delta_i U_l$ if he deviated from bidding low in the auction. If such deviation is available, then bribe t_i is *either* insufficient to preclude i's deviation, *or* irrational from l's perspective as it exceeds his gain from cooperation $\Delta_i U_l$.

For bidder i, receiving $\Delta_i U_l$ is equivalent to winning the auction at price $\hat{b}$, if $\hat{b}$ is lower than his valuation v_i. We show next that this allocation can be implemented with a so-called *gap rule*. This rule favors the second-highest bidder under a condition that turns out to be equivalent to being pivotal to the cartel surplus (that is, equivalent to $\Delta_i U_l > 0$). By selling the object to bidder i whenever the condition is met, the auctioneer guarantees him a payoff that cannot be surpassed by any rational transfer promise the leader can make.

The challenge to auction design is that, in general, it is *not* in the seller's interest to reject the highest bid in favor of a lower one. We should thus ensure that the highest bid is rejected only in the case of collusion. To identify colluders we introduce 'target bidding' that allows the cartel members to use the knowledge of the leader's identity to construct profitable deviations. The targeting would trigger the allocation by gap rule in the two-stage auction. We start with considering the gap rule separately in Section 3 and then as part of the two-stage auction with target bids in Section 4.

3 Gap Rule

By the gap rule, the good is sold to the highest-valuation bidder at the second price, if the difference between the 1^{st} and the 2^{nd} bid is larger than the difference between the 2^{nd} and the 3^{rd} bid. Otherwise, the object is sold to the second highest bidder at the third price (See Fig. 2). Put differently, the winner is one of the two highest bidders, who enjoys the larger "revealed surplus" from winning at the next-highest price.

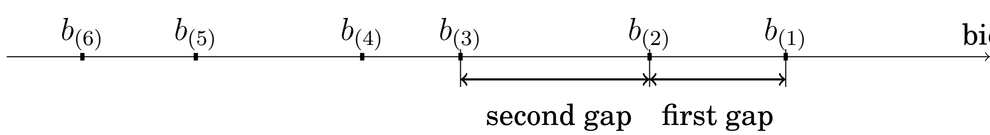


Figure 2: *Gap Rule.* $b_{(k)}$ *denotes* k^{th}*−highest bid. If the second gap is larger than the first gap, the second bidder wins at price* $b_{(3)}$.

Although inefficient, this assignment rule is interesting for the following reason. Under the gap rule, a cartel member i can obtain the equivalent of $\Delta_i U_l$, his contribution to the collusion surplus when he withdraws his bid in the Vickrey auction. To obtain the equivalent surplus he places his bid at the average of his own and the leader's value (Figure 3), to which we will refer as *midpoint* bidding. The following lemma makes the formal statement.

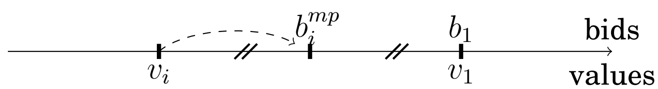
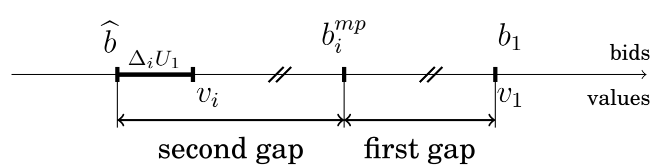


Figure 3: *Midpoint bidding* $b_i^{mp} = \frac{v_i+v_1}{2}$

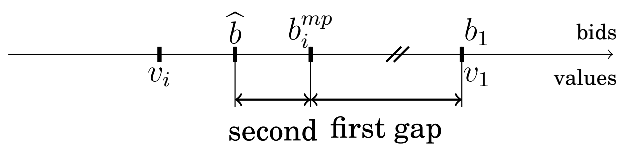
Lemma 1 Fix any valuations profile such that $v_i < v_l$ and let $\hat{b} = \max\{v_j\}_{j \in N/\{l,i\}}$, where N is the set of all bidders, $|N| \geq 3$. Under the gap rule, bidding $b_i^{mp} = \frac{v_i+v_l}{2}$ yields payoff $\Delta_i U_l$ to bidder i.

Proof Bidder i's payoff from using strategy b_i^{mp} depends on the realized value $\hat{b}$ of the maximal residual bid. First, suppose that $\hat{b}$ is less than i's valuation v_i. In this case, the first gap is less than the second. Therefore the defecting bidder i wins the object, paying the third price $\hat{b}$. Payoff $v_i - \hat{b}$ coincides with his contribution to the collusive surplus in the Vickrey auction: when v_i is *de facto* the second-highest valuation, the withdrawal of v_i reduces the Vickrey price by the difference between v_i and the next-highest valuation $\hat{b}$.



Suppose now that the highest residual bid $\hat{b}$ is greater than i's valuation v_i.

If $\hat{b} \leq b_i^{mp}$, then the first gap is larger, and if $\hat{b} > b_i^{mp}$, then i's bid is only third-highest; in either case, the payoff to bidder i is zero. Observe that whenever $\hat{b} > v_i$ i's true value v_i is *not* second-highest, and thus the withdrawal does not reduce the selling price in the Vickrey auction and therefore creates no surplus. We obtain that also in case $\hat{b} > v_i$ bidder i's payoff coincides with her contribution $\Delta_i U_l = 0$. ■



4 The Auction with Target Bids

4.1 Auction Rules

The auction with target bids is a modification of the Vickrey mechanism that uses the gap rule. In the first round each bidder places a preliminary bid β_i and chooses a target bidder; self-targeting is permitted. In the second round, each bid is revised in the target direction: If the target preliminary bid is higher, then final bid moves *half-way up* to target (Equation 1 below). If the target is lower, final bid moves *all the way down*.

If bidder i's target bidder, in turn, targets another bidder, we have to consider bidder i's entire *target set* that we define as follows.[9] Let bidder i's (first-degree) target be denoted $T(i)$. The target of bidder $T(i)$ is bidder i's second-degree target, denoted $T^2(i) = T(T(i))$. Recursively define $T^{k+1}(i) = T\left(T^k(i)\right)$. Bidder

[9]We consider bidder sets in order to rule out collusive manipulations where the cartel leader targets one of the cartel members.

i's *target set* is then defined as $\mathbb{T}(i) = \cup_{k=1}^{n} T^k(i)$, and the target preliminary bid as the minimal over the target set: $\underline{\beta_i} = \min\{\beta_j, j \in \mathbb{T}(i)\}$.

Formally, we have the following.

Round 1 Each bidder i submits a preliminary bid $\beta_i \geq 0$ and a target bidder identity $j \in N$ (self-targeting is permitted).

Round 2 For i the final bid is determined as follows:

$$b_i = \min\left\{\frac{1}{2}\left(\beta_i + \underline{\beta_i}\right); \underline{\beta_i}\right\} \tag{1}$$

If two highest bidders self-target, then the Vickrey rule is implemented. Otherwise, the gap rule is implemented.

If there is a tie at the top, and M is the set of tying bidders, the winner is picked at random from the intersection of target sets $\cap_{i \in M}\mathbb{T}(i)$ and the winner pays his bid.

4.2 Examples

As illustration to the rules, consider the following examples.

Example 1. Suppose there are three bidders: B1, B2, and B3, who bid respectively 1, 2 and 3 dollars in the first round and target their own bids. The Vickrey rule is applied and B3 wins at price 2.

Example 2. As in example 1, except that B2 targets B3. The final bids are $\left(1, 2\frac{1}{2}, 3\right)$. In this case, gap rule applies and B2 wins at price 1.

Example 3. As in example 1, except that B2 targets B1. The final bids are $(1, 1, 3)$. B3 wins at price 1.

Example 4. As in example 1, except that B3 targets B2. The final bids are $(1, 2, 2)$, there is a tie. $\mathbb{T}(B2) \cap \mathbb{T}(B3) = \{B2\}$, hence B2 wins at price 2.

Example 1 illustrates the non-cooperative equilibrium of the auction with target bids (see Section 4.4, Proposition 1). In this equilibrium, the outcome of the auction with target bids coincides with the outcome of the standard Vickrey auction. Deviation from self-targeting to targeting another bidder pays off only if one has enough information to make the correct choice of target. This is illustrated by examples 2-4. Example 2 is the only case for successful targeting, where pointing at the highest bidder (B3) results in winning the auction. Examples 3 and 4 illustrate faulty targeting. In 3, bidder B2 makes an incorrect choice of target, while in Example 4 bidder B3 is better off self-targeting than targeting any other bidder.

Consider a bidder who has only prior information about his competitors. If they are symmetric and sufficiently likely to have low valuation it is risky to target either of them at random. Compared to self-targeting, targeting another bidder can reduce the final bid, if target turns out to be low, and increase, but only partially, if the target turns out to be high. Deviation from the self-targeting equilibrium can pay off only if the target bid is the highest. Self-targeting is the most profitable strategy in the absence of information about the competitors. This is the first of our main results stated in proposition 1. It states, moreover, that truthful bidding in the auction with target bids is optimal.

In contrast, if a bidder knows that one of his competitors is about to win, he will find it profitable to target the likely winner. By doing so, the bidder has the opportunity to win the auction himself, if his final bid appears high compared to the rest of the participants. In particular, a cartel member who is not designated to win the auction will target the cartel leader because his reward for cooperation cannot be greater than his own winning payoff. If unsuccessful, the deviation will remain undetected. If successful, the defection payoff will match the price reduction that would occur in case the bidder followed the collusive manipulation. In either case, the defection strategy brings the cartel member the same payoff as his contribution to the collusive surplus $\Delta_i U_l$. The total surplus is then not sufficient to preclude deviations. Proposition 2 formalizes the impossibility of successful collusion in any cartel of size less than n. In section 5 we describe a random mechanism that precludes collusion in cartels of all sizes.

4.3 Alternative representation: Japanese auction.

The Japanese auction is an equivalent representation of the Vickrey mechanism in case of a single object.[10] The auction starts at price zero when all bidders are active. As the price goes up on a clock, the bidders quit the auction; the last remaining bidder pays the price at which the second-to-last dropped out. This version is equivalent to Vickrey auction in the sense that, in equilibrium, the bidders do not quit until the price reaches their true valuation. The winner gets

[10] See, for instance, Milgrom, 2004.

the good at the price of the second-highest value.

Similarly to the Vickrey auction, the Japanese auction is prone to collusion. The prospective winner can promise a reward to some other bidders for dropping out early, in order to reduce the competition and price. In order to restore competition using the same targeting technique, one could modify the Japanese auction as follows.

Before the start, each bidder chooses a target. If his target drops out, the targeting bidder drops out as well. If only two bidders remain and one of them targets the other, the targeted bidder drops out. Otherwise, the clock keeps going until only one bidder remains active.

Such procedure will be robust to collusion in cartels of size less than n. Since there is at least one bidder whose valuation is unknown to the cartel, the cartel leader will not use any manipulation other than self-targeting. This gives a cartel runner-up the opportunity to win the auction by targeting the cartel leader.

4.4 Equilibrium Analysis

The game is summarized by the following timeline.

t=0	Coalition structure is fixed. Bidders privately observe their valuations.
t=1	Cartel members communicate.
t=2	The seller runs the auction and announces the winner. Contingent side transfers are paid.

Players The seller's valuation for the object at sale is 0. There is a set N of bidders, $|N| \equiv n \geq 3$. The bidders' valuations are independent draws from a distribution with cumulative function $F(\cdot)$ on $\mathbb{R}_+$. $F(\cdot)$ has mass point at 0: $\int_0^c dF(x) \geq \frac{1}{2}$, $\forall c > 0$. We allow for mixed strategies and introduce, for each bidder i, a random variable ξ_i that implements mixing.[11] ξ_i is associated with a c.d.f. $\Phi(\cdot)$ with full support on $\mathbb{R}$. Similarly, the seller (player 0) uses a tie-breaking device $\xi_0 \sim \Phi(\cdot)$. The state space Ω is the product of the space of players' valuations and the space of randomization outcomes: $\Omega = \left(\{0\} \times \mathbb{R}_+^n\right) \times \mathbb{R}^{n+1}$. The typical element of Ω is denoted $\omega = (v_0, v_1, .., v_n, \xi_0, \xi_1, ..\xi_n) = (v_i, \xi_i)_{i \in N \cup \{0\}}$, where $v_0 = 0$. v_i is privately observed at $t = 0$, and ξ_i is privately observed at $t = 2$.

Coalitions The coalition structure is defined by a partition[12] $\pi_N = \{C_1, C_2, ..C_m\}$ of the player set N. A *cartel* is any coalition $C \in \pi_N$ with at least two members. π_N is exogenous and independent of state ω. The partition defines the boundaries for pre-play communication: there is no communication between the members of distinct coalitions in π_N. Each player only observes the composition of his coalition and assumes that no (further) cartels have formed. That is, for bidder $i \in C$ the perceived partition is $\left\{C, \{j\}_{j \in N/C}\right\}$. Observe that the finest partition $\pi_N = \{\{1\}, \{2\}, .. \{n\}\}$ corresponds to the standard non-cooperative environment, or *fair play*.

[11] Bidder i's mixed strategy that involves playing $s_i^1, s_i^2, ..s_i^k$ with positive probabilities $p_i^1, p_i^2, ..p_i^k$, respectively, is implemented as follows. Fix a set of thresholds $a^1 < a^2 < ... < a^{k-1}$, such that $\int_{-\infty}^{a^1} d\Phi(\xi_i) = p_i^1$, $\int_{-\infty}^{a^1} d\Phi(\xi_i) = p_i^1$, $\int_{a^1}^{a^2} d\Phi(\xi_i) = p_i^2$,... $\int_{a^{k-1}}^{+\infty} d\Phi(\xi_i) = p_i^k$, and draw ξ_i. If $\xi_i \leq a^1$ then s_i^1 is played, if $a^1 < \xi_i \leq a^2$ then s_i^2 is played, and so on. Player i's mixed strategy becomes a pure strategy - function of the random outcome ξ_i.

[12] That is, π_N is a collection of subsets of N such that $\forall i \neq j$, $C_i \cap C_j = \emptyset$; $\cup_{i=1..m} C_i = N$.

Consider cartel $C \in \pi_N$. The purpose of the cartel is to define a bidding manipulation $s_C = (s_i)_{i \in C}$ designating one of its members, w.l.o.g. $l \in C$, to win the auction and enforce transfers (bribes) $t_C = (t_i)_{i \in C}$ in the event $1[W_l]$ where the designated bidder wins. s_i is the recommended auction strategy for $i \in C$, $t_i \in \mathbb{R}_+$ is the reward to $i \in C/l$ in $1[W_l]$.

Thus a *side contract* is a tuple (s_C, t_C) of bidding manipulation and transfers characterized by the following.

(i) Bidder l is the designated winner under s_C: (i.1) $W_l\left(s_C, s_{N/C}^{nc}\right) \supseteq W_l\left(s_N^{nc}\right)$, (i.2) $W_j\left(s_C, s_{N/C}^{nc}\right) = \emptyset$, $j \in C/l$.

(ii) There is no external financing: $\sum_{i \in C} t_i = 0$, hence $t_l = -\sum_{i \in C/l} t_i$.

Information structure We define players' knowledge in terms of Aumann (1976). For a given cartel C, its side contract (s_C, t_C) and a member $i \in C$, the knowledge is given by a $\sigma-$algebra σ_i of events in Ω. The realized valuation profile v pins down the set $\sigma_i(v) = \underset{\mathcal{E} \in \sigma_i, v \in \mathcal{E}}{\cap} \mathcal{E}$ of states that are possible from i's viewpoint. In contrast, any state in $\Omega/\sigma_i(v)$ is assigned zero probability. We assume that:

(1) Every agent observes his type: $\forall i \in N$, if $\sigma_i(v) = \sigma_i(\hat{v})$, then $v_i = \hat{v}_i$.

(2) Beliefs are updated according to the Bayes rule. The expectations operator associated with the updated belief is denoted $\mathbb{E}^{|\sigma_i(v)}$.

A strategy s_i is a $\sigma_i(v)-$measurable function that maps the player's information into target bidder identities and bids: $s_i = (j, \beta_i)$. Define the *Vickrey strategy* as self-targeting and bidding one's true valuation: $s_i^V = (i, v_i)$. Let s_N^{nc} denote the non-cooperative equilibrium of the auction. If s_C is the cartel's bidding manipulation, then $\left(s_C, s_{N/C}^{nc}\right)$ the $n-$tuple of strategies such that the members of C play s_C and N/C play the non-cooperative equilibrium.

Collusion incentives Side contract (s_C, t_C) is *collusion incentive compatible* (CIC) if and only if it is individually incentive compatible (IIC) and C-Pareto improving (CPI). A contract is *individually incentive compatible* (IIC) if there is no member of C can profitably deviate from s_C:[13]

$$\forall i \in C,\, \forall \hat{s}_i, \forall v \qquad \begin{aligned} & \mathbb{E}^{|\sigma_i(v)} \left[(U_i + t_i 1 [W_l]) \left(s_C, s_{N/C}^{nc} \right) \right] \\ \geq \; & \mathbb{E}^{|\sigma_i(v)} \left[(U_i + t_i 1 [W_l]) \left(\hat{s}_i, s_{C_{-i}}, s_{N/C}^{nc} \right) \right]. \end{aligned} \tag{2}$$

A contract is *C-Pareto improving* (CPI) if the members of C are better off colluding than playing fair:

$$\forall i \in C,\, \forall v \qquad \begin{aligned} & \mathbb{E}^{|\sigma_i(v)} \left[(U_i + t_i 1 [W_i]) \left(s_C, s_{N/C}^{nc} \right) \right] \\ \geq \; & \mathbb{E}^{|\sigma_i(v)} \left[U_i \left(s_N^{nc} \right) \right], \end{aligned} \tag{3}$$

where at least one inequality is strict.

Payoffs

[13] We use the following shortcut notation: $(U_i + t_i l [W_l]) (s) \equiv U_i (s) + t_i l [W_l] (s)$.

For a singleton player i, the payoff equals:

$$(v_i - p)\, 1\, [W_i] \equiv U_i. \tag{4}$$

$W_i = W_i(s_N) \in \Omega$ indicates the event (set of states) where bidder i wins the auction given the strategy profile s_N. $1\,[W_i]$ is the indicator function that takes value 1 is the realized state is in W_i and 0 otherwise. Equation (4) says that bidder i with valuation v_i receives payoff $v_i - p$ if he wins at price p.

The auction payoff to cartel member $i \in C$ is given by the following:

$$U_i + t_i 1\, [W_l]\,, \tag{5}$$

When the designated cartel member wins, bidder $i \in C/l$ receives transfer t_i. If bidder l wins, he pays transfers to the cartel members and pays the price to the seller.

The collusion surplus due to the withdrawal of i's bid is given by the following:

$$\Delta_i U_l = \left[\min\{v_l, v_i\} - \hat{b}\right]_+ 1\,[W_l]\,. \tag{6}$$

where $\hat{b} = \max\{b_j\}_{j \in N/C}$ is the highest residual bid, and $[x]_+ = \max\{x, 0\}$. Equation (6) says that the designated bidder's extra surplus from collusion with i equals the price reduction if $v_l > v_i$, or the entire payoff of winning the good at price $\hat{b}$ if $v_l \le v_i$ (in this case bidder i would win if they played non-cooperatively).

Proposition 1 The profile of Vickrey strategies constitutes a non-cooperative

(Bayes-Nash) equilibrium in the auction with target bids:

$$s_i^{nc} = s_i^V, \forall i \in N.$$

The proofs of Propositions 1 and 2 are given in the Appendix. To prove that the Vickrey profile is a non-cooperative equilibrium, we proceed in three steps. First, we show that the Vickrey strategy is best reply in the class of self-targeting strategies. Second, in the class of other-targeting strategies, the target is chosen at random due to ex ante symmetry and the absence of further information about the competitors. The optimal preliminary bid while targeting another bidder is also truthful. This implies that, irrespective of the chosen target, the preliminary bid equals one's true valuation. Finally, we compare both strategies to show that the Vickrey strategy is best reply. Intuitively, any other-targeting strategy entails higher risk and yields lower expected payoff than self-targeting (and in particular Vickrey) if one does not have extra information about his competitors. A simple corollary follows.

Corollary The outcomes of the auction with target bids and the Vickrey auction coincide ex post.

The outcome equivalence is a consequence of truthful bidding at the first stage and self-targeting in equilibrium, which implies that the assignment is made according to the standard Vickrey rule.

Our next step is to study the side contract with respect to the collusion incentive compatibility constraints (2) and (3) in the auction with target bids. Our main result states that it is impossible to find a collusive bidding manipulation that satisfies collusion incentives for at least some valuations profile, and cartel, as long as the cartel does not include all bidders. (The extension to all-inclusive cartel is given in the next section)

Proposition 2 Fix any $\pi_N \neq \{N\}$ and $C \in \pi_N$. There exists no side-contract (s_C, t_C) that is collusion incentive compatible (CIC) in the auction with target bids.

The proof is by contradiction: we assume that a collusion incentive compatible contract (s_C, t_C) exists. We study the possible deviations from s_C to show that (IIC) implies the violation of (CPI) and vice versa. Recall from Lemma 1 that if the cartel has two members, the surplus due to the bid withdrawal in the Vickrey auction $\Delta_i U_l$ equals the exact payoff from the midpoint strategy under the gap rule. With three or more members, the unilateral deviation by bidder i brings him *at least* $\Delta_i U_l$: the gain is strictly higher if i is second-highest and the third-highest value belongs to a ring member who sticks to bidding low and bidder l places the highest bid. Then in order to preclude deviations, the leader should pay a reward that exceeds his own gain from collusion. And in the other direction, any reward that leaves the leader with a positive surplus is insufficient to preclude deviations.

The proof relies on the fact that in cartels of size less than n the leader will never target another bidder. Targeting an outsider (non-member of C) is not a best reply, and thus not part of equilibrium at the collusion stage. The same is true for targeting another cartel member, whether or not that member targets himself. In case of the all-inclusive cartel (size n), this is no longer true. In fact there is a manipulation where the cartel leader targets another member who self-targets. In order to rule out such scheme also in the case of the all-inclusive cartel, we construct the following random extension of the mechanism.

4.5 Extension: Random Mechanism

The baseline mechanism described in the beginning of the section is non-robust to a particular cartel, $C = N$. In order to counter this possibility we can adjust the mechanism in the following way. After the first round of bidding a coin is tossed, and if it flips tails, the preliminary bids become final and the Vickrey rule is applied. If however the coin flips heads, we proceed to the second round of the auction with target bids and its assignment rule.

Introducing the random clause helps the seller rule out the collusive manipulation that the grand coalition can implement in the baseline mechanism. Namely, they could agree that one of the cartel members places a preliminary bid higher than the leader while the leader targets that member. Given that the auction can end after the preliminary bidding round, the cartel will not employ such manipulation. At the same time, the incentive to deviate will still be in place to prevent collusion. The defecting cartel member will either win the auction or receive his reward since the deviation will be unobserved. This implies that the total sum of rewards that the leader can rationally promise will be insufficient to preclude deviations.

5 Conclusion

The standard second price auction has numerous desirable properties, but is highly susceptible to bidder collusion. This paper shows how collusion can be precluded with a modification that features the gap rule and how it can be used by a bidder who has additional information about his opponents. This rule induces a conflict within the cartel, as a runner-up can win the auction using a targeting strategy against the designated winner. Proposing collusion in this auction bears risk to the proposer, because the mere disclosure of his identity gives cartel members the opportunity for a profitable deviation. Since the total incentives to deviate exceed the potential collusive surplus, the transfers fall short of sustaining a collusive manipulation.

The auction with target bids has two main virtues: (i) it replicates the revenue of the collusion-free Vickrey auction without the reserve price (ii) it requires very little information on part of the seller. The auction's rules are invariant in the value distributions (F), the number of players (n) and the identities of the colluding buyers (C). Furthermore, the knowledge of F is not required to construct the optimal deviation from the collusive plot. When the value distributions cannot be identified or the exact number of players is unknown, the seller can use the auction with target bids to get second-highest valuation as the sale price.

Note that the ex ante symmetry is an essential condition for the Vickrey equilibrium of the auction with target bids. In settings where some bidders are a priori known to be stronger than others, targeting can be used in ways undesirable for the seller.

The present mechanism lends itself to further extensions. First, the gap rule and bid targeting can be extended to auction settings with divisible goods and multi-unit auctions. Second, the implicit whistle-blowing tools, such as target bidding, can be used to preclude collusion in other mechanism design problems, such as the provision of public goods. In auctions, targeting is a way to disseminate information among bidders, thus it can be applied in common-value environments.[14]

A Appendix

A.1 Proof of Proposition 1.

The proof proceeds in three steps. In steps 1 and 2 we first identify best-replies within the classes of self-targeting (step 1) and other-targeting strategies (step 2). In both classes bidding the true valuation in the first round is best reply to the residual profile of Vickrey strategies. Next, we calculate payoffs to both strategies under the equilibrium assumption. Finally, in step 3, we compare the payoffs to conclude that the optimal self-targeting is best reply to the Vickrey residual profile. Thus we obtain that the Vickrey profile is a non-cooperative equilibrium.

First, we introduce the following notation.

$s_i^{\to j} = s_i^{\to j}(v_i) = (j, v_i)$, $s_i^V = s_i^V(v_i) = (i, v_i)$, $s_{N/i}^V = s_{N/i}^V\left(v_{N/i}\right) = \left(s_l^V, ..s_{i-1}^V, s_{i+1}^V, ..s_n^V\right)$

Step 1. Self-targeting

[14] I am grateful to Rida Laraki for this observation.

If bidder i targets his own bid, then all bidders self-target and thus the Vickrey rule is applied. By Vickrey (1961) truthful bidding is weakly dominant and therefore best reply.

Payoff ex post $U_i\left(s_i^V, s_{N/i}^V\right)$

$$= [v_i - \max\{v_k,\ k \in N_{-i}\}]_+$$

Payoff in expectation $\mathbb{E}U_i\left(s^V, s_{N/i}^V\right)$

$$= \int_0^{v_i} (v_i - w)\, dF^{n-1}(w)$$

Step 2. Other-targeting

First we prove that bidding the true value v_i in the first round is best reply conditional on targeting a random bidder j. If bidder i targets another bidder, then the gap rule applies. If i's target's preliminary bid β_j is lower than the highest residual bid $\hat{b}$ then bidder i loses the auction irrespective of his own preliminary bid, since $b_i = b_j = \beta_j < \hat{b}$ such that the first gap (positive) is greater than the second gap (zero) and the highest bidder wins. If target β_j is higher than both the highest residual bid $\hat{b}$ and β_i, then i wins (at price $\hat{b}$) if and only if $b_i - \hat{b} > b_j - b_i$ or equivalently if $\beta_i > \hat{b}$. In that case the winning payoff is positive if and only if $v_i - p = v_i - \hat{b} > 0$. If target β_j is higher than the highest residual bid $\hat{b}$ but lower than β_i, then i and j's final bids tie and j wins by the tie breaking rule. The winning payoff is positive if and only if $v_i - \hat{b} > 0$, thus truthful bidding $\beta_i = v_i$ is best-reply whether or not target is above or below v_i.

Payoff ex post $U_i\left(s_i^{\to j}, s_{N/i}^V\right)$

$$= \begin{cases} v_i - \max\{v_k,\, k \in N_{-ij}\}, & if\ (\max\{v_k,\, k \in N_{-ij}\} < v_i < v_j) \\ 0, & otherwise \end{cases}$$

Payoff in expectation $\mathbb{E}U_i\left(s_i^{\to j}, s_{N/i}^V\right)$[15]

$$= \int_{v_i}^{+\infty} \int_0^{v_i} (v_i - w)\, dF^{n-2}(w)\, dF(v_j) =$$

$$= (1 - F(v_i)) \int_0^{v_i} (v_i - w)\, dF^{n-2}(w)$$

Step 3. We show that $\mathbb{E}U_i\left(s_i^V, s_{N/i}^V\right) > \mathbb{E}U_j\left(s^{\to j}, s_{N/i}^V\right)$ for all $i, v_i, j \neq i$. The difference in the expected payoffs between the two strategies for type v_i:

$$\mathbb{E}\left[U_i\left(s_i^V(v_i), s_{N/i}^V\right) - U_i\left(s^{\to j}(v_i), s_{N/i}^V\right)\right] =$$

$$= \int_0^{v_i} (v_i - w)\, dF^{n-1}(w) - (1 - F(v_i)) \int_0^{v_i} (v_i - w)\, dF^{n-2}(w) =$$

$$= \int_0^{v_i} (v_i - w)\left(\frac{n-1}{n-2}F(w) - 1 + F(v_i)\right) dF^{n-2}(w) \geq$$

$$\geq \int_0^{v_i} (v_i - w)\left(\frac{n-1}{n-2}F(w) - 1 + F(w)\right) dF^{n-2}(w) =$$

$$= \int_0^{v_i} (v_i - w)\left(\frac{2n-3}{n-2}F(w) - 1\right) dF^{n-2}(w) > 0,$$

[15]Due to type independence, for any i, the prior and interim (conditional on v_i) probability distributions over $v_{N/i}$ coincide.

since $F(v_i) \geq \frac{1}{2}$ for $v_i > 0$. We obtained that the Vickrey strategy yields a greater expected payoff, and s^V is thus a best reply to $s^V_{N/i}$, and s^V_N is a strict Bayes-Nash equilibrium. ■

A.2 Proof of Proposition 2.

The proof is by contradiction: Suppose there exist $\pi_N \neq \{N\}$, cartel $C \in \pi_N$ and a collusion incentive compatible side-contract (s_C, t_C). Denote $i*$ the identity of the colluder for whom Pareto condition (3) holds as strict inequality.

Step 1. First we consider the IIC condition (2) that states that for any $i \in C_{-l}$ there is no profitable deviation from s_C:

$$\begin{aligned} \forall i \in C, \forall \hat{s}_i, \forall v \qquad & \mathbb{E}^{|\sigma_i(v)}\left[\left(U_i + t_i 1\left[W_l\right]\right)\left(\hat{s}_i, s_{C_{-i}}, s^{nc}_{N/C}\right)\right] \\ \leq \quad & \mathbb{E}^{|\sigma_i(v)}\left[\left(U_i + t_i 1\left[W_l\right]\right)\left(s_C, s^{nc}_{N/C}\right)\right]. \end{aligned} \tag{7}$$

Consider the game restricted to player set C. In the restricted game the payoffs U_i^V are defined as in the original game, where the members of N/C play the non-cooperative equilibrium (Vickrey) strategies.[16] We write, for $i \in C$:

$$U_i^V(s_C) \equiv U_i\left(s_C, s^V_{N/C}\right).$$

[16]By Proposition 1.

s_C denotes a strategy profile of C, and $s^V_{N/C}$ is the residual profile of Vickrey strategies. Similarly, we define $W^V_l(s_C) \equiv W_l\left(s_C, s^V_{N/C}\right)$ as the event that the designated bidder l wins given that the bidders in N/C play Vickrey. In Lemma 2 (see A.3) we characterize the bidding manipulations s_C, and show that the designated bidder plays the Vickrey strategy. Given that i is not the designated winner, his auction surplus $U^V_i(s_C)$ is zero. Therefore (7) can be rewritten as follows:

$$\begin{aligned} \mathbb{E}^{|\sigma_i(v)}\left[U^V_i\left(\hat{s}_i, s_{C/i}\right)\right] &+ t_i \Pr{}^{|\sigma_i(v)}\left[W^V_l\left(\hat{s}_i, s_{C/i}\right)\right] \\ &\leq t_i \Pr{}^{|\sigma_i(v)}\left[W^V_l(s_C)\right], \end{aligned} \tag{8}$$

for all i, v, $\hat{s}_i$. In particular, Equation 8 must hold for $\hat{s}_i = s^{\to l}_i$:

$$\begin{aligned} \mathbb{E}^{|\sigma_i(v)}\left[U^V_i\left(s^{\to l}_i, s_{C/i}\right)\right] &+ t_i \Pr{}^{|\sigma_i(v)}\left[W^V_l\left(s^{\to l}_i, s_{C/i}\right)\right] \\ &\leq t_i \Pr{}^{|\sigma_i(v)}\left[W^V_l(s_C)\right], \end{aligned} \tag{9}$$

for all $i \in C_{-l}$, v. Since $\Pr[\cdot] \in [0,1]$, this implies:

$$\mathbb{E}^{|\sigma_i(v)}\left[U^V_i\left(s^{\to l}_i, s_{C/i}\right)\right] \leq t_i \Pr{}^{|\sigma_i(v)}\left[W^V_l(s_C)\right], \tag{10}$$

where the inequality is strict for $i*$. Next, we integrate over v to obtain the following inequality w.r.t. the prior distribution:

$$\mathbb{E}\left[U_i^V\left(s_i^{\to l}, s_{C/i}\right)\right] \leq t_i \Pr\left[W_l^V\left(s_C\right)\right], \tag{11}$$

where the inequality is strict for $i*$. Sum across $i \in C/l$ to obtain:

$$\sum_{i \in C_{-l}} \mathbb{E}\left[U_i^V\left(s_i^{\to l}, s_{C/i}\right)\right] \leq \Pr\left[W_l^V\left(s_C\right)\right] \sum_{i \in C_{-l}} t_i. \tag{12}$$

Condition *(12)* holds as a strict inequality if $i* \in C/l$.

Step 2. By Lemma 1 deviation strategy $s_i^{\to l}$ brings payoff equal to bidder i's contribution to the collusive surplus $\Delta_i U_l^V(s)$. In case the deviator's valuation exceeds the designated bidder's value, strategy $s_i^{\to l}$ yields payoff strictly greater than $\Delta_i U_l^V(s)$. Therefore:

$$U_i^V\left(s_i^{\to l}, s_{C/i}\right) \geq \Delta_i U_l^V(s), \tag{13}$$

for all $i \in C_{-l}$, all $v \in \mathbb{R}_+^n$. Summing the inequalities (13) across ring members $i \in C_{-l}$ we obtain:

$$\sum_{i \in C_{-l}} U_i^V\left(s_i^{\to l}, s_{C/i}\right) \geq \sum_{i \in C_{-l}} \Delta_i U_l^V\left(s_C\right). \tag{14}$$

for all $v \in \mathbb{R}^n_+$. Lemma 3 (see A.4) states that the sum of individual contributions to the collusive surplus is at least as large as the total collusive surplus: $\sum_i \Delta_i U_l^V(s_C) \geq \Delta_C U_l^V(s_C)$, where the total surplus is the change in leader's surplus if the members of C switch from non-cooperative to collusive play: $\Delta_C U_l^V(s_C) = U_l^V(s_C) - U_l^V(s^V)$. Lemma 3 and Equation 14 imply that:

$$\sum_{i \in C_{-l}} U_i^V\left(s_i^{\to l}, s_{-i}\right) \geq \Delta_C U_l^V(s_C). \tag{15}$$

for all $v \in \mathbb{R}^n_+$. Taking expectations with respect to v on both sides of equation (15) we obtain:

$$\mathbb{E}\left[\sum_{i \in C_{-l}} U_i^V\left(s_i^{\to l}, s_{-i}\right)\right] \geq \mathbb{E}\left[\Delta_C U_l^V(s_C)\right]. \tag{16}$$

By the Pareto condition (3) for bidder l, $\mathbb{E}\left[\Delta_C U_l^V(s_C)\right] \geq \Pr\left[W_l^V(s_C)\right] \sum_{i \in C_{-l}} t_i$. Therefore:

$$\mathbb{E}\left[\sum_{i \in C_{-l}} U_i^V\left(s_i^{\to l}, s_{-i}\right)\right] \geq \Pr\left[W_l^V(s_C)\right] \sum_{i \in C_{-l}} t_i. \tag{17}$$

Condition (17) holds as a strict inequality if $i* = l$. We arrive at a contradiction between equations (17) and (12), hence the premise of existence is false. ■

A.3 Lemma 2.

Statement In any CIC side contract (s_C, t_C), $|C| < n$, bidder l targets himself and bids his true valuation v_i.

Proof

Suppose that bidder l target another bidder with positive probability. Four cases are possible.

Case 1. Suppose s_C is such that $T(l) \in N/C$ with a positive probability. Then condition (i.1) (see page 17) is violated, because for any β_l there exist states in which bidder l wins under non-cooperative play, but loses under the collusive manipulation s_C.

Case 2.1. Suppose s_C is such that $T(l) = j \in C$ with a positive probability and $T(j) \in N/C$ with a positive probability. Due to minimization over the target set this is outcome equivalent to $T(l) \in N/C$, i.e. to Case 1.

Case 2.2. Suppose s_C is such that $T(l) = j \in C$ with a positive probability and $T(j) = j$ with a positive probability. If $\beta_j \leq \beta_l$ then condition (i.1) is violated. If $\beta_j > \beta_l$, condition (i.2) is violated.

Case 2.3. Suppose s_C is such that $T(l) = j \in C$ with a positive probability and $T(j) \in C$ with probability 1. If l's target set is a subset of C, this case is equivalent to 2.2. Otherwise, if the target set includes non-members, to case 1.

Therefore bidder l targets himself with probability 1. Since IIC condition requires that s_C is an equilibrium at the collusion stage, the leader's preliminary bid must be a best reply. From the proof of Proposition 1 (step 1), the only best reply under self-targeting is truthful bidding $\beta_l = v_l$. Thus the leader self-targets and bids truthfully in any collusion incentive compatible manipulation s_C. ∎

A.4 Lemma 3.

Statement $\sum_{i \in C_{-l}} \Delta_i U_l^V(s) \geq \Delta_C U_l^V(s)$.

Proof Let $\{i_k\}_{k=1..(|C|-1)}$ be an arbitrary permutation of the elements of C_{-l}. By definition,

$$\Delta_C U_l^V(s) = U_l^V(s) - U_l^V\left(s^V\right) \tag{18}$$

Adding and subtracting equivalent terms in the right-hand side of Equation (18), we obtain the following:

$$= \sum_{k=1..|C|-1} \left(U_l^V\left(s_{1..i_k}, s^V_{(i_k+1)..(|C|-1)}, s_l\right) - U_l^V\left(s_{1..(i_k-1)}, s^V_{i_k..(|C|-1)}, s_l\right)\right) + \tag{19}$$
$$+ \underbrace{U_l^V\left(s^V_{-l}, s_l\right) - U_l^V\left(s^V\right)}_{\leq 0,\, Eq.}$$

Observe that the last term in equation (19) is non-positive, since the Vickrey profile is equilibrium. For every k, the expression in the parenthesis contains the increase in the leader's utility when player i_k switches from the Vickrey strategy

$s_{i_k}^V$ to the collusive strategy s_{i_k} given that the residual profile is such that the players ranked 1 to $k-1$ in the permutation and the leader play the collusive strategy, while players ranked i_k+1 to $|C|-1$ play Vickrey. Observe that the increase in leader's utility is greater if the strategy switch happens when *all* other cartel members play the collusive strategy (since the collusive profile never entails bidding more than one's true value):

$$\begin{aligned} & U_l^V\left(s_{1..i_k}, s^V_{(i_k+1)..(|C|-1)}, s_l\right) - U_l^V\left(s_{1..(i_k-1)}, s^V_{i_k..(|C|-1)}, s_l\right) \leq \\ \leq \quad & U_l^V\left(s\right) - U_l^V\left(s_{-i_k}, s_{i_k}^V\right) \end{aligned} \tag{20}$$

Recall that $U_l^V\left(s\right) - U_l^V\left(s_{-i_k}, s_{i_k}^V\right) = \Delta_{i_k} U_l^V\left(s\right)$ by definition, we obtain from (19) and (20) that:

$$\Delta_C U_l^V\left(s\right) \leq \sum_{k=1..|C|-1} \Delta_{i_k} U_l^V\left(s\right) = \sum_{i\in C_{-l}} \Delta_i U_l^V\left(s\right), \tag{21}$$

completing the proof of the statement. ■

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