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Abstract

A robust feature of models of electoral competition between two opportunistic, purely office-motivated parties is that both parties become indistinguishable in equilibrium. In this short note, I show that this strong connection between the office motivation of parties and their equilibrium choice of identical platforms depends on the following two - possibly counterfactual - assumptions: 1. Issue spaces are uni-dimensional and 2. Parties are unitary actors whose preferences can be represented by expected utility functions. The main goal here is to provide an example of a two-party model in which parties offer substantially different platforms in equilibrium even though no exogenous asymmetries are assumed. In this example, some voters' preferences over the 2-dimensional issue space are assumed to exhibit non-convexities and parties evaluate their actions with respect to a set of beliefs on the electorate.

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1 Introduction

Two parties never run on the exact same platform in an electoral campaign. This stands in sharp contrast to the predictions of models of two-party electoral competition in the tradition of Downs (1957) and Hotelling (1929). One of the equilibrium predictions of these models is that both parties announce the same platform. This equilibrium convergence is often seen

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as a result of the assumptions that voters have single peaked preferences and that parties are uniquely office-motivated, can freely choose and fully commit to their platforms.

In this article I provide an example of political competition a la Downs-Hotelling in which equilibrium platforms may diverge even if the aforementioned tenets hold true. The following two assumptions of the Downs-Hotelling model are relaxed here: Issue spaces are uni-dimensional and parties act as unitary actors with a single belief on the voters' preferences. Let me discuss each of these assumptions in turn:

There is wide agreement that the assumption that political issue spaces are uni-dimensional is counterfactual. The main merit of this assumption is that it simplifies the analysis of political competition. However, the predictions of the Downs-Hotelling model change dramatically when one replaces the assumption of a single dimensional issue space with that of a multi-dimensional one: in the first case, an equilibrium always exists; in that equilibrium, both parties announce the median voter's preferred platform. In the alternative case, an equilibrium only exists under very stringent assumptions on the distribution of voter preferences. So the simplification associated with the assumption of a uni-dimensional issue space can hardly be viewed as benign.

Just as in the classic Downs-Hotelling model, I assume that the goal of the parties is to maximize their respective vote shares. However, the parties in my model do not hold a single belief on the distribution of voter preferences. Instead, they are assumed to hold multiple beliefs. They calculate their vote share according to each one of these beliefs and only change their platform if such a change looks favorable according to each of these calculations.

The first story to justify these assumptions on the preferences and behavior of parties goes as follows: say there is a group of party leaders. These leaders all share the goal of winning as many votes as possible, but do have different beliefs on the electorate. In addition, a party's platform will only be changed if the entire leadership unanimously agrees on such a change. A second story motivates the same model of decision making as one of subjective uncertainty. It holds that parties might not know enough about all voters' preferences over all policies to assign *objective* probabilities to their odds of obtaining votes for *all* possible constellations of platforms. The decision-theoretic model of the present paper arises if we assume that in the face of such subjective uncertainty parties do not act as expected utility maximizers, but follow in Bewley's model of Knightian uncertainty.

Let me now explain why party platforms might diverge in the equilibrium of a modified Downs-Hotelling model. Consider a profile of two different platforms $\mathbf{x}$ and $\mathbf{y}$ and a voter V who prefers platform $\mathbf{x}$ to platform $\mathbf{y}$. Would V also prefer the intermediate platform $\lambda\mathbf{x} + (1 - \lambda\mathbf{y})$ (for $\lambda \in (0, 1)$) to the platform $\mathbf{y}$? If the issue space is uni-dimensional, the assumption that V's preferences are single-peaked implies their convexity. In that case, the V must exhibit the named preference. Consequently, either party can only gain by moving its platform closer to that of the other party if the issue space is uni-dimensional. However, if the issue space is multi-dimensional, single-peaked preferences need not be convex. So, with a multidimensional issue space, V might well prefer platform $\mathbf{y}$ to some intermediate platform $\lambda\mathbf{x} + (1 - \lambda\mathbf{y})$. In that case, a party might lose votes by moving its platform closer to that of the opponent. Moving closer is not necessarily a better response in the modified Downs-Hotelling model.

This argument of non-convexities of single-peaked preferences over multidimensional issue spaces does not suffice to obtain divergent equilibria. To see this, observe that, in each equilibrium of a game of electoral competition a la Downs-Hotelling with a known distribution of voter preferences, the two parties must obtain exactly half the vote share. If not, the party with the lower vote share has an incentive to adopt the platform of its opponent.

The multiple-beliefs model breaks this feature of political competition a la Downs-Hotelling: in that model, a change of platform is only preferred if it increases the party's vote share according to *all* its beliefs on the electorate. If a platform profile is associated with a vote share below one half for party one according to *some* belief, the adoption of the other parties' platform increases party one's vote share according to *that* belief. This alone does not imply that party one prefers to run on the opponent's platform. For that, the adoption of the opponent's platform would have to increase party one's vote share according to *all* of party one's beliefs on the distribution of voter preferences. If party one holds some belief according to which its vote share is higher than one half for the current profile of platforms, party one does not prefer to offer the platform of the opponent. In a nutshell, the standard model differs from the multiple-beliefs model in the number of tests a deviation has to pass to be considered preferred.

Since violations of convexity play a major role in my arguments, let me argue now that the assumption of convexity does not appear to be intuitive for preferences over a multidimensional issue space. Take the example of a mayor who could try to lure the European soccer cup and/or the Olympic games to his city. Consider the following three profiles of money spent towards the candidacy for the two events: $(10, 0), (0, 10)$, and $(5, 5)$. Consider an inhabitant of the city under question who is indifferent between the first two platforms. Convexity would demand that this citizen would prefer the third platform to the other two. While this might hold for some citizens, I surmise that the opposite preference is equally plausible. To see this, let's assume that the citizen under consideration would like to have a big event in his home town and believes that any campaign for such an event must spend at least 7 to have a positive chance to succeed. Clearly this citizen should strictly prefer the "extreme" platforms $(10, 0)$ and $(0, 10)$ to the intermediate platform $(5, 5)$.

This is an example with a natural unit of measurement: euros. However, in many other typical problems in political economy, the use of cardinal rankings is questionable. It might, for example, be possible to order foreign policy on a scale from dovish to hawkish and abortion politics on a scale from liberal to conservative. In either case, such statements as "twice as hawkish", "half as liberal" seem to be devoid of any meaning. But the definition of convex preferences presupposes that these statements are filled with meaning. If we remain agnostic on the measurement units of the axes of the political issue space, we should only use conditions that are robust to a rescaling of the axes. Without cardinal measures of politics, we can, in particular, not impose that the voters' preferences should be convex.

Of course, this is not the first model of electoral competition that explains the divergence of party platforms. However, as far as I am aware, the emergence of different platforms is usually owed to the assumption of some exogenous ideological allegiance of parties and/or politicians. This definitely holds for the models by Wittman (1973), Osborne and Slivinski (1996), Besely and Coate (1997), and Roemer (1999). Another set of models goes a different route by prefacing Downsian competition among two parties by a stage in which these two competitors are selected (or threatened by the entry of a third). Palfrey (1984) and more recently Brusco, Dziubinski, and Roy (2010) fall into that group.

2 Electoral Competition

I model political competition as a two-stage game played by two different types of actors, two political parties, and a large set of voters. First, the two parties simultaneously choose their platforms within some (non-empty) convex issue space $X \subset \mathbb{R}^n$, $n \geq 1$. I denote generic elements of X by $\mathbf{x} = (x_1, x_2, \cdots, x_n)$. Profiles of platforms are denoted by $\langle \mathbf{x}, \mathbf{y} \rangle$. Then, the voters, whose preferences are defined over that same issue space X, cast their votes. I assume throughout that parties credibly commit to their platforms, and that voters only care about platforms. In particular, no voter has any ideological attachment or bias towards either party. So there are no a priori differences between the two parties. Any differences between their equilibrium positions arise endogenously.

2.1 Voters

I assume that all voter preferences are single-peaked, in the sense that each voter has some most preferred policy in the issue space and that his utility decreases as platforms move further and further away from this ideal point. Formally, some preferences $\succsim$ are considered **single-peaked** if there exists an **ideal point a** $\in X$, such that for any two platforms $\mathbf{x}$ and $\mathbf{y}$ in X with $(a_i - x_i)(a_i - y_i) \geq 0$, $\mid a_i - x_i \mid \leq \mid a_i - y_i \mid$ for all issues i and $\mid a_{i'} - x_{i'} \mid < \mid a_{i'} - y_{i'} \mid$ for at least one i' it holds that $\mathbf{x} \succ \mathbf{y}$. Barbera, Gul, and Stachetti (1993) propose the same

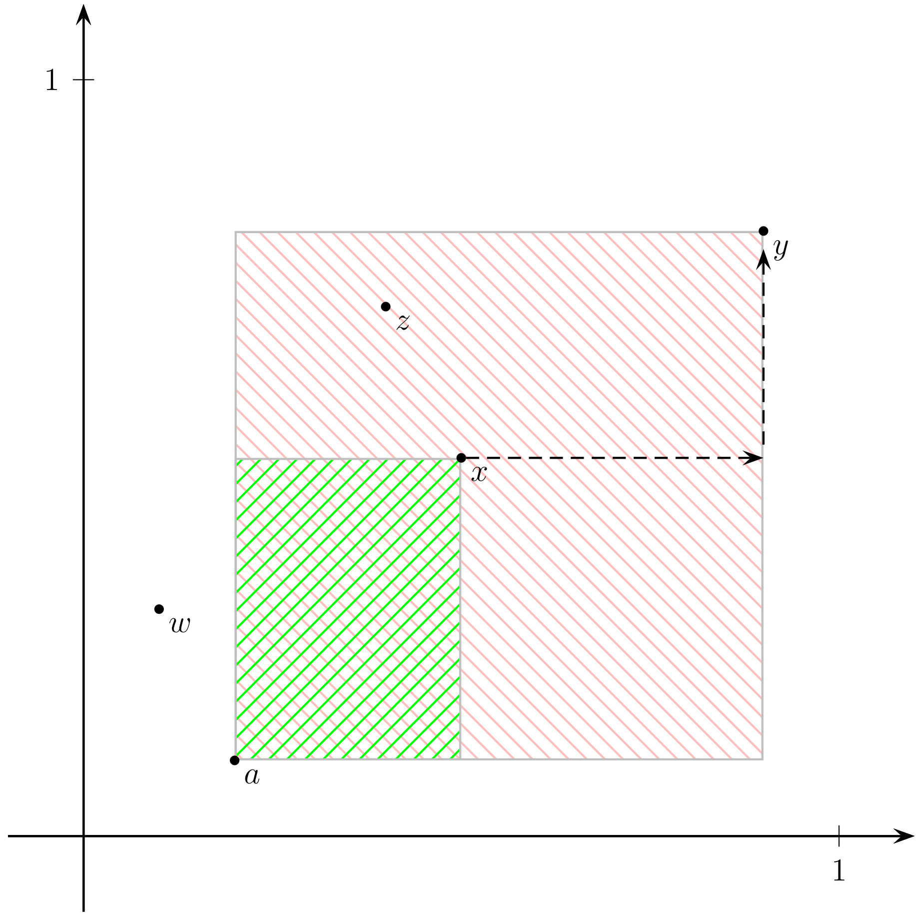

Figure 1: The Condition of single-peakedness

definition of single-peakedness.[1]

To understand this definition, consider the platforms **a**, **x**, **y**, **w**, and **z** in the two-dimensional issue space illustrated in Figure 1. The criterion of single-peakedness imposes that a voter with the ideal point **a** prefers platforms **x** and **z** to platform **y**. These three platforms lie northeast of **a**, so $(a_i - x_i)(a_i - y_i) \geq 0$, $(a_i - x_i)(a_i - z_i) \geq 0$ and $(a_i - z_i)(a_i - y_i) \geq 0$ for $i = 1, 2$, the necessary condition for single-peakedness to rank these platforms holds. Moreover, the two arrows indicate that **y** is more distant from **a** than **x** with respect to both axes. The same holds for the relation between **z** and **y**. Conversely, **z** and **x** are not ranked by the criterion, as $\mid a_1 - x_1 \mid > \mid a_1 - z_1 \mid$, whereas $\mid a_2 - x_2 \mid < \mid a_2 - z_2 \mid$. Finally, the criterion does not rank **w** with respect to any of the other three platforms, since $(a_1 - w_1)(a_1 - t_1) < 0$ for $t = x, y, z$.

Note that the platforms **x** and **z** both lie in the rectangle with **a** and **y** as its south-west

[1]If we impose that the issue space is uni-dimensional ($n = 1$), then the present definition reduces to the standard definition of single peakedness. Any voter's ideal point is unique and no voter prefers any platform to his ideal point.

and north-east corners, hatched in pink in Figure 1. In fact, the criterion of single-peakedness ranks two platforms $\mathbf{x}$ and $\mathbf{y}$ if and only if one of the two lies in the rectangle spanned by the other and the agent's ideal point $\mathbf{a}$. To formalize this statement, I define the **area in between two platforms** $\mathbf{x}$ and $\mathbf{y}$ as

$$\lfloor \mathbf{x}, \mathbf{y} \rceil := \{\mathbf{z} \in X \setminus \{\mathbf{x}, \mathbf{y}\} \mid \min(x_i, y_i) \leq z_i \leq \max(x_i, y_i)\}$$

and say that some platform $\mathbf{z}$ **lies in between** the two platforms $\mathbf{x}$ and $\mathbf{y}$ if $\mathbf{z} \in \lfloor \mathbf{x}, \mathbf{y} \rceil$. Now the requirement of single-peakedness can be reformulated as follows: preferences are single-peaked if there exists an (ideal point) $\mathbf{a}$ such that $\mathbf{x} \in \lfloor \mathbf{a}, \mathbf{y} \rceil$ implies that $\mathbf{x} \succ \mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in X$. To see that the criterion of single-peakedness does not rank $\mathbf{x}$ with respect to $\mathbf{z}$ or $\mathbf{w}$ in the example given by Figure 1, observe that $\mathbf{w}, \mathbf{z} \notin \lfloor \mathbf{a}, \mathbf{x} \rceil$, the area hatched in green, holds as well as $\mathbf{x} \notin \lfloor \mathbf{a}, \mathbf{w} \rceil$ and $\mathbf{x} \notin \lfloor \mathbf{a}, \mathbf{z} \rceil$. Similar arguments can be made to show that single peakendess does not impose any ranking between $\mathbf{w}$ and the other platforms singled out in this figure.

Figure 2.1 takes up the same platforms $\mathbf{x}$, $\mathbf{y}$, $\mathbf{w}$, and $\mathbf{z}$ and provides two examples of indifference curves corresponding to different single-peaked preferences with ideal point $\mathbf{a}$. As a first example, consider Euclidean preferences in which each voter measures the disutiliy of a platform $\mathbf{x}$ as the Euclidean distance between his ideal point $\mathbf{a} \in X$ and that platform $\mathbf{x}$. Such preferences over a two-dimensional issue space X are represented through the function $v^a : X \to \mathbb{R}$ with $v^a(\mathbf{x}) = -(x_1 - a_1)^2 - (x_2 - a_2)^2$, where $\mathbf{a}$ is the ideal point of the preferences. The black circle in Figure 2 represents an indifference curve of an agent with ideal point $\mathbf{a}$ and Euclidean preferences. Such an agent prefers $\mathbf{w}$ to $\mathbf{x}$, which he, in turn, prefers to $\mathbf{z}$. Next, consider the example of preferences over a two-dimensional issue space X that can be represented by a function $u^a : X \to \mathbb{R}$ with $u^a(\mathbf{x}) = -\sqrt{|x_1 - a_1|} - \sqrt{|x_2 - a_2|}$. The green line represents an indifference curve corresponding to this kind of preferences. Note that these preferences are not convex, the agent is indifferent between platforms $\mathbf{s}$ and $\mathbf{t}$ and strictly prefers each of these to the intermediate platform $\frac{1}{2}\mathbf{t} + \frac{1}{2}\mathbf{s}$. To see that the convexity of preferences depends on the scaling of the axes, consider a voter whose preferences are represented by $u^{(0,0)}$. Rescale both axes with the strictly monotonic function $t \mapsto t^4$. In the rescaled issue space, the same preferences are represented by $v^{(0,0)}$.

In terms of voter behavior, I assume that any voter who strictly prefers the platform of one party to that of the other votes for his strictly preferred platform. Any indifferent voter votes for each party with equal probability.

2.2 Electorates

Formally an electorate is identified with some distribution ψ on the set of single-peaked preference $\succsim$ over X. The probability that any voter that is randomly drawn from an

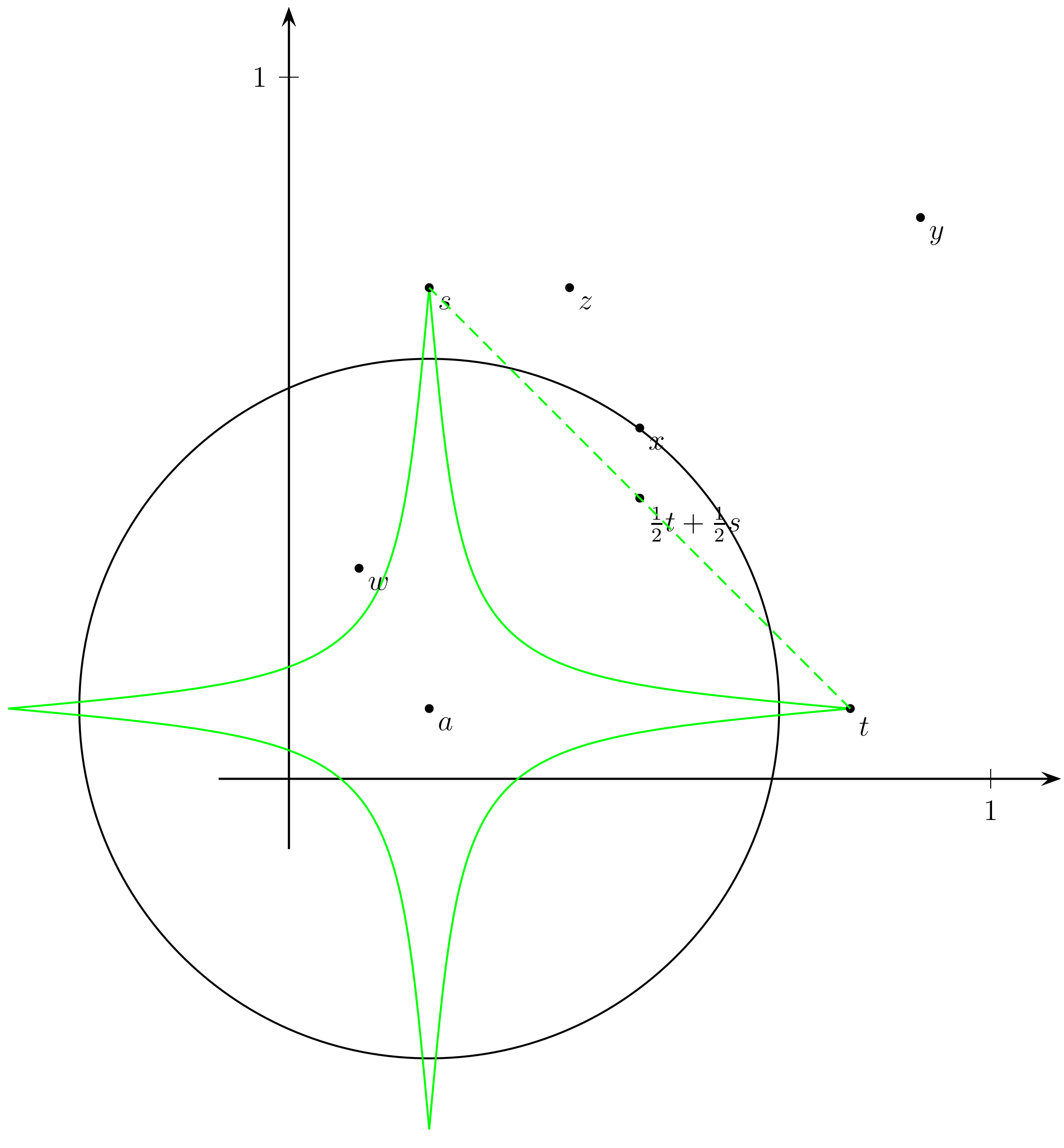

Figure 2: Examples of Single-peaked Preferences

electorate ψ satisfies some property Θ is denoted by $\psi(\{\succsim \;:\; \succsim \text{ satisfies } \Theta\})$. To avoid formal complications, I only consider distributions ψ with finite support. Bearing the assumption over voter behavior in mind, the expected vote share of party one at platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ and electorate ψ can be calculated as: $\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle) :=\psi(\{\succsim : \mathbf{x} \succ \mathbf{y}\}) + \frac{1}{2}\psi(\{\succsim : \mathbf{x} \sim \mathbf{y}\})$. Party two's vote share is $1 - \pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle) = \pi_\psi(\langle \mathbf{y}, \mathbf{x} \rangle)$

Observe that π_ψ was defined with respect to a known distribution of voter preferences. In general this distribution will not be known, decision makers will have to form some expectation on the distribution of voter preferences. In this context, it is important to note that the expected vote share $E(\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle))$ for some distribution over the set of possible electorates equals the vote share according to the expected electorate $\pi_{E(\psi)}(\langle \mathbf{x}, \mathbf{y} \rangle)$, where I assume that the expectation is taken over some distribution p over distributions of voter preferences with finite support: $p := (p_1 : \psi^1; p_2 : \psi^2; \cdots ; p_m : \psi^m)$. Formally $E(\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle)) = \sum_{j=1}^m p_j \pi_{\psi^j}(\langle \mathbf{x}, \mathbf{y} \rangle)) = \sum_{j=1}^m p_j(\psi^j(\{\succsim : \mathbf{x} \succ \mathbf{y}\}) + \frac{1}{2}\psi^j(\{\succsim : \mathbf{x} \sim \mathbf{y}\})) = \sum_{j=1}^m p_j \psi^j(\{\succsim : \mathbf{x} \succ \mathbf{y}\}) + \frac{1}{2}\sum_{j=1}^m p_j \psi^j(\{\succsim : \mathbf{x} \sim \mathbf{y}\})) = E(\psi)(\{\succsim : \mathbf{x} \succ \mathbf{y}\}) + \frac{1}{2}E(\psi)(\{\succsim : \mathbf{x} \sim \mathbf{y}\}))$ where $E(\psi)$ is a distribution of voter ideal points that attributes probability $\sum_{j=1}^m p_j \psi^j(\succsim)$ to any preferences $\succsim$. Given that the support of p and of each ψ^j is finite, the support of $E(\psi)$ is finite. Moreover, since $\psi^j(\succsim)$ is only positive for single-peaked preferences, $E(\psi)(\succsim) = 0$ for all $\succsim$ that are not single-peaked.[2]

2.3 Parties

I assume that the goal of each party is to maximize its vote share. A party's strategy variable is its platform, and hence the issue space X is its strategy space. If the electorate were known to be ψ, the objective of party one would simply be to maximize $\pi_\psi(\langle x, y \rangle)$. By the arguments in the preceding section, the same formula would apply to the case that parties are expected vote-share maximizers; in that case, ψ would have to be interpreted as the expected electorate.

However, the parties in the present model are assumed to follow a model of decision making that deviates from expected utility maximization. They are assumed to take a set of different distributions Ψ into account when deciding where to locate their platforms in the issue space. I assume that parties one and two have incomplete preferences represented by

$$\langle \mathbf{x}, \mathbf{y} \rangle \mapsto (\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle))_{\psi \in \Psi} \quad \text{and} \quad \langle \mathbf{x}, \mathbf{y} \rangle \mapsto (1 - \pi_\psi(\langle \mathbf{y}, \mathbf{x} \rangle))_{\psi \in \Psi} = (\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle))_{\psi \in \Psi} \tag{1}$$

for some set of priors Ψ, where $(\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle))_{\psi \in \Psi}$ denotes the vector of vote shares $\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle)$ for all $\psi \in \Psi$. Party one prefers some platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ to another platform profile

[2] In Bade (2010), I provide the technically more complex proof that the equality $E(\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle)) = \pi_{E(\psi)}(\langle \mathbf{x}, \mathbf{y} \rangle)$ holds for non-atomic distributions.

$\langle \mathbf{x'}, \mathbf{y'} \rangle$ if and only if party one's vote share under $\langle \mathbf{x}, \mathbf{y} \rangle$ is not lower than its vote share under $\langle \mathbf{x'}, \mathbf{y'} \rangle$ for any electorate ψ in Ψ. This preference is strict if, in addition, there exists some electorate $\psi' \in \Psi$ such that $\pi_{\psi'}(\langle \mathbf{x}, \mathbf{y} \rangle) > \pi_{\psi'}(\langle \mathbf{x}', \mathbf{y}' \rangle)$. Party one is indifferent between the two profiles if and only if $\pi_{\psi}(\langle \mathbf{x}, \mathbf{y} \rangle) = \pi_{\psi}(\langle \mathbf{x}', \mathbf{y}' \rangle)$ holds for all $\psi \in \Psi$. In all other cases, party one cannot rank the two platform profiles. The standard case of vote-share-maximizing parties obtains upon assuming that Ψ is a singleton ($\Psi = \{\psi\}$).

Given that preferences are incomplete, party behavior is not (fully) determined by preferences. I assume that parties only change their platform if this leads to a strictly preferred outcome. If party one cannot rank profile $\langle \mathbf{x}, \mathbf{y} \rangle$ and $\langle \mathbf{x}', \mathbf{y} \rangle$, then that party would not deviate from $\mathbf{x}$ to $\mathbf{x}'$ given that party two's platform is $\mathbf{y}$. The larger the set of party beliefs Ψ, the more incomplete the preferences and the more important is the behavioral assumption.

2.4 Stories

Let me tell three different stories to justify such a model of party preferences and behavior: According to the first story, parties are subjectively uncertain about the electorate, where their preferences can be represented following Bewley's Model of Knightian uncertainty (Bewley, 2002). In justification of the assumption of subjective uncertainty, observe that real parties do indeed face choices with uncertain consequences. Electoral outcomes are generally hard to predict as elections are usually held in ever-changing environments involving new issues, turnover of party elites, and an evolving composition of the electorate. Taking the 2008 US presidential election as an example, observe that neither the experience with past elections nor any amount of polling would have been sufficient to determine objective winning probabilities for Barack Obama and John McCain *for all possible combinations of platforms*, given that they where dealing with some new issues (how to respond to bankruptcy of Lehman Brothers?), a new party elite (Barack Obama was the first African American to be nominated for President by a major political party), and a large share of first-time voters.

It is well documented that individuals violate expected utility maximization in the face of such subjective uncertainty. A large range of experimental studies demonstrates a bias against uncertainty (cf. Camerer and Weber (1992)). These experimental results have spurred a large literature on the representation of uncertainty-averse preferences.

Bewley's (2002) multiple prior model stands out as one of the path-breaking representations in that literature.[3] According to his model, agents calculate expected utilities for a set of priors Ψ on the subjectively uncertain environment. An agent prefers some action f over some other action g if f yields a higher expected utility according to *all* priors $\psi \in \Psi$.

[3]Other approaches towards modeling uncertainly aversion have been proposed by Gilboa and Schmeidler (1989), Schmeidler (1989), Klibnoff, Marinacci, and Mukerji (2005) and Cerreira-Vioglio, Maccheroni, Marinacci, and Montrucchio (2009).

The resulting preferences are incomplete: if for both actions f and g there exist priors ψ^f and ψ^g in Ψ such that f is associated with strictly higher expected utility than g according to the prior ψ^f and the inverse holds true when according to ψ^g, the agent cannot rank f and g. To determine behavior in the face of incompleteness, Bewley's framework contains an *inertia assumption* which corresponds precisely to the assumption made here.[4] In terms of individual behavior, the inertia assumption can be justified following Kahneman and Tversky (1982), who argue that people tend to regret losses resulting from actions more than losses resulting from inaction. Analogously to their stock market example, we could expect that party members would feel more upset about losing an election as a result of switching their platform than losing an election by keeping their platform.

For the second story on party preferences, observe that parties generally consist of many members or factions with different interests or beliefs. To decide on a platform, the various party leaders have to come to some form of compromise or agreement. We could assume that all these leaders share a common goal: they all hope to maximize their party's vote share. However, the leaders might have different expectations on the preferences of the electorate. The preferences posited here obtain if some platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ is preferred to another profile $\langle \mathbf{x}', \mathbf{y}' \rangle$ if it yields a higher vote share to party one according to the beliefs of all party leaders. John Roemer (1999, 2001) and Gilat Levy (2004) tell similar stories of parties as non-unitary actors. Both posit that a party prefers platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ over another profile $\langle \mathbf{x}', \mathbf{y}' \rangle$ if it is preferred from the vantage point of every faction or member of that party. The main difference between their stories and the story told here is that different leaders are set apart by different beliefs in the present model, whereas they are set apart by different goals in their models.

In terms of this story, the inertia assumption can be derived from a party decision process that requires unanimity for deviations. To change any of party's platform, the leaders all have to agree. If no compromise can be reached, the status quo always remains as a fallback option. John Roemer (1999, 2001) and Gilat Levy (2004) rely on precisely this argument to introduce a status quo bias into their respective models of platform positioning. In both cases, parties consist of many factions or members and they move from one platform to another only if this move is considered worthwhile by all the factions or party members. In other words: every party faction or member has veto power against any change of a party's position.

The same type of preferences could be derived from the following set of entirely different

[4]There is one difference between the model proposed here and Bewley's: I did not specify any particular sets of beliefs Ψ. In contrast, the representation by Bewley demands closed and convex sets of beliefs. The centerpiece of the article consists in an example with a finite set of beliefs Ψ. However, this example can easily be amended to fit Bewley's framework, by replacing the set of beliefs in that example with the convex hull of this set. The equilibrium set of the example does not change.

assumptions. Remaining fully within the framework of standard Downs-Hotelling models, we could assume that parties know everything about the preferences of voters. Deviating from that standard setup, we would have to posit that the preferences of voters are incomplete. So we would have to assume that voters would not be able to rank all platforms. Finally, we could assume that a party prefers platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ to platform profile $\langle \mathbf{x}', \mathbf{y}' \rangle$ if the first platform profile yields the party a higher vote share according to all possible theories on the choices made by indecisive voters. If we assume, in addition, that parties would only change their platform in the case of such strict preference, we could obtain the same formulation of a party's preferences without ever referring to their uncertainty about the preferences of voters.

Some further justification of the inertia assumption lies in the political value of stability and applies to all three stories: the image of a party being true to itself carries a positive connotation, whereas a party that is fickle, that goes with every trend, can be seen as a turncoat, definitely a negative association. The model presented here is a static one; consequently, such dynamic considerations cannot play any role within the model. However, within a more general model that would analyze elections within a dynamic framework, long-run considerations might bias a party in favor of their status quo platform. An explicit introduction of credibility concerns could provide a foundation of inertia. The credibility of a commitment to any particular platform could, in a dynamic model, be modeled as dependent upon the continuity with which the party has been advocating that same platform.

2.5 The Game of Platform Positioning

A two-player game of electoral competition is then characterized by the triplet (n, X, Ψ), where n is the dimension of the issue space X and Ψ is the set of party beliefs. This is a normal-form game with the two parties as the players, the issue space X as the strategy space of either player, and where the payoff functions of parties one and two are defined in expression (1). I assume here that both parties subscribe to the same set of priors Ψ on the electorate. On the one hand, this keeps notation at a minimum. On the other hand, this strengthens the argument that divergence arises purely endogenously; no exogenous differences between parties are assumed. The results can easily be modified to comprise games in which parties hold different sets of priors about the electorate. Standard Downsian games are embedded in the present framework, they constitute the set of games, with unidimensional issue spaces and singleton priors: $(1, X, \{\psi\})$.

A platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ is a **(political) equilibrium** of the game (n, X, Ψ) if and only if

$$\mathbf{x} \in \operatorname*{arg\,max}_{\mathbf{z} \in X} \left(\pi_\psi(\langle \mathbf{z}, \mathbf{y} \rangle)\right)_{\psi \in \Psi} \quad \text{and} \quad y \in \operatorname*{arg\,max}_{\mathbf{z} \in X} \left(1 - \pi_\psi(\langle \mathbf{x}, \mathbf{z} \rangle)\right)_{\psi \in \Psi}.$$

Note that the platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ is not an equilibrium if there exists a platform $\mathbf{x}'$ such that $\langle \mathbf{x}', \mathbf{y} \rangle$ leads to a higher expected vote share for party one for every $\psi \in \Psi$ as $\mathbf{x} \in \arg\max_{\mathbf{z} \in X} (\pi_\psi(\langle \mathbf{z}, \mathbf{y} \rangle))$ would then be violated. The definition of equilibrium reflects the notion of inertia, insofar as *any* maximizer of $(\pi_\psi(\langle \mathbf{z}, \mathbf{y} \rangle))_{\psi \in \Psi}$ is considered a best reply to $\mathbf{y}$.[5]

2.6 Divergence

To present an example of electoral competition with a divergent equilibrium, some notion of "divergence" is needed. As a first pass, one might consider any equilibrium $\langle \mathbf{x}, \mathbf{y} \rangle$ with $\mathbf{x} \neq \mathbf{y}$ as divergent. To see that this definition is not stringent enough, recall that even classical Downsian games $(1, X, \{\psi\})$ can have such equilibria: any profile of platforms $\langle \mathbf{x}, \mathbf{y} \rangle$ with both $\mathbf{x}$ and $\mathbf{y}$ medians of the distribution of voter ideal points is an equilibrium in such a game. To rule out such cases, an equilibrium $\langle \mathbf{x}, \mathbf{y} \rangle$ is only called divergent if "some voters' ideal points lie in between the platforms $\mathbf{x}$ and $\mathbf{y}$". In the framework of the classical Downsian model, this requirement is easily formalized as $\langle \mathbf{x}, \mathbf{y} \rangle$ is divergent if and only if $\psi(\{\succsim : a(\succsim) \in (\min(\mathbf{x}, \mathbf{y}), \max(\mathbf{x}, \mathbf{y}))\}) > 0$. According to this definition, the classical Downsian model has no divergent equilibria. In terms of the general model, the same requirement is formalized as

Definition: An equilibrium $\langle \mathbf{x}, \mathbf{y} \rangle$ is called **divergent** if $\psi(\{\succsim : a(\succsim) \in \lfloor \mathbf{x}, \mathbf{y} \rceil\}) > 0$ for all $\psi \in \Psi$.

Here "in between" is interpreted following the definition of the area in between two platforms given in Section 2.1. The statement "Θ holds for some voters" is formally translated to "according to *any* $\psi \in \Psi$, Θ holds for a set of voters that has positive probability mass". In sum, a platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ is considered divergent if the probability that the ideal point of a randomly drawn voter lies in between $\mathbf{x}$ and $\mathbf{y}$ is positive according *every* belief $\psi \in \Psi$.

The notion of divergence plays a crucial role in the two main results of the article: Theorem 1 shows that divergent equilibria may exist in the present model of Downsian competition, while Theorem 2 shows that the multi-dimensionality of the political issue space is a necessary condition for divergent equilibria to exist. One could obtain a stronger version of Theorem 1 if one was to use a more restrictive notion of divergence,; the opposite holds true for Theorem 2. I therefore now consider a strengthening and a weakening of the notion of divergence.

A more restrictive notion of divergence could be based on a more restrictive interpretation of some ideal point lying "in between" two platforms. To this end, one could define the set

[5] In Bade (2005), I provide a characterization of equilibrium sets of such games with incomplete preferences.

of all convex combinations of two platforms as the area in between these two platforms. I opted against this definition since the set of platforms between two platforms is not robust to rescaling the axes of the issue space X. The notion of "in between", defined above, is not vulnerable to this criticism, the set $\lfloor \mathbf{x}, \mathbf{y} \rceil$ is robust to rescaling.[6] Moreover, if one considers non-atomic distributions of voter ideal points, there are no profiles of divergent platforms since the set of convex combinations of two platforms has measure zero. Still, once I prove Theorem 1, I go on to show that the result also holds when using this much stronger notion of divergence. This is important since any *reasonable* weaker notion of divergence would probably also demand that any convex combination of the platforms is considered to be lying in between these two platforms. Conversely, the notion of divergence could be weakened if one was to interpret "Θ holds for some voters" as "according to *some* $\psi \in \Psi$, Θ holds for a set of voters that has positive probability mass". If one requires absolute continuity of the set Ψ, the two notions of divergence coincide. Theorem 2 can be amended to a stronger version, by using the less restrictive notion of divergence.

3 The Existence of Divergent Equilibrium

In this section, I state and prove the claim that multidimensional games with uncertainty averse parties can have divergent equilibria.

Theorem 1: *A game of electoral competition* (n, X, Ψ) *can have divergent equilibria.*

Proof: Take the game $(2, [0,1]^2, \Psi)$ with $\Psi = \{\psi^1, \psi^2\}$ defined by

$$\begin{aligned}
\psi^1(\{\succsim : a(\succsim) = (0,0)\}) &= \psi^2(\{\succsim : a(\succsim) = (1,1)\}) = \frac{1}{3}, \\
\psi^1(\{\succsim : a(\succsim) = (1,1)\}) &= \psi^2(\{\succsim : a(\succsim) = (0,0)\}) = \frac{1}{9}, \\
\psi^1(\{\succsim : a(\succsim) = (1,0)\}) &= \psi^2(\{\succsim : a(\succsim) = (0,1)\}) = \frac{1}{6}, \\
\psi^1(\{\succsim : a(\succsim) = (0,1)\}) &= \psi^2(\{\succsim : a(\succsim) = (1,0)\}) = \frac{1}{6} + \frac{4}{27}, \\
\psi^i(\{\succsim : a(\succsim) = (.5,.5)\}) &= \; = \frac{2}{27} \text{ for } i = 1, 2.
\end{aligned}$$

Let the preferences of voters with ideal points $(1,0)$ and $(0,1)$ be represented by non-convex utility functions u^a, as defined in Section 2.1. All other voters have Euclidean preferences. The electorate is illustrated in Figure 3. All indifference curves are drawn in green. The

[6] Note that the set $\lfloor \mathbf{x}, \mathbf{y} \rceil$ only involves notions of "more" or "less" and is therefore unaffected by any rescaling of the axes of the political issue space. This does not hold for the set of convex combinations of two platforms.

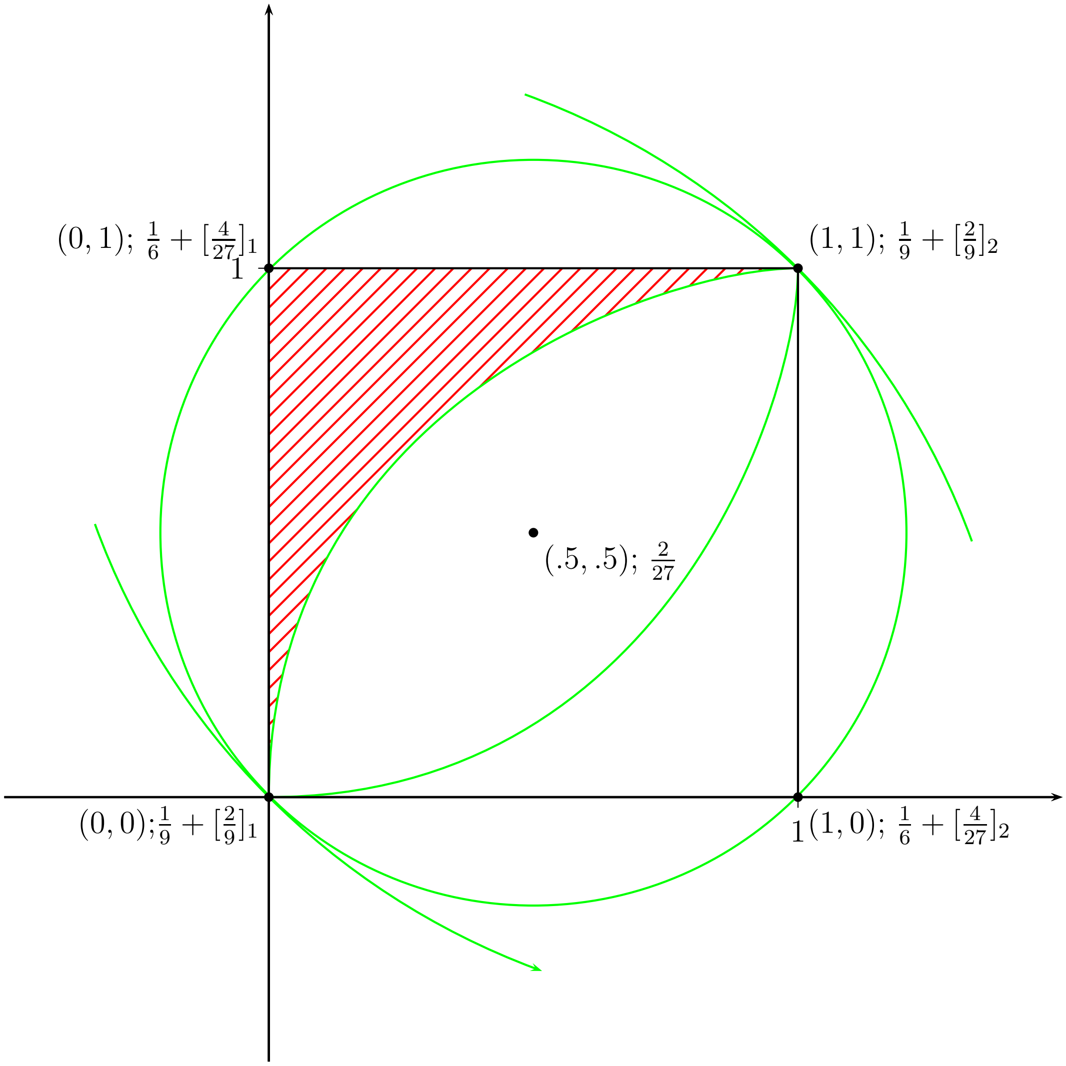

Figure 3: The Electorate

fractions right next to the ideal points denote the probability mass of voters at the respective ideal points, where the label $\frac{1}{6}+[\frac{4}{27}]_2$ of point $(0,1)$ is to be read as follows: under either distribution, at least $\frac{1}{6}$ of the electorate is expected to have their idea point at $(0,1)$; under distribution ψ^2, the probability mass at $(0,1)$ increases to $\frac{1}{6}+\frac{4}{27}$.

Note that parties are unsure about $\frac{10}{27}$ of the electorate: these voters might have their ideal points at $(0,1)$, $(1,0)$, $(0,0)$ and $(1,1)$. For the remaining $\frac{17}{27}$, the two distributions ψ^1 and ψ^2 agree. In particular, both distributions assign a probability of $\frac{2}{27}$ to the event that a randomly drawn voter has his ideal point at $(.5,.5)$. All voters with ideal points at $(0,1)$ and $(1,0)$ and $(.5,.5)$ are indifferent between the "extreme policies" $(0,0)$ and $(1,1)$. The preferences of voters with their ideal point at $(0,1)$ and $(1,0)$ are not convex; these voters prefer platforms $(0,0)$ and $(1,1)$ to any of the intermediate policies (x,x) for any $x \in (0,1)$.

The set of platforms in $[0,1]^2$ which voters with their ideal point at $(0,1)$ prefer to platform $(1,1)$ is hatched in red.

I claim that $\langle(0,0);(1,1)\rangle$ is a divergent equilibrium of this game.

I start by showing that party one's platform $(0,0)$ is a best response to $(1,1)$, the second party's platform. To see this, observe that $\pi_{\psi^1}((0,0),(1,1)) = \frac{1}{3} + \frac{1}{2}(\frac{2}{27} + \frac{1}{6} + \frac{1}{6} + \frac{4}{27}) > \frac{1}{2}$ and $\pi_{\psi^2}((0,0),(1,1)) = \frac{1}{9} + \frac{1}{2}(\frac{2}{27} + \frac{1}{6} + \frac{1}{6} + \frac{4}{27}) < \frac{1}{2}$. Since there is a prior, namely ψ^1, such that party one gets more than half the vote according to the profile under consideration, it is not a best reply for party one to offer party two's platform.

Next, observe that for any $\mathbf{x} \notin \{(0,0),(1,1)\}$, either voters with their ideal point at $(0,1)$ or the voters with their ideal point at $(1,0)$ (or both) strictly prefer platform $(1,1)$ to $\mathbf{x}$. Graphically this claim corresponds to the observation that the upper-contour sets of $(1,1)$ of voters with their ideal point at $(0,1)$ and $(1,0)$ do not intersect.

To prove this claim formally suppose there was a platform $\mathbf{x}$ that would win the favor of the voters with ideal point at $(1,0)$ [or $(0,1)$] while keeping voters with the ideal point $(1,0)$ [or $(0,1)$] at least indifferent. For such a platform $\mathbf{x}$, we would have to have that:

$$\begin{aligned}
-\sqrt{|x_1-1|} - \sqrt{|x_2-0|} &\geq -\sqrt{|1-1|} - \sqrt{|1-0|} \text{ and} \\
-\sqrt{|x_1-0|} - \sqrt{|x_2-1|} &\geq -\sqrt{|1-0|} - \sqrt{|1-1|} \\
&\text{or, expressed differently:} \\
|x_1-1| + 2\sqrt{|x_1-1|\,|x_2|} + |x_2| &\leq 1 \text{ and} \\
|x_1| + 2\sqrt{|x_1|\,|x_2-1|} + |x_2-1| &\leq 1
\end{aligned}$$

with at least one of the inequalities holding strictly. Adding both constraints up and taking into account that at least one is strict, we obtain:

$$2\sqrt{|x_1-1|\,|x_2|} + 2\sqrt{|x_1|\,|x_2-1|} < 2 - |x_1-1| - |x_1| - |x_2| - |x_2-1|$$

But $2 - |x_1-1| - |x_1| - |x_2| - |x_2-1| \leq 0$ for all x_1 and x_2.

By single-peakedness, voters with their ideal point at $(0,0)$ strictly prefer $\mathbf{x}$ to $(1,1)$, and voters with their ideal point at $(1,1)$ have the opposite preference. So, the question is whether party one can gain the votes of all voters with ideal points at $(1,0)$ and $(.5,.5)$ (or at $(0,1)$ and $(.5,.5)$) instead of gaining half the votes of the voters at $(0,1),(1,0)$ and $(.5,.5)$. According to both distributions, ψ^1 and ψ^2, $\frac{1}{6} + \frac{1}{6} + \frac{4}{27} + \frac{2}{27}$ of the electorate have their ideal points at $(0,1),(1,0)$, and $(.5,.5)$. However, according to ψ_1, only $\frac{1}{6} + \frac{2}{27}$ of the electorate have their ideal points at $(1,0),(.5,.5)$, so a deviation that only gains the favor of these voters cannot be preferred. The existence of a preferred deviation that gains the support of voters with ideal points $(1,0),(.5,.5)$ is ruled out mutatis mutandis.

We can conclude that $(0,0)$ is indeed a best reply to $(1,1)$. Since the example was constructed fully symmetric it also holds that $(1,1)$ is a best reply to $(0,0)$, and consequently $\langle(0,0),(1,1)\rangle$ is an equilibrium. Finally $\langle(0,0),(1,1)\rangle$ is divergent as as $(.5,.5) \in \lfloor(0,0),(1,1)\rceil$ and $\psi^i(\{\succsim\colon a(\succsim) = (.5,.5)\}) = \frac{2}{27} > 0$ for both $i = 1$ and $i = 2$. ■

Remarks:

(A) Following Section 2.6, an equilibrium $\langle\mathbf{x},\mathbf{y}\rangle$ is called divergent if some voters have their ideal points between the two platforms $\mathbf{x}$ and $\mathbf{y}$. This notion of divergence relies on the definition of the area in between $\mathbf{x}$ and $\mathbf{y}$ as $\lfloor x, y\rceil$. Here, I would like to note that the proof of Theorem 1 goes through when replacing the weaker notion of divergence by the (much) stronger one which calls two platforms $\mathbf{x}$ and $\mathbf{y}$ divergent if $\psi(\{\succsim\colon a(\succsim) = \lambda\mathbf{x} + (1-\lambda)\mathbf{y}$ for some $\lambda \in (0,1)\})$ holds for all $\psi \in \Psi$ as $\psi^i(\{\succsim\colon \mathbf{a}(\succsim) = (x,x)$ with $0 < x < 1\}) = \psi^i(\{\succsim\colon \mathbf{a}(\succsim) = (.5,.5)\})$ holds for $i = 1, 2$.

(B) The only other equilibrium of the game is $\langle(1,1),(0,0)\rangle$. The proof of this observation is tedious and therefore relegated to the Appendix.

(C) Not all aspects of the specification of Ψ were needed to prove that $\langle(0,0),(1,1)\rangle$ is an equilibrium. In particular, I did not use the assumption that voters with ideal points $(0,0), (.5,.5)$, and $(1,1)$ have Euclidean preferences. The proof goes through when only assuming that their preferences are single-peaked. Given that the preferences of all voters in the example are representable, separable and symmetric, the result also holds if we require that all voters satisfy any combination of these properties, which are defined as follows: Some preferences $\succsim$ on X are representable if there exists a function $u : X \mapsto \mathbb{R}$ such that $\mathbf{x} \succsim \mathbf{y}$ holds if and only if $u(\mathbf{x}) \geq u(\mathbf{y})$. Some representable preferences on $X \subset \mathbb{R}^n$ are considered separable if there exist functions u^i for $i = 1, \cdots, n$ such that $u(\mathbf{x}) = \sum_{i=1}^n u_i(x_i)$.[7] Some single-peaked preferences $\succsim$ with ideal point $\mathbf{a}$ are considered symmetric if $\mathbf{x} \sim 2\mathbf{a} - \mathbf{x}$ for $\mathbf{x}, 2\mathbf{a} - \mathbf{x} \in X$. In addition, there is some leeway as to the distribution of probability mass across the different voter types.

(D) The profile $\langle(0,0),(1,1)\rangle$ is an equilibrium of any game $(2,[0,1]^2,\Psi')$ with $\Psi \in \Psi'$. To see this, observe that, for $(0,0)$ to be a best response to $(1,1)$, it only needs to hold that for any deviation $\mathbf{x} \neq (0,0)$ there exists some belief $\psi \in \Psi$ such that $\pi_\psi(\langle\mathbf{x},(1,1)\rangle) < \pi_\psi(\langle(0,0),(1,1)\rangle)$. Since this inequality holds either for ψ^1 or ψ^2, as was shown in the proof of Theorem 1, and since $\psi^i \in \Psi \subset \Psi'$ for $i = 1, 2$, there cannot be any better response

[7] Separability can also be defined without the assumption of representability. Since that more general definition is tedious to state and since separability without representability is not important to the development of the arguments here, I chose to stick with the simpler case.

$\mathbf{x}$ to $(1,1)$ for the larger set of beliefs Ψ'. To say it more generally: making preferences of players more incomplete can only increase the equilibrium set of a game. Games might have convergent and divergent equilibria. To see this, consider the game $(2,[0,1]^2,\Psi'')$ with $\Psi''=\{\psi^1,\psi^2,\psi^3\}$ and $\psi^3(\{\succsim : a(\succsim)=(.5,.5)\})=1$, so according to ψ^3 the parties are sure that all voters have their ideal point at $(.5,.5)$. Note that $\langle(.5,.5),(.5,.5)\rangle$ is an equilibrium of the game, for any deviation from this profile the deviating party would leave the *entire* electorate to the other party according to ψ^3. By the preceding argument $\langle(0,0),(1,1)\rangle$ is also an equilibrium of $(2,[0,1]^2,\Psi'')$.

(E) In this article, I assume that parties maximize their respective vote shares. Let me now investigate how Theorem 1 would change if one was to follow the equally popular assumption that parties maximize their probability of winning the election. Formally, party one's objective function is $w(\pi_\psi(\langle\mathbf{x},\mathbf{y}\rangle))$ with $w:[0,1]\mapsto[0,1]$ with $w(x)=0$ for $x<\frac{1}{2}$, $w(\frac{1}{2})=\frac{1}{2}$ and $w(x)=1$ for $x<\frac{1}{2}$. Party two's objective function is $1-w(\pi_\psi(\langle\mathbf{x},\mathbf{y}\rangle))$. In the classic Downs-Hotelling model, the two assumptions turn out to be equivalent, in the sense that a profile is an equilibrium for one of the two assumptions if and only if it is an equilibrium for the other assumption. This does not hold for the present model:

If we change the present model to assume that $(w(\pi_\psi(\langle\mathbf{x},\mathbf{y}\rangle)))_{\psi\in\Psi}$ and $(1-w(\pi_\psi(\langle\mathbf{x},\mathbf{y}\rangle)))_{\psi\in\Psi}$, respectively, are party one and two's objective functions, then $\langle(0,0),(1,1)\rangle$ is not an equilibrium of (n,X,Ψ) as defined in the proof of Theorem 1. Party one's payoff vector for the profile $\langle(0,0),(1,1)\rangle$ would then be $(1,0)$. For a deviation to $(.1,0)$, party one would win the election under both beliefs, so it would increase its payoff vector to $(1,1)$.

This does not mean that the assumption of parties that maximize their probability of winning rules out the existence of divergent equilibria. Consider the game (n,X,Ψ^*) where $\Psi^*=\{\psi^{1*},\psi^{2*}\}$ and ψ^{1*},ψ^{2*} differ from ψ^1,ψ^2 only insofar as that $\psi^i(\{\succsim : a(\succsim)=(0,0)\})=\psi^i(\{\succsim : a(\succsim)=(1,1)\})=\frac{2}{9}$ for both $i=1,2$. It is easy, but tedious, to check that $\langle(0,0),(1,1)\rangle$ is a (divergent) equilibrium of that game when assuming that parties aim to maximize their probability of winning. Interestingly, $\langle(0,0),(1,1)\rangle$ is also an equilibrium of that game when assuming parties that maximize their vote shares. In fact, $\langle(0,0),(1,1)\rangle$ remains an equilibrium in any version of (n,X,Ψ^*) that assumes that parties aim to maximize some convex combination of their probability of winning and their vote share. This last assumption is unquestionably the most realistic representation of party goals. I chose to work with the assumption of vote-share-maximizing parties since it renders the analysis significantly less cumbersome and since the main insights of the model are not hampered by this more simplistic assumption on party goals.

4 Necessary Conditions for Divergence

The model presented here deviates from the classical Downs-Hotelling model in two respects: the issue space is multidimensional and parties hold multiple beliefs on the electorate. In this section, I argue that both differences are essential to obtaining divergent equilibria.

Theorem 2: *The multi-dimensionality of the issue space is a necessary condition for the existence of divergent equilibria.*

Proof: Suppose some game (n, X, Ψ) had a divergent equilibrium $\langle \mathbf{x}, \mathbf{y} \rangle$ even though the preferences of all voters are convex (formally: $\psi(\{\succsim\}) > 0$ for some $\psi \in \Psi$ implies that $\succsim$ is convex). Observe that the party proposing $\mathbf{x}$ is at least as well off proposing $\lambda\mathbf{x} + (1-\lambda)\mathbf{y}$ for any $\lambda \in (0,1)$. Consider a voter with preferences $\succsim$ who prefers $\mathbf{x}$ to $\mathbf{y}$. By the convexity assumption, he also prefers $\lambda\mathbf{x} + (1-\lambda)\mathbf{y}$ to $\mathbf{y}$. So we have that the set of voters preferring $\lambda\mathbf{x} + (1-\lambda)\mathbf{y}$ to $\mathbf{y}$ is a superset of the set of voters preferring $\mathbf{x}$ to $\mathbf{y}$ and therefore, $\pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle) \leq \pi_\psi(\langle \lambda\mathbf{x} + (1-\lambda)\mathbf{y}, \mathbf{y} \rangle)$ for all $\psi \in \Psi$.

To show that there exists a *strictly* preferred deviation for the first party, we need to show that for at least one $\psi^* \in \Psi$ and some $\lambda^* \in (0,1)$ we have $\pi_{\psi^*}(\langle \mathbf{x}, \mathbf{y} \rangle) < \pi_{\psi^*}(\langle \lambda^*\mathbf{x} + (1-\lambda^*)\mathbf{y}, \mathbf{y} \rangle)$. By the supposition that $\langle \mathbf{x}, \mathbf{y} \rangle$ a divergent equilibrium we know that $\psi(\{\succsim: a(\succsim) \in \lfloor \mathbf{x}, \mathbf{y} \rceil\}) > 0$ for all $\psi \in \Psi$. Assume w.l.o.g. that under platform profile $\langle \mathbf{x}, \mathbf{y} \rangle$ some voters in the area $\lfloor \mathbf{x}, \mathbf{y} \rceil$ are in favor of platform $\mathbf{y}$, formally: $\psi^*(\{\succsim^*\}) > 0$ for some $\psi^* \in \Psi$ where $y \succ^* x$ and $a(\succsim^*) \in \lfloor \mathbf{x}, \mathbf{y} \rceil$. Now observe that for some small enough $\lambda^* \in (0,1)$ $\lambda^*\mathbf{x} + (1-\lambda^*)\mathbf{y}$ lies in between $a(\succsim^*)$ and $\mathbf{y}$. This together with single peakedness implies that $\pi^*_\psi(\langle \lambda^*\mathbf{x} + (1-\lambda^*)\mathbf{y},\ \mathbf{y} \rangle) \geq \psi^*(\{\succsim^*\}) + \pi^*_\psi(\langle \mathbf{x}, \mathbf{y} \rangle)$. Which in turn implies that party 1 is better off deviating to $\lambda^*\mathbf{x} + (1-\lambda^*)\mathbf{y}$.

The multi-dimensionality of the issue space is a necessary condition for the existence of divergent equilibria since single peaked preferences over a uni-dimensional issue space are automatically convex. ■

Remark: Just like Theorem 1 is strengthened by assuming a more restrictive notion of divergence, Theorem 2 is strengthened by assuming a weaker one. Following the discussion in Section 2.6, let me weaken the concept of divergence to require only that $\psi^i(\{\succsim: a(\succsim) \in \lfloor x, y \rceil\})$ holds for *some* $\psi \in \Psi$ for $\langle \mathbf{x}, \mathbf{y} \rangle$. The proof of Theorem 2 directly applies to the stronger version of Theorem 2 which builds on the weakened notion of divergence: in fact, in the proof, I only showed that the preferred deviation is strictly preferred according to *some* prior $\psi^* \in \Psi$.

I was not able to show that games in which parties base their decision on unique priors $(n, X, \{\psi\})$ never have any divergent equilibria. What certainly holds, is that for such an equilibrium to exist, some very specialized conditions would have to hold true. To start with, it has to hold true that $\psi(\langle \mathbf{x}, \mathbf{y} \rangle) = \frac{1}{2}$, since otherwise the party with the lower vote share would be able to increase its vote share by offering the platform of the opponent. Of course, to be an equilibrium $\psi(\langle \mathbf{x}', \mathbf{y} \rangle) \leq \frac{1}{2}$ as well as $\psi(\langle \mathbf{x}, \mathbf{y}' \rangle) \leq \frac{1}{2}$ would have to hold for all platforms $\mathbf{x}'$ and $\mathbf{y}'$. This would, in turn, imply that $\langle \mathbf{x}, \mathbf{x} \rangle$ and $\langle \mathbf{y}, \mathbf{y} \rangle$ are also equilibria of the game. It would certainly not be possible to draw much empirical relevance from the example of a game $(n, X, \{\psi\})$ with a divergent equilibrium, as the conditions for the existence are very tight. Thus, it can be concluded that a multiple-prior model is at least necessary for divergence to be more than a knife-edge phenomenon. The question whether the multiplicity is mathematically necessary for divergent equilibria remains open.

5 Prior Results and a Note on Equilibrium Existence

Games of the form $(n, X, \{\psi\})$,, that is games with known electorates or, equivalently, with expected utility-maximizing parties, have been studied widely in the literature. For games with uni-dimensional issue spaces Downs (1957) showed that equilibria always exist and that in equilibrium both parties will announce the policy preferred by some median voter. No game $(1, X, \{\psi\})$ has any divergent equilibria.

On the contrary, games of multidimensional political competition with expected utility maximizing parties hardly ever have any equilibria. Davis, Hinich, and de Groot (1972), Grandmont (1978), and Plott (1967) all provide negative results on the existence of equilibria in games $(n, X, \{\psi\})$ with $n \geq 2$. A study on the characterization of these equilibria of multidimensional political games would therefore be quite meaningless: it would be a study of objects that hardly ever exist. In any case, the results we have mostly predict that, if an equilibrium exists, both parties should announce the same platform (see Davis, Hinich, and de Groot (1972) and Grandmont (1978)).

The games under review in this study, that is, games of the form (n, X, Ψ), are not plagued by the same non-existence problems as are games of multidimensional political competition with utility-maximizing or certain parties. To see this, consider games $(n, X, \Psi)^*$ which differ from the games defined in the present paper only insofar as party preferences over platform profiles $\langle \mathbf{x}, \mathbf{y} \rangle$ are complete and can be represented by $\min_{\psi \in \Psi} \pi_\psi(\langle \mathbf{x}, \mathbf{y} \rangle)$.[8] In Bade (2010), I described conditions on the set of party beliefs Ψ, under which games $(n, X, \Psi)^*$ have an

[8] In terms of the motivational story on parties as non-unitary actors, the assumption that parties only change their position if it looks advantageous from the point of view of all factions would have to be replaced by the assumption that any party changes its platform if it looks advantageous from the point of view of the faction that foresees the lowest share for the party. In terms of the story of uncertainty aversion, Bewley's model of Knightian uncertainty would have to be replaced by Gilboa and Schmeidler's maxmin expected utility representation.

equilibrium.

Now observe that the parties in games (n, X, Ψ) are less prone to change their platforms than the parties in games $(n, X, \Psi)^*$. In a game $(n, X, \Psi)^*$, party one would deviate from $\mathbf{x}'$ to $\mathbf{x}$ if $\min_{\psi\in\Psi} \pi_\psi(\langle \mathbf{x}', \mathbf{y}\rangle) > \min_{\psi\in\Psi} \pi_\psi(\langle \mathbf{x}, \mathbf{y}\rangle)$. In contrast, in a game (n, X, Ψ), party one would only deviate if the following decisively stronger condition held: $\pi_\psi(\langle \mathbf{x}', \mathbf{y}\rangle) \geq \pi_\psi(\langle \mathbf{x}, \mathbf{y}\rangle)$ for all $\psi \in \Psi$ with $\pi_{\psi'}(\langle \mathbf{x}', \mathbf{y}\rangle) > \pi_{\psi'}(\langle \mathbf{x}, \mathbf{y}\rangle)$ for some $\psi' \in \Psi$. Therefore, the equilibrium set of any game $(n, X, \Psi)^*$ is always contained in the corresponding game (n, X, Ψ). It holds, in particular, that (n, X, Ψ) has an equilibrium if $(n, X, \Psi)^*$ does. Consequently, the results on equilibrium existence in Bade (2010) can directly be transferred to the present framework.[9]

6 Conclusion

In the profession, there seems to be a deeply ingrained belief that political equilibria with parties that maximize their vote share would always have to be convergent. But given the Theorems of this paper, it should be clear that office motivation can be reconciled with divergent platforms in equilibrium. The crucial argument for convergence is that moving closer to the other party's platform can never hurt. But this argument only makes sense in the context of convex preferences on the part of voters. Once we drop the assumption of convexity, there is no reason to believe that any party should always want to move closer to the other parties' platform. The dimensionality of the issue space comes into play since single-peaked preferences over a uni-dimensional issue space are automatically convex.

Observe that the same argument holds true for certain parties: it is not always better for a party to move closer to the other party's platform when voter preferences exhibit non-convexities. It might well be that some party's best response is to diverge from the other party's platform, even when both parties have perfect information about the electorate. Consider, for example, the case of a game $(2, [-2, 2], \{\psi^2\})$ with ψ^2 as defined in the proof of Theorem 1. Fix the platform of party two at $(.1, .1)$. It is a best reply for party one to offer platform $(1, 1)$. Voters with ideal points at $(0, 1), (1, 0)$ and $(1, 1)$ all prefer $(1, 1)$ to $(.1, .1)$, yielding a vote share of $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{4}{27} = \frac{7}{9}$ to party one. There is no platform $\mathbf{x}$ in that game such that $\pi_{\psi^1}(\langle \mathbf{x}, (.1, .1)\rangle) > \frac{7}{9}$ would hold. Moreover, the platforms $(.1, .1)$ and $(1, 1)$ diverge (even according to the strictest notion of divergence proposed here). However, the profile $\langle (1, 1), (.1, .1)\rangle$ does not constitute an equilibrium in $(2, [-2, 2], \{\psi^2\})$, as party two would be better off to match party one's platform.

[9] Of course, there are some more differences between the setup of the preceding paper and the present one: in that paper, I study convex belief sets and non-atomic electorates. None of these turn out to be relevant to obtaining the existence result.

In short, the result that equilibrium platforms must be identical is actually the artificial result of the assumption that issue spaces are uni-dimensional and that parties act as expected vote-share-maximizers. Without those two - quite possibly counterfactual - assumptions, it is easy to give examples of models in which parties announce divergent platforms in equilibrium. The centerpiece of this paper is such an example.

This is just a paper on the *existence* of divergent equilibria in games of electoral competition a la Downs-Hotelling. It sets the stage for a set of interesting *and* difficult questions on the characterization of the equilibrium sets in such games: what are sufficient conditions for the existence of divergent equilibria? How can the connection between party uncertainty and equilibrium divergence be quantified? Can we relate the "extent" of the non-convexities in voter preferences to the divergence of equilibrium platforms? How do the equilibrium platforms relate to the distributions of ideal points that are implied by the sets of priors on voter preferences Ψ?

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8 Appendix

Here I show that the game defined in the proof of Theorem 1 does not have any equilibria outside $\langle(0,0),(1,1)\rangle$ and $\langle(1,1),(0,0)\rangle$. To see this, assume that $\langle\mathbf{x},\mathbf{y}\rangle \notin \{\langle(0,0),(1,1)\rangle, \langle(1,1),(0,0)\rangle\}$ was an equilibrium of the game. The proof proceeds by showing that for any $\mathbf{y}$ and any best reply $\mathbf{x}^*$ to $\mathbf{y}$, it holds that $\pi_{\psi^i}(\langle\mathbf{x}^*,\mathbf{y}\rangle) > \frac{1}{2}$ for $i = 1,2$. Applying this observation to the conjectured equilibrium profile, we obtain that $\pi_{\psi^i}(\langle\mathbf{x},\mathbf{y}\rangle) > \frac{1}{2}$ holds for $i = 1,2$. But this implies that party two would be able to increase its vote share according to ψ^1 and ψ^2 by changing its platform from $\mathbf{y}$ to $\mathbf{x}$. To show that $\pi_{\psi^i}(\langle\mathbf{x}^*,\mathbf{y}\rangle) > \frac{1}{2}$ indeed holds for $i = 1,2$ for any best reply $\mathbf{x}^*$, I separately consider the case in which $\mathbf{y} \in (0,1)^2$ and the remainder (for which $y_1(1-y_1)y_2(1-y_2) = 0$ holds).

First consider the case in which $\mathbf{y} \in (0,1)^2$ and $y_1 + y_2 \leq 1$ hold. Define $\epsilon > 0$ such that $(y_1 - \epsilon, y_2 - \epsilon) \in [0,1]^2$. Observe that voters with ideal points $(0,1)$ and $(1,0)$ prefer $(y_1 - \epsilon, y_2 - \epsilon)$ to $\mathbf{y}$. To see this, compare the utility that a voter with ideal point $(0,1)$ derives from the platforms $\mathbf{y}$ and $(y_1 - \epsilon, y_2 - \epsilon)$ and observe that

$$\begin{aligned} u^{(0,1)}(\mathbf{y}) < u^{(0,1)}((y_1-\epsilon, y_2-\epsilon)) &\Leftrightarrow \\ -\sqrt{y_1} - \sqrt{1-y_2} < -\sqrt{y_1-\epsilon} - \sqrt{1-(y_2-\epsilon)} &\Leftrightarrow \\ y_1 + 1 - y_2 + 2\sqrt{y_1(1-y_2)} > y_1 - \epsilon + 1 - y_2 + \epsilon + 2\sqrt{(y_1-\epsilon)(1-y_2+\epsilon)} &\Leftrightarrow \\ \epsilon(\epsilon + 1 - y_2 - y_1) > 0&, \end{aligned}$$

where the last line holds for any permissible ϵ. By symmetry, any voter an with ideal point $(1,0)$ also prefers $(y_1 - \epsilon, y_2 - \epsilon)$ to $\mathbf{y}$. Single-peakedness of preferences implies that voters who have their ideal point at $(0,0)$ prefer $(y_1 - \epsilon, y_2 - \epsilon)$ to $\mathbf{y}$ and that voters with ideal points $(.5,.5)$ and $(1,1)$ have the inverse preference. In sum, we have $\psi^i(\{\succsim : a(\succsim) \in \{(0,0),(1,0),(0,1)\}\}) = \pi_{\psi^i}(\langle(y_1-\epsilon, y_2-\epsilon),\mathbf{y}\rangle)$. For the two beliefs, the expression takes the values $\pi_{\psi^1}(\langle(y_1-\epsilon, y_2-\epsilon),\mathbf{y}\rangle) = \frac{22}{27} > \frac{1}{2}$ and $\pi_{\psi^2}(\langle(y_1-\epsilon, y_2-\epsilon),\mathbf{y}\rangle) = \frac{16}{27} > \frac{1}{2}$.

Since the agents with ideal points $(0,1)$ and $(1,0)$ have preferences that are symmetric around the line of profiles $\mathbf{z}$ with $z_1 + z_2 = 1$, these agents are indifferent between $(y_1-\epsilon, y_2-\epsilon)$ and $(1-y_2+\epsilon, 1-y_1+\epsilon)$. Since ϵ was chosen such that $y_i - \epsilon \geq 0$ for both $i = 1,2$ the platform $(1-y_2+\epsilon, 1-y_1+\epsilon)$ is in the issue space $[0,1]^2$. The voters with ideal point at $(1,1)$ prefer $(1-y_2+\epsilon, 1-y_1+\epsilon)$ to $\mathbf{y}$ by single-peakedness. By the symmetry of the setup, the vote share to party one in profile $\langle(1-y_2+\epsilon, 1-y_1+\epsilon),\mathbf{y}\rangle$ is $\frac{16}{27}$ and $\frac{22}{27}$ according to ψ^1 and ψ^2 respectively.

Now, observe that if another platform $\mathbf{x}$ attracted any other set of voters S at least one of the three following conditions holds for both $i = 1,2$: either $\psi^i(\{\succsim \in S\}) > \frac{1}{2}$, or $\psi^i(\{\succsim \in S\}) < \pi_{\psi^i}(\langle(y_1-\epsilon, y_2-\epsilon),\mathbf{y}\rangle)$, or $\psi^i(\{\succsim \in S\}) < \pi_{\psi^i}(\langle(1-y_2+\epsilon, 1-y_1+\epsilon),\mathbf{y}\rangle)$. Now, if the one of the latter two conditions holds $\mathbf{x}$ cannot be a best reply, as either $(y_1-\epsilon, y_2-\epsilon)$ or $(1-y_2+\epsilon, 1-y_1+\epsilon)$ would be strictly better replies than $\mathbf{x}$. In sum, it holds that $\pi_{\psi^i}(\langle \mathbf{x}^*, \mathbf{y}\rangle) > \frac{1}{2}$ holds for both $i = 1, 2$ at all best responses $\mathbf{x}^*$ to some platform $\mathbf{y} \in (0,1)^2$ with $y_1 + y_2 \leq 1$. By symmetry, the same holds for the case in which $y_1 + y_2 \geq 1$ and $\mathbf{y} \in (0,1)^2$. Applying the introductory argument we can conclude that there is no equilibrium $\langle \mathbf{x}, \mathbf{y}\rangle$ with $\mathbf{y} \in (0,1)^2$.

So next, consider a strategy profile $\langle \mathbf{x}, \mathbf{y}\rangle$ with $y_1 = 0$. And let us assume that $y_2 \neq 0$. First, consider the case in which $\mathbf{y} \succ \mathbf{x}$ holds true for voters with ideal point $(0,1)$. In this case, party one gains the support of all voters when playing a best reply $\mathbf{x}^*$ against $\mathbf{y}$. To see this, consider the platform $(\epsilon, y_2 - \epsilon)$ for some small ϵ and observe that voters with ideal points at $(0,0)$ and $(1,1)$ and $(.5,.5)$ prefer the platform $(\epsilon, y_2-\epsilon)$ to $\mathbf{y}$ as

$$\begin{aligned} v^{(\alpha,\alpha)}(\mathbf{y}) < v^{(\alpha,\alpha)}((\epsilon, y_2-\epsilon)) &\Leftrightarrow \\ -\alpha^2 - (\alpha - y_2)^2 < -(\alpha-\epsilon)^2 - (\alpha - y_2 + \epsilon)^2 &\Leftrightarrow \\ -\alpha^2 - (\alpha-y_2)^2 < -\alpha^2 + 2\epsilon\alpha - \epsilon^2 - (\alpha-y_2)^2 - 2\epsilon(\alpha - y_2) - \epsilon^2 &\Leftrightarrow \\ 0 < 2\epsilon\alpha - \epsilon^2 - 2\epsilon(\alpha-y_2) - \epsilon^2 &\Leftrightarrow \\ 0 < 2(\epsilon y_2 - \epsilon^2). \end{aligned}$$

In addition, observe that by single-peakedness a voter with ideal point $(1,0)$ prefers $(\epsilon, y_2-\epsilon)$ to $\mathbf{y}$. Only voters with ideal point $(0,1)$ prefer $\mathbf{y}$ to $(\epsilon, y_2 - \epsilon)$. Since $\psi^i(\{\succsim\colon a(\succsim) \in \{(0,0),(1,0),(1,1),(.5,.5)\}\}) = \pi_{\psi^i}(\langle(\epsilon, y_2-\epsilon), \mathbf{y}\rangle) > \frac{1}{2}$ for $i = 1,2$ the platform profile $\langle(\epsilon, y_2-\epsilon), \mathbf{y}\rangle$ cannot be an equilibrium by the arguments presented in the introduction of the proof. So it cannot be that $\mathbf{y}$ gains the support of $(0,1)$.

So let us consider the alternative case in which $\mathbf{x}$ gains the support of $(0,1)$. It cannot be that $\mathbf{x} \in (0,1)^2$, since we already ruled out above that either party offers platform in $(0,1)^2$ in equilibrium. For $\mathbf{x} \notin (0,1)^2$ to gain the support of the voters with ideal point $(0,1)$ it must hold that either $\mathbf{x} = (0, x_2)$ with $x_2 > y_2$ or $\mathbf{x} = (x_1, 0)$ with $x_1 < 1 - y_2$. If only voters with ideal point $(0,1)$ vote for party one according to the profile $\langle \mathbf{x}, \mathbf{y}\rangle$, party one would be better off to offer $\mathbf{y}$. If not, then party two can deviate in a way to reduce the partisans of party one to the set of all voters with ideal point $(0,1)$ (where the deviations are constructed like the deviation against $(0, y_2)$ above). By symmetry, the same logic applies to all other cases with $y_i \in \{0,1\}$ for $i = 1$ or $i = 2$ and $\mathbf{y} \notin \{(0,0),(1,1)\}$.

There remain two last candidates for equilibria $\langle(0,0),(0,0)\rangle$ and $\langle(1,1),(1,1)\rangle$. Of course, all voters are indifferent for these platform profiles and therefore each party obtains a vote share of $\frac{1}{2}$ according to either belief for either one of these platform profiles. To see that a deviating party can do better than that consider the profile $\langle(0,0),(0,0)\rangle$ and a deviation to $(0,.5)$ by party one. Voters with ideal points $(0,1), (.5,.5)$ and $(1,1)$

all prefer $(0,.5)$ to $(0,0)$, the remaining voters have the inverse preference - all by single-peakedness. Now, observe that $\psi^1(\{\succsim\colon a(\succsim) \in \{(0,1)(.5,.5),(1,1)\}\}) = \frac{1}{6} + \frac{4}{27} + \frac{2}{27} + \frac{1}{9} = \frac{1}{2}$ and $\psi^2(\{\succsim\colon a(\succsim) \in \{(1,0)(.5,.5),(1,1)\}\}) = \frac{1}{6} + \frac{2}{27} + \frac{1}{3} > \frac{1}{2}$. Therefore, $(0,.5)$ is a better reply for party one and $\langle(0,0),(0,0)\rangle$ cannot be an equilibrium. By symmetry, the profile $\langle(1,1),(1,1)\rangle$ cannot be one either. ■