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PROCRASTINATION IN TEAMS, CONTRACT DESIGN, AND DISCRIMINATION

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Abstract

We study a dynamic model of team production with moral hazard. We show that the players begin to invest effort only shortly before the time limit when the reward for solving the task is shared equally. We explore how the team can design contracts to mitigate this form of procrastination and show that the second-best optimal contract is discriminatory. We investigate how limited liability or the threat of sabotage influences the team's problem. It is further shown that players who earn higher wages can be worse off than teammates with lower wages and that present-biased preferences can mitigate procrastination.

JEL Classification: D82, L22, J71, M52.
Keywords: moral hazard, team production, partnerships, procrastination, contract design, discrimination.

1 INTRODUCTION

Team production is widely used in organizations (Katzenbach and Smith 1993) and it has become increasingly important over the last decades (Wuchty, Jones, and Uzzi 2007). A common problem of team production is that individual contributions cannot be identified, which causes moral hazard (Holmström 1982, Prendergast 1999).

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This paper contributes to the understanding of the dynamic aspects of moral hazard in teams. We examine how the externalities from team production induce not only static, but also rich dynamic effects. We consider a simple model where a team, consisting of perfectly rational and time-consistent players, works on a joint task, which yields a reward when solved.[1]

Our first result is that when the reward is shared equally among players and the time limit for solving the task is sufficiently long, the team procrastinates in the sense that members invest effort only shortly before the time limit. Consider a simple example with two players, who have two periods to solve a task. The task is solved in a period with probabilities 1/2, 1/4, or 0 when respectively both, one, or no player(s) invest(s) effort in this period. Effort causes private costs of 1. Solving the task yields a per-capita reward of 5. In period 2, a player's additional expected payoff from investing is $1/4 \times 5 - 1 > 0$. Hence, both players invest effort, conditional that the task is not yet solved. The expected payoff from period 2 is $1/2 \times 5 - 1 = 3/2$ when the task is not yet solved and zero otherwise. In period 1, a player's additional expected payoff from investing is $1/4 \times (5 - 3/2) - 1 < 0$. Thus, players do not invest in period 1. The key intuition is that team production in the future causes externalities which make investing effort in the present less worthwhile. Procrastination is socially inefficient, but individually rational.[2]

As a benchmark case, we consider the game with a single player. Because the externalities from team production are absent the player does not procrastinate. Consider the example from before. The player invests in period 2 (conditional that the task is not yet solved) because the additional expected payoff from investing is $1/4 \times 5 - 1 > 0$. The expected payoff from period 2 is $1/4 \times 5 - 1 = 1/4$ when the task is not yet solved. The player also invests in period 1 because the additional expected payoff from investing is $1/4 \times (5 - 1/4) - 1 > 0$.

Motivated by the finding that a team may procrastinate when players are remu-

[1] See Cohen and Bailey (1997) for examples how organizations use so called project teams, which are usually time-limited and non-repetitive (p. 242).

[2] O'Donoghue and Rabin (1999) define procrastination as "wait when you should do it" (p. 104). In the example, if both players invest also in period 1, the team's expected payoff is $1/2 \times 10 - 2 + 1/2 \times 3 = 9/2$ instead of just $2 \times 3/2 = 3$ when players procrastinate.

nerated equally, we explore how the team can design a wage contract that mitigates procrastination. Our investigation primarily concerns two questions: which effort profile is implemented, and what contract leads to its implementation? While effort is noncontractible, we allow the team to condition each player's wage in each period on whether the task is solved or not. We require that the team's budget is balanced, which can be understood as a consequence of feasibility and renegotiation-proofness. We show that when the first-best (where all players invest, as long as the task is not solved) is implementable, one can implement it with the equal-sharing contract, where all players receive the same share of the team's reward. When the time limit is sufficiently long, we know from before that players procrastinate with the equal-sharing contract. In this case the first-best is not implementable.

We then investigate the properties of second-best contracts. We first show that the equal-sharing contract is not second-best. The idea is that by discriminating between players (in the sense of remunerating them differently) one can induce a player, or some players, to invest effort in an early period, at a time when under the equal-sharing contract no player would invest. As a general result, we show that a second-best contract has to be discriminatory. To see that discriminatory contracts mitigate procrastination in teams, consider again the previous example. Suppose that when the task is solved in period 1, player 1 receives the team's reward of 10, while player 2 receives nothing. When the task is solved in period 2 each player still receives 5. Then player 1 invests effort in period 1 because her additional expected payoff from investing is $1/4 \times (10 - 3/2) - 1 > 0$. Player 2 does not invest in period 1 and (when the task is not solved) both players invest in period 2. By the aforementioned change of the first-period rewards, the team's expected payoff improves from 3 to $1/4 \times 10 - 1 + 3/4 \times 3 = 15/4$.

We specify a wage contract, called "handsome contract", which induces all players except one to invest in early periods (i.e., periods where the team does not invest with the equal-sharing contract) and all players to invest in late periods. We show that the handsome contract is second-best and renegotiation-proof. There may also exist other contracts which are second-best. But when we require the contract to be renegotiation-proof, all contracts which are second-best implement the same effort profile (i.e., the same number of players to invest in each period) as the handsome contract. We then explore how limited liability or the threat of sabotage restricts the team's contract design problem. We show that both issues can seriously limit the team's possibilities to mitigate procrastination, especially when there are more than two players.

Since our model is easily tractable, we can gain several additional insights. First, when we allow the team to hire a principal the first-best can be implemented when the team members are fully liable. When the team members are protected by limited liability and the time limit is sufficiently long, this is no longer true. The idea is that with a long time limit the wages in early periods have to be huge to convince the team members to invest effort. The principal would then earn a negative expected profit, which makes implementing the first-best impossible.

Second, discriminatory wage contracts often lead to differences in players' well-being. Interestingly, players who earn *higher* wages can be *worse* off than teammates with lower wages. The reason is that players with higher wages may be motivated to invest effort, but teammates with lower wages may not be motivated. Due to team production, the latter benefit from the formers' investments, while they do not have to bear effort costs. This effect can overcompensate the wage differences. The finding has the severe implication that an outside party (e.g., a judge) is in general not able to infer that player A is disadvantaged compared to B when A receives lower wages than B.

Third, the model we consider can also be interpreted as a public-good game. Numerous experiments show that in such games people contribute more than they should if they were selfish, which indicates that people are often not purely selfish.[3] In an extension of our model, we suppose that players have social preferences, in the sense that they take other players' well-being (at least partly) into account. We first show that players begin to invest effort earlier than when they are selfish. That is, procrastination is alleviated by social preferences. Unless players are fully altruistic, however, the players still procrastinate when rewarded equally and the time limit is sufficiently long. We further consider the case where players differ with respect to their social preferences. For this case it is better to have heterogenous teams (where players

[3]For stronger evidence, see, for example, Andreoni (1995).

have different social preferences) than having homogenous teams (where players have the same social preferences).[4]

Fourth, present-bias preferences can mitigate procrastination. The idea is that present-biased players put less weight on the future externalities arising from team production. This insight is highly interesting because many models explain procrastination by means of present-bias preferences (e.g., Akerlof 1991, Laibson 1997, and O'Donoghue and Rabin 1999, 2001).

Related Literature.— Our paper is related to the huge literature on moral hazard in teams, which is pioneered by Alchian and Demsetz (1972) and Holmström (1982).[5] Most of this literature either considers static models or dynamic models with a repetition of a one-period game. A remarkable exception is Bonatti and Hörner (forthcoming). Independently of our work, they developed a model where teams procrastinate. They put a strong emphasize on the effects of uncertainty and learning (they assume that players do not know the production technology) and show that observability of effort highly influences their results. In our model the production technology is known[6] which has the consequence that it is not important whether effort is observable or not. Their main interest is the infinity horizon case, where they characterize the Markov perfect equilibrium when effort costs are linear. We briefly study the infinite horizon case and are able to characterize the Markov perfect equilibrium for our model also for the case when effort costs are nonlinear. An interesting result of our analysis is that a team may benefit from a lower discount factor. Our main focus, however, is the case with a finite horizon, where we study in depth the question how contracts are optimally designed. Contract design is not the focus of Bonatti and Hörner. They briefly study mechanisms for the infinite horizon case and assume that wages have to be nondiscriminatory.[7] As our analysis reveals, discrim-

[4]Interestingly, Hamilton, Nickerson, and Owan (2003) show empirically that teams which are more heterogenous with respect to the abilities of their members are more productive.

[5]We especially contribute to the branch of the literature on moral hazard in teams which explores optimal sharing rules for teams when there is no principal and the team's budget has to be balanced; see Holmström (1982), Radner (1986), Radner, Myerson, and Maskin (1986), Rasmusen (1987), Legros and Matthews (1993), and Strausz (1999).

[6]This is in line with most agency models in the literature.

[7]Another difference is that they require that the team's budget has to be balanced only on average

inatory contracts are key instruments to alleviate procrastination. We also study how renegotiation-proofness, limited liability, or the threat of sabotage influences the team's design problem. Bonatti and Hörner put a strong emphasize on deadlines. We show that in our model designing a discriminating contract is a better measure to mitigate procrastination than a deadline, at least when the discount factor is sufficiently close to one.[8] Because our model allows for a very simple characterization of the solution, we are able to extend our baseline model in several interesting ways. Next to the extensive study how contracts are optimally designed, we explore issues like present-biased preferences and social preferences and team design.

We also contribute to the economic literature on discrimination, which started with Becker (1957). He argues that market forces are effective means to repress discrimination because firms which discriminate are less efficient than those who do not. A problem with this argument is, however, that it cannot explain why discrimination persists (Stiglitz 1973, Arrow 1998). In our model discrimination is necessary to mitigate procrastination in teams and mandatory when implementing the second-best. Winter (2004) provides a static model of team production with moral hazard and an increasing returns to scale production technology. His main result is that when one requires the Nash equilibrium to be unique, the optimal contract is discriminatory. For surveys, see Darity and Mason (1998) and Holzer and Neumark (2000, 2006).

The paper proceeds as follows. In Section 2, we present the basic model, which is then analyzed in Section 3. In Section 4, we explore how the team can design a wage contract that mitigates procrastination and study how constraints like renegotiation-proofness, limited liability, and the threat of sabotage influences the team's problem. We also investigate the case where the team can hire a principal. In Section 5, we offer some extensions. We conclude in Section 6. Proofs are relegated to Appendix A.

(which requires a third party).

[8]Moreover, a problem of deadlines is that they are not renegotiation-proof: players prefer to ignore a deadline once it is reached.

2 THE MODEL

A set of risk-neutral and time-consistent players $\mathcal{N} := \{1, ..., n\}$ works on a joint task. Time is discrete and indexed by $t \in \{1, 2, ...\}$. The team has $T \geq 2$ periods of time to solve the task. The time limit T is potentially very large, but finite.[9] It can be interpreted as the time when solving the task is no longer economically interesting; see Section 5.4.

When the task is solved the team obtains a joint reward of $Z > 0$, which is immediately paid. Initially we assume that the reward is shared equally so that each player receives $z := Z/n$.[10] When the task is not solved in period t, then each player receives 0 in t.

At the beginning of period t, each player i decides how much effort to invest: $e_{t,i} \in \{0, 1\}$, where 0 denotes no effort and 1 effort.[11] Effort costs are private and normalized to $e_{t,i}$. It is useful to define $e_t := \sum_{\mathcal{N}} e_{t,i}$ as the team's investment in effort in period t. The vector $\boldsymbol{e} = (e_1, ..., e_t, ..., e_T)$ is denoted as the effort profile.

The probability that the task is solved in t is $p \times e_t$, where $0 < p < 1/n$.[12] The idea is that when more players invest effort in a period, it is more likely that the task is solved in this period. The effort decisions made by players are not verifiable and therefore not contractible. Because the production technology is known and our arguments rely on backward induction, it is not important whether effort is observable or not by other players.

When the task is solved at the end of period t or we are at the end of period T, the game ends. Otherwise, the next period begins and players decide again how much effort they want to invest. We start with the simplest case where there is no discounting.[13]

Player i's strategy is the plan which describes how much effort she invests in

[9] In Section 5.2, we explore the case when there is no time limit for solving the task.

[10] In Section 4, we investigate how the team optimally shares the reward.

[11] Alternatively, each player chooses effort from the continuum $[0, 1]$. When marginal effort costs are constant, this does not change our analysis.

[12] In Section 5.3, we examine more general success functions.

[13] We consider discounting in Section 3.3.

which period, conditional that this period is reached.[14] Formally, i's strategy is $\boldsymbol{s_i} = (e_{1,i}, ..., e_{t,i}, ..., e_{T,i})$. The solution concept we use is subgame perfect Nash equilibrium.

3 ANALYSIS

Denote player i's continuation payoff of reaching period t by $C_{t,i}$. The continuation payoff can also be interpreted as the expected payoff of the player, from t to T. It depends on the parameters of the game (p, z, n, T) as well as on the players' strategies. Because the game ends, at the latest, after period T, we have $C_{t,i} = 0 \; \forall t > T, i \in \mathcal{N}$.

Player i's continuation payoff at period t is

$$C_{t,i} = e_t pz + (1 - e_t p) C_{t+1,i} - e_{t,i}. \tag{1}$$

The interpretation of this Bellman equation is as follows: the player's expected payoff equals the probability of success in t times the per-capita reward which is then paid, plus the probability of failure in t times the continuation payoff of the next period minus the effort costs which eventually accrue.

We assume as a tie-breaking rule that a player invests in case of indifference.[15] Player i maximizes her expected payoff over her effort choice. From (1) the following lemma is immediate.

L E M M A 1: *Player i invests effort in t if and only if*

$$p\left(z - C_{t+1,i}\right) \geq 1.$$

Intuitively, player i's incentive constraint is satisfied if and only if the additional payoff from success, namely $(z - C_{t+1,i})$, times the additional probability that there is success when she invests effort, namely p, is at least as great as her private effort costs of 1.

Iterative use of (1) yields

$$C_{t,i} = \sum_{s=t}^{T} \left[(e_s pz - e_{s,i}) \prod_{r=t}^{s-1} (1 - e_r p) \right]. \tag{2}$$

[14] We can allow for mixed strategies, but this does not contribute to the point we want to make.
[15] This assumption is not crucial. Instead, one can also assume that a player does not invest in case of indifference.

Because a player cannot obtain z for sure and effort costs are nonnegative, $C_{t,i} < z$ for all $t \in \{1, ..., T\}, i \in \mathcal{N}$. From (1) we see that this implies that a player always benefits when another player invests effort.

It is useful to denote the continuation payoff of reaching period t, given that all players always invest from t until T, as $\hat{C}_{t,i}$. Throughout the paper we mean by investing in some future period, investing conditional that this period is reached because the task is not yet solved. From (2) we get that

$$\hat{C}_{t,i} = (npz - 1)\frac{1 - (1 - np)^{T-t+1}}{np}. \tag{3}$$

Because a player can always decide not to invest, a continuation payoff is never negative. Together with Lemma 1, this implies that a necessary condition for investment is the following.

A S S U M P T I O N 1: *$pz \geq 1$.*

Throughout the paper we maintain Assumption 1. We briefly discuss it in Section 4.6.3. Assumption 1 implies that in the static game ($T = 1$) all players invest effort.

We first look at the benchmark case, where the social planner solves the problem.

P R O P O S I T I O N 1: *For the special case where $n = 1$ and $pZ = 1$ welfare is zero for all investment profiles. Otherwise, in the first-best all players always invest effort. Moreover, $\sum_{\mathcal{N}} C_{t,i}$ increases with a higher level of effort e_s, where $s \geq t$.*

Intuitively, welfare is maximized when players use every chance to solve the task, i.e., always invest effort when the task is not solved.

3.1 SINGLE PLAYER

Suppose there is a single player. Because $C_{T+1,1} = 0$, Lemma 1 implies that the player invests in T if and only if $pz \geq 1$, which holds because of Assumption 1.

To determine the player's decisions in the other periods we have to determine whether or not the following inequality holds for all $t \in \{1, ..., T\}$

$$p\left(z - \hat{C}_{t,1}\right) \overset{?}{\geq} 1. \tag{4}$$

For the limit case we yield, using (3),

$$p\left(z - \lim_{T\to\infty} \hat{C}_{t,1}\right) = 1. \tag{5}$$

Note that $\hat{C}_{t,1}$ is increasing in T. Therefore, for any finite T we have

$$p\left(z - \hat{C}_{t,1}\right) \geq 1. \tag{6}$$

From before, we already know that the player will invest in T. Given this, $C_{T,1} = \hat{C}_{T,1}$. Therefore, Lemma 1 and (6) imply that it is optimal for the player to invest in $T-1$. Then $C_{T-1,1} = \hat{C}_{T-1,1}$ and it is again optimal for the player to invest in $T-2$. These arguments can be repeated and imply the following proposition.

PROPOSITION 2: *A single player always invests effort.*

The intuition is that the player always invests, because there are no externalities from team production and so her continuation payoff is rather small in early periods also.

3.2 SEVERAL PLAYERS

Consider now the case with $n \geq 2$ players. From Lemma 1 we see that players invest in T because $C_{T+1,i} = 0$. Similar to the case with a single player, we are interested in whether or not the following inequality holds:

$$p\left(z - \hat{C}_{t,i}\right) \overset{?}{\geq} 1. \tag{7}$$

In the limit case we have, using (3),

$$p\left(z - \lim_{T\to\infty} \hat{C}_{t,i}\right) = p\left(z - (npz-1)\frac{1}{np}\right) = \frac{1}{n} < 1. \tag{8}$$

Therefore, when the time limit is sufficiently long it cannot hold that players always invest effort.

We want to explore exactly when players invest effort and when not. The following lemma is useful.

LEMMA 2: *(i) Players choose the same investment in period t. This holds for all $t \in \{1, ..., T\}$.*

(ii) When players invest no effort in t *then* $C_{t,i} = C_{t+1,i}$*.*

(iii) When players invest effort in t *then* $C_{t,i} > C_{t+1,i}$*.*

(iv) $C_{t,i} = \hat{C}_{t,i}$ *when players always invest from* t *on and* $C_{t,i} < \hat{C}_{t,i}$*, otherwise.*

The intuition for part (i) is that players are symmetric and therefore always choose the same investment. That is, either all players invest effort or not in a specific period. The reason why part (ii) holds is that when nothing happens in period t, then the expected payoff at the beginning of period t is the same as at the beginning of period $t+1$. Because investing effort by all agents is beneficial, in the sense that all are better off, part (iii) holds. The intuition for part (iv) is that when players invest less than in the first-best, their expected payoffs suffer.

PROPOSITION 3: *(i) When the time limit is sufficiently short, namely* $T \leq \bar{x}+1$*, where*

$$\bar{x} = \frac{\ln(n-1) - \ln(npz-1)}{\ln(1-np)},$$

then players always invest effort.

(ii) When the time limit is sufficiently long, namely $T > \bar{x}+1$*, then players do not invest effort in periods* $t < T - \bar{x}$ *and invest effort in the periods* $t \geq T - \bar{x}$*.*

So when the time limit is long, players will invest if and only if there are sufficiently few periods left. This is in stark contrast to the findings with a single player, who always invests effort. Intuitively, when there is team production, each player knows that shortly before the time limit all players will invest effort. Then team production generates externalities. Therefore, not investing is very attractive in early periods, because this allows the player to exploit the future externalities arising from team production. Technically, due to the externalities of team production the continuation payoffs are too large to always sustain investment in effort.

Note that Proposition 1 implies that all players would be better off if all invest at all times. Because players' inactivity is collectively harmful to them, we also call inactivity procrastination. Proposition 3 can be rephrased as follows: when the time limit is sufficiently long, the team procrastinates. Procrastination is socially inefficient, but individually rational because in the early periods of a game, a player is individually better off when she does not invest.

Note that the reason why it is optimal for a player not to invest in the early periods is not that the player hopes that some other player will invest effort and eventually solve the task. Each player is fully aware that the other players will, like her, not invest effort in early periods.

We are interested in the comparative statics.

PROPOSITION 4: *$d\bar{x}/dz > 0$. Fixing z, $d\bar{x}/dn < 0$. Fixing Z, $d\bar{x}/dn < 0$.*

In words, with a higher per-capita reward z procrastination is less likely[16] and players begin to invest earlier. Intuitively, the advantage of not investing today is that future externalities arising from team production can be exploited. The drawback is that it is less likely that the reward is ever received. With a higher per-capita reward the latter aspect becomes more important.

The intuition why procrastination is more likely and players begin to invest later when the number of players increases is that then the aforementioned externalities from team production become more important. Technically, the continuation payoff when all players invest is increasing in the size of the team. The effect of a higher success parameter p on $\bar{x}$ is ambiguous.

One can yield the following result regarding the likelihood that the task is ever solved.

PROPOSITION 5: *A single player is more likely to solve the task than $n \geq 2$ players if T is sufficiently large. It is the other way round when T is sufficiently small.*

The intuition is simple. Because a single player always invests, the likelihood that she solves the task sometimes approaches 1 when the time limit is very long. With several players this is not true, because they invest only in the last few periods.

3.3 DISCOUNTING

In this section, we explore how players invest when they discount future payoffs. We first consider the case where players are exponential discounters. Then we study

[16]Less likely in the sense that it occurs for a smaller parameter set.

the case where players are time inconsistent and their intertemporal preferences are described by quasi-hyperbolic discounting.

3.3.1 EXPONENTIAL DISCOUNTING

Suppose that players are time consistent and use a per-period discount factor of $\delta \in (0, 1]$. As in the case without discounting a social planner directs players to always invest effort.

P R O P O S I T I O N 6: *In the first-best players always invest effort.*

In period t, player i's expected utility is

$$U_{t,i} = E_t \left[\sum_{s=t}^{T} \delta^{s-t} u_{s,i} \right], \tag{9}$$

where $\delta \in (0, 1]$ and $u_{s,i}$ is the instantaneous utility experienced in period s.

The Bellman equation (1) gets

$$C_{t,i} = e_t pz + (1 - e_t p)\delta C_{t+1,i} - e_{t,i}. \tag{10}$$

Iteratively using it yields

$$C_{t,i} = \sum_{s=t}^{T} \left[(e_s pz - e_{s,i})\, \delta^{s-t} \prod_{r=t}^{s-1} (1 - e_r p) \right] \tag{11}$$

and

$$\hat{C}_{t,i} = (npz - 1) \frac{1 - (\delta(1 - np))^{T-t+1}}{1 - \delta(1 - np)}. \tag{12}$$

From (10), it is optimal for player i to invest in period t if and only if

$$p(z - \delta C_{t+1,i}) \geq 1. \tag{13}$$

The next proposition says that discounting weakly improves the players incentives to invest.

P R O P O S I T I O N 7: *With discounting players invest in all periods where they would also invest without discounting and they may invest in more periods.*

The intuition is that discounting lowers the present value of reaching future periods. Thereby investing effort in the present gets more attractive. Proposition 7 implies together with Proposition 2 that a single player always invests effort also when there is discounting.

Another result, see the next proposition, is that for low discount factors δ there is no procrastination. The intuition is that for low discount factors players care little about the future externalities arising from team production.

P R O P O S I T I O N 8: *When* $\delta \leq \bar{\delta} := \frac{(pz-1)}{(pz-1)+p(n-1)}$ *then players always invest effort. When* $\delta > \bar{\delta}$ *and* T *is sufficiently large, players do not always invest effort.*

The following result says that with discount factors close to 1 nothing changes compared to the case without discounting.

P R O P O S I T I O N 9: *Generically, for every game* (p, z, n, T) *there exists a* $\grave{\delta} < 1$ *such that for all* $\delta \in (\grave{\delta}, 1)$ *the equilibrium is the same as without discounting* ($\delta = 1$)*.*

When the periods (or even the time limit) are short it is reasonable to assume that the discount factor is close to 1. Then discounting does, at least qualitatively, not change much, cf. Propositions 8 and 9.

3.3.2 QUASI-HYPERBOLIC DISCOUNTING

We next consider the case where players are time-inconsistent and present-biased. Formally, we assume that players have $\beta - \delta$ preferences, see Phelps and Pollak (1968) and Laibson (1997):

$$U_{t,i} = E_t \left[u_{t,i} + \beta \sum_{s=t+1}^{T} \delta^{s-t} u_{s,i} \right], \tag{14}$$

where $\beta, \delta \in (0, 1]$. The Bellman equation (1) gets

$$C_{t,i} = e_t pz + (1 - e_t p)\delta\beta C_{t+1,i} - e_{t,i}. \tag{15}$$

Hence, player i invests in period t if and only if

$$(p(z - \delta\beta C_{t+1,i}) \geq 1. \tag{16}$$

We first consider the case where players are aware of the time-inconsistency problems. Then players' planned investments in effort and their actual investments coincide. From (16) we see that player i is, all else equal, more eager to invest effort when $\beta < 1$ than when $\beta = 1$. The reason therefore is that, because the reward Z is payed instantaneously to players in case of success, quasi-hyperbolic discounting (at least weakly) improves players incentives to invest today.[17]

PROPOSITION 10: *When $\beta \leq \bar{\beta} := \frac{(z-1/p)(1-\delta(1-np))}{\delta(npz-1)}$ players always invest effort. When $\beta > \bar{\beta}$ and T is sufficiently large, then players do not always invest effort.*

The threshold is minimized—i.e., most difficult to undercut—for $\delta = 1$, in which case $\bar{\beta} = \frac{pz-1}{pz-1/n}$. When for example, $z = 12$, $n = 2$, and $p = 1/4$, then $\bar{\beta} = 4/5$, which is above the level estimated by Laibson, Repetto, and Tobacman (2007). Although this is not the case for all parameter constellations, quasi-hyperbolic discounting is clearly a factor which can mitigate or even prevent procrastination in teams.

When players are not aware of their time-inconsistency problem they believe that their future behavior coincides with the one of players who have an exponential discount function (i.e., $\beta = 1$), see Section 3.3.1. The same holds true for the believes about the other players' investments when a player is not aware of the other players' time-inconsistency problems. All this provides another channel why a player invests: she erroneously believes that she and the other players will not invest in some future periods; this wrong belief leads to an underestimation of the continuation payoff; as we see from (16), this has the effect that the player is more eager to invest effort.

[17] When the reward is payed with delay this argument no longer holds because then also the reward is discounted with factors δ and β.

4 CONTRACT DESIGN

Previously, we assumed that the players' rewards are exogenously given. We now assume that the team can fix at period 0 a contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ with

$$\boldsymbol{w^S} = \begin{pmatrix} w^S_{1,1} & \dots & w^S_{1,n} \\ \dots & \dots & \dots \\ w^S_{T,1} & \dots & w^S_{T,n} \end{pmatrix}, \ \boldsymbol{w^F} = \begin{pmatrix} w^F_{1,1} & \dots & w^F_{1,n} \\ \dots & \dots & \dots \\ w^F_{T,1} & \dots & w^F_{T,n} \end{pmatrix}, \tag{17}$$

where $w^S_{t,i}$ is the wage which player i receives when there is success in period t and $w^F_{t,i}$ when there is failure. For the special case of equal sharing, $w^S_{t,i} = Z/n$ and $w^F_{t,i} = 0$.

How does the team optimally design such a contract? Are there contracts which alleviate procrastination? We consider the case where there is no discounting, because then procrastination is most likely and therefore most difficult to mitigate.

Player i's continuation payoff at t is given by the following Bellman equation:

$$C_{t,i} = e_t p w^S_{t,i} + (1 - e_t p)\left(w^F_{t,i} + C_{t+1,i}\right) - e_{t,i}. \tag{18}$$

Iterative using this equation yields

$$C_{t,i} = \sum_{s=t}^{T} \left[\left(e_s p w^S_{s,i} + (1 - e_s p) w^F_{s,i} - e_{s,i}\right) \prod_{r=t}^{s-1} (1 - e_r p) \right]. \tag{19}$$

From (18) we see that it is optimal for player i to invest effort in t if and only if

$$p\left(w^S_{t,i} - w^F_{t,i} - C_{t+1,i}\right) \geq 1, \tag{20}$$

which is a generalization of Lemma 1.

So that the wage contract is feasible we must have

$$\sum_{\mathcal{N}} w^S_{t,i} \leq Z, \ \sum_{\mathcal{N}} w^F_{t,i} \leq 0. \tag{21}$$

Because it is ex post not in the interest of the players to waste resources (cf. Holmström 1982, p. 327), any contract which does this would be renegotiated by the players. Therefore, we require that the team's budget is not only feasible, but balanced:

$$\sum_{\mathcal{N}} w^S_{t,i} = Z, \ \sum_{\mathcal{N}} w^F_{t,i} = 0. \tag{22}$$

From (18) we get that with a balanced budget

$$\sum_{\mathcal{N}} C_{t,i} = e_t pZ + (1 - e_t p) \sum_{\mathcal{N}} C_{t+1,i} - e_t. \tag{23}$$

Iterative use of this equation yields

$$\sum_{\mathcal{N}} C_{t,i} = (pZ - 1) \sum_{s=t}^{T} \left[e_s \prod_{r=t}^{s-1} (1 - e_r p) \right]. \tag{24}$$

The first part is the expected payoff when one additional player invests in one period, which consists of the additional expected payoff of pZ minus the effort costs of 1. The sum is the expected number of players who invest from t until T, taking into account that the task may be solved before some period s.

We suppose that the team designs the contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ such that the team's expected payoff $\sum_{\mathcal{N}} C_{1,i}^{(\boldsymbol{w^S},\boldsymbol{w^F})}$ is maximized. When two contracts yield different expected payoffs, then the team is able to make transfers such that all players are better off under the contract with the higher expected payoff. Alternatively, the players' names can be drawn from an urn; then all players are in expectation better off under the contract with the higher expected payoff.

DEFINITION 1: *A contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ is better than another contract $(\boldsymbol{w^{S'}}, \boldsymbol{w^{F'}})$ if $\sum_{\mathcal{N}} C_{t,i}^{(\boldsymbol{w^S},\boldsymbol{w^F})} > \sum_{\mathcal{N}} C_{t,i}^{(\boldsymbol{w^{S'}},\boldsymbol{w^{F'}})}$.*

Note that because wages are just transfers between players, Proposition 1 stays valid also when the team can design the wage contract. That is, the team's expected payoff improves when more players invest and is maximized when all players always invest.

4.1 THE FIRST-BEST

We now explore when the first-best is implementable and when not.

PROPOSITION 11: *When the first-best is implementable with some contract it is also implementable with the equal-sharing contract $w_{t,i}^S = Z/n$ and $w_{t,i}^F = 0$. The first-best is implementable if and only if $T \leq \bar{x} + 1$, where $\bar{x}$ is given in Proposition 3.*

That is, when the first-best is implementable the team can use the simple equal-sharing contract. The idea of the proof is that when the equal-sharing contract does not implement the first-best, we cannot find a contract that implements the first-best because the team's budget has to be balanced and therefore not all players can be incentivized to invest effort in all periods.

4.2 THE SECOND-BEST

When $T > \bar{x} + 1$, the first-best is not implementable and we can only look for a second-best contract. What does the second-best contract look like? We are first interested in whether or not the equal-sharing contract is second-best. Suppose that initially the equal-sharing contract $w^S_{t,i} = Z/n$ and $w^F_{t,i} = 0$ is used. Then, because the first-best is not implementable the team does not invest in the early periods $t < T - \bar{x}$, see Proposition 3. When the equal-sharing contract is modified for period 1 and players 1 and 2, so that $w^S_{1,1}$ is huge and $w^S_{1,2} = -w^S_{1,1} + 2Z/n$, player 1 will invest effort in $t = 1$. Due to this modification, the investments in all periods are the same as with the initial equal-sharing contract, except for period 1 where player 1 invests instead of nobody. From Proposition 1 we know that this is an improvement for the team.

DEFINITION 2: *A wage contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ is nondiscriminatory if $w^S_{t,i} = w^S_{t,j}$ and $w^F_{t,i} = w^F_{t,j}$ holds for all $t \in \{1, ..., T\}$ and for all $i, j \in \mathcal{N}$. Otherwise, the contract is discriminatory.*

Because the only nondiscriminatory contract which fulfills budget balance is the equal-sharing contract, the aforementioned insights imply that the second-best contract has to be discriminatory.[18] Proposition 12 summarizes.

PROPOSITION 12: *When the first-best is not implementable, the equal-sharing contract is not second-best. A second-best contract has to be discriminatory.*

[18]Note that although our theory predicts that it is possibly optimal to discriminate between players, it does not predict that discrimination has to condition on personal characteristics like gender, age, race, religion, sexual orientation, or social group.

An attractive contract is the following. For the periods $t \geq T - \bar{x}$ impose the equal-sharing contract. For early periods, set $w^S_{t,i} - w^F_{t,i}$ sufficiently large for all players $\{2, ..., n\}$, so that these players invest effort, and set player 1's wages $w^S_{t,1}$ and $w^F_{t,1}$ to balance the budget. Then, in the early periods all except one player invest and for later periods all players invest. We call the aforementioned contract *handsome contract*. Can one find another contract which is better? The answer is no.

PROPOSITION 13: *The handsome contract is second-best. It implements $e_t = n$ for periods $t \geq T - \bar{x}$ and $e_t = n - 1$ for periods $t < T - \bar{x}$.*

In a handsome contract player 1 acts, in a sense, as a principal in the early periods $t < T - \bar{x}$, where she breaks the budget conditions for the subteam consisting of players $\{2, ..., n\}$. Thereby she can induce all players of the subteam to invest. In the late periods $t \geq T - \bar{x}$, breaking the budget balance condition is no longer necessary and player 1 invests effort herself.

Consider the example from the beginning ($Z = 10$, $n = 2$, $p = 1/4$, $T = 2$). An example for a handsome contract (there are in general many handsome contracts) is the following:

$$\boldsymbol{w^S} = \begin{pmatrix} 3 & 7 \\ 5 & 5 \end{pmatrix}, \ \boldsymbol{w^F} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{25}$$

It is readily verified that player 1 only invests in $t = 2$, while player 2 always invests. The effort profile is $(1, 2)$.

There are also wage contracts which induce a different effort profile and which are second-best as well, for example:

$$\boldsymbol{w^S} = \begin{pmatrix} 5 & 5 \\ 3 & 7 \end{pmatrix}, \ \boldsymbol{w^F} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{26}$$

Then player 1 only invests in $t = 1$, while player 2 always invests. The effort profile is then $(2, 1)$. This contract is, however, not renegotiation-proof. At the beginning of period 2 players can improve. With contract (26) each player's expected payoff in period 2 is $3/4$. By writing a new contract $w^S_{2,1} = w^S_{2,2} = 5$ and $w^F_{2,1} = w^F_{2,2} = 0$ both players invest effort in period 2 and each player's expected payoff improves to $3/2$.

When players are able to renegotiate contracts, considering renegotiation-proof contracts is without loss of generality. The reason is that, because the players foresee renegotiations, one can directly set wages so that renegotiations are superfluous.

PROPOSITION 14: *The handsome contract is renegotiation-proof. Any second-best contract which is renegotiation-proof induces the same effort profile as the handsome contract.*

Although there are usually several second-best contracts which are renegotiation-proof, this result says that all these contracts implement the same effort profile.

4.3 LIMITED LIABILITY

In this section, we explore whether or not limited liability restricts the team's optimization problem. Suppose that players have no wealth, which causes their liability to be limited, and therefore requires wages to be nonnegative: $w^S_{t,i}, w^F_{t,i} \geq 0 \; \forall t \in \{1, .., T\}, i \in \mathcal{N}$. This implies, together with the budget balance condition (22) that $w^F_{t,i} = 0 \; \forall t \in \{1, .., T\}, i \in \mathcal{N}$.

Observe that when the first-best is implementable without limited liability, it is implementable with the equal-sharing contract; see Proposition 11. Because this contract respects limited liability, it is then also implementable with limited liability.

PROPOSITION 15: *When the first-best is implementable without limited liability, it is also implementable with limited liability.*

Next, suppose that the first-best is not implementable even without limited liability. That is, $T > \bar{x}+1$. Is the second-best without limited liability also implementable when there is limited liability? We require the contract to be renegotiation-proof. By Proposition 14, we therefore can concentrate on the second-best contract where all players invest in the late periods $t \geq T - \bar{x}$ and $n-1$ players invest effort in the early periods $t < T - \bar{x}$. The following proposition says that when there are two players the limited liability constraint does not matter.

PROPOSITION 16: *With two players, the second-best without limited liability is also implementable with limited liability.*

The idea is that limited liability is no problem when one designs a contract which lets players invest alternatingly in the early periods.

For the case with three or more players we have the following result.

PROPOSITION 17: *When there are $n \geq 3$ players, T is sufficiently large, and*

$$Z < \underline{Z} := n(n-2)/p + 1/p - n(n-1)(n-2)/2,$$

then the second-best without limited liability is not implementable with limited liability.

For example, when $n = 3$ and $p = 0.1$, then $\underline{Z} = 37$. Assumption 1 only requires that $Z \geq 30$. So when, for example, $Z = 35$, Proposition 17 says that the second-best is not implementable when T is sufficiently large.

For many parameter constellations $Z \geq \underline{Z}$ is not only a necessary condition, but also sufficient for the second-best without limited liability to be implementable with limited liability. This is, however, not true in general. This can be seen from the following example.

An Example.— Suppose $n = 3$, $p = 0.3$, and $T = 2$. Then $\underline{Z} = 10\frac{1}{3}$. In the second-best without limited liability, we have $e_2 = 3$ and $e_1 = 2$. Suppose, without loss of generality, that player 1 only invests in period 2, while the other players always invest. Then the incentive constraint (20) requires that in period 2 we have $w^S_{2,i} \geq 10/3$ for $i = 1, 2, 3$. The continuation payoff is then $C_{2,i} = e_2 p w^S_{2,i} - 1 \geq 2$ for $i = 1, 2, 3$. Given this, the incentive constraint of period 1 requires that $w^S_{1,2}, w^S_{1,3} \geq 16/3$. Because due to limited liability $w^S_{1,1} \geq 0$, this implies that $\sum_{\mathcal{N}} w^S_{1,i} \geq 32/3 = 10\frac{2}{3} > 10\frac{1}{3} = \underline{Z}$.

4.4 SABOTAGE

Sharing information and experiences are often central elements to improve the productivity in teams. Up to now we have assumed that these aspects do not play a role or are not influenced by the form of the contract. We now suppose that a player can deteriorate the productivity of the team by strategically withholding information and experiences. We look at the extreme scenario where a single player can make success impossible in a period. See Lazear (1989) for a static model of team production with sabotage and Itoh (1991) for a model with helping effort, which can be interpreted as negative sabotage.

We suppose that each player can destruct a success of the team by sabotage. This causes private effort costs for the player of $\kappa \geq 0$.[19] It is optimal for player i not to sabotage if and only if the expected payoff from doing so is at least as great as from sabotage:[20]

$$w_{t,i}^S \geq C_{t+1,i} + w_{t,i}^F - \kappa, \tag{27}$$

which can be rewritten as

$$w_{t,i}^S - w_{t,i}^F + \kappa \geq C_{t+1,i}. \tag{28}$$

We call this the "no sabotage constraint".

When some player will sabotage it is no longer optimal for the other players to invest effort because effort is costly. Given that no player will sabotage, it is optimal for player i to invest effort if and only if (20) is satisfied, which can be rewritten as

$$w_{t,i}^S - w_{t,i}^F - \frac{1}{p} \geq C_{t+1,i}. \tag{29}$$

Observe that when this inequality is satisfied, (28) is satisfied, too, and player i has no incentive to sabotage.

When there is sabotage, then no success is possible. Therefore, we concentrate on contracts $(\boldsymbol{w^S}, \boldsymbol{w^F})$ where there is no sabotage. Summing (28) and (29) over all players we get that

$$\sum_{\mathcal{N}} w_{t,i}^S - \sum_{\mathcal{N}} w_{t,i}^F + (n - e_t)\kappa - e_t \frac{1}{p} \geq \sum_{\mathcal{N}} C_{t+1,i}. \tag{30}$$

Using the budget balance conditions (22) yields

$$e_t \leq \frac{Z + n\kappa - \sum_{\mathcal{N}} C_{t+1,i}}{\kappa + 1/p}. \tag{31}$$

Players maximize the team's expected payoff by setting e_t as high as possible. The highest e_t which is compatible with (31) and $e_t \leq n$ is

$$e_t = \min\left\{ \lfloor \frac{Z + n\kappa - \sum_{\mathcal{N}} C_{t+1,i}}{\kappa + 1/p} \rfloor, n \right\}, \tag{32}$$

[19]One can easily reformulate the problem and assume that a player has to invest in sabotage without yet knowing whether or not there would be success in this period without sabotage.

[20]It is not important for our results how cases of indifference are solved.

where $\lfloor a \rfloor$ is the floor function which yields the largest integer not greater than a. The team can indeed implement this level of e_t because it can construct a contract such that the incentive constraint holds for e_t players and the no sabotage constraint for $n - e_t$ players.

When we are close to the time limit, then $\sum_{\mathcal{N}} C_{t+1,i}$ is relatively low and (31) does not have much bite. For example, when $t = T$, then all players invest effort with an equal-sharing contract and no player has a reason to sabotage. On the other extreme, when T is large relative to t, $\sum_{\mathcal{N}} C_{t+1,i}$ is close to $Z - 1/p$ when from t until T at least one player invests in every period.[21] Then (31) gets

$$e_t \lessapprox \frac{np\kappa + 1}{p\kappa + 1}. \tag{33}$$

The right-hand-side is in the interval $[1, n)$. This implies that when there are two players, the team can only implement that one player invests effort. For larger teams the same holds true when the costs of sabotage are sufficiently low: the right-hand-side is lower than 2 for $\kappa < \frac{1}{p(n-2)}$. In contrast, without sabotage (and with full liability) at least $n - 1$ players can be induced to invest effort in every period. To sum up, the possibility that players can sabotage the team's success has the effect of seriously restricting the team's desire to implement high levels of effort e_t, especially when there are more than two players.

An Example.— Suppose that $Z = 24$, $n = 3$, $p = 1/4$, and $\kappa = 0$. We solve the problem by backward induction, which ensures renegotiation-proofness. Throughout we set $w^F_{t,i} = 0$ and show that this is optimal.

Period $t = T$: Set $w^S_{T,i} = Z/3 = 8$ for all players. Then the incentive constraint (29) is satisfied for all players. Hence, $e_T = 3$, $C_{T,i} = 5$, and $\sum_{\mathcal{N}} C_{T,i} = 15$.

Period $t = T - 1$: From (31) we see that $e_{T-1} \leq 9/4$. Hence, we can at most implement $e_{T-1} = 2$. We set $w^S_{T-1,1} = 6$ and $w^S_{T-1,2} = w^S_{T-1,3} = 9$. For player 1 the no sabotage constraint (28) is satisfied. For players 2 and 3 the incentive

[21]To see this, observe that with $e_t = 1$ for all t we get with (3) that $\sum_{\mathcal{N}} C_{t,i} = \hat{C}^{n=1}_{t,1} = (pZ - 1)\frac{1-(1-p)^{T-t+1}}{p}$. For $\lim_{T\to\infty} C^{n=1}_{t,1} = Z - 1/p$. Moreover, from Proposition 1 we know that higher investments increase welfare which implies that when $e_t \geq 1$ for all t and sometimes $e_t > 1$, $\sum_{\mathcal{N}} C_{t,i} > \hat{C}^{n=1}_{t,1} = (pZ - 1)\frac{1-(1-p)^{T-t+1}}{p}$. Finally, Proposition 1 and (3) imply that $\sum_{\mathcal{N}} C_{t,i}$ cannot exceed $Z - 1/p$.

constraint (29) is satisfied. Hence, $e_{T-1} = 2$, $C_{T-1,1} = 5.5$, $C_{T-1,2} = C_{T-1,3} = 6$, and $\sum_{\mathcal{N}} C_{T-1,i} = 17.5$.

Periods $t = T-2$: From (31) $e_{T-2} \leq 13/8$. Therefore, we can at most get $e_{T-2} = 1$. We set $w^S_{T-2,1} = 5.5$, $w^S_{T-2,2} = 6$, and $w^S_{T-2,3} = 12.5$. For players 1 and 2 the no sabotage constraint (28) is satisfied. For player 3 the incentive constraint (29) is satisfied. Hence, $e_{T-2} = 1$, $C_{T-2,1} = 5.5$, $C_{T-2,2} = 6$, $C_{T-2,3} = 53/8$, and $\sum_{\mathcal{N}} C_{T-2,1} = 145/8$.

Periods $t < T-2$: Because $\sum_{\mathcal{N}} C_{t,i}$ is nonincreasing in t, we cannot get $e_t \geq 2$, see (31). Using the wage contracts from period $T-2$ we get that players 1 and 2 do not sabotage. Player 3 always invests effort because one can show that $C_{t,3} \leq w^S_{T-2,3} - 1/p = 8.5$ even when T is large and so the incentive constraint (29) is satisfied. Hence, $e_t = 1$ for all $t < T-2$.

Finally, when we set $w^F_{t,i} \neq 0$ in some periods for some players (31) stays unaffected for a balanced budget. Hence, we cannot yield a better contract.

4.5 THE MODEL WITH A PRINCIPAL

Suppose the team hires a principal.[22] This allows the team to get rid of the budget balance constraints. That is, it is possible to have a contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ with $\sum_{\mathcal{N}} w^S_{t,i} \gtreqless Z$ and $\sum_{\mathcal{N}} w^F_{t,i} \gtreqless 0$. We assume that the team has all the bargaining power and that the contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ is conducted when the team hires the principal. Furthermore, the principal is not wealth-constraint, risk-neutral, and has an outside option of zero. We suppose that the principal receives the reward Z when the task is solved. This is without loss of generality, because the wage contract can be specified in a way that the principal pays some or all of the reward back to the team. We consider two cases. First the case with full liability of the team members, then the case with limited liability. We explore whether or not it is possible to implement the first-best.

[22]When the principal hires a team this does not change our results when team members are fully liable. When their liability is limited, it is easier to obtain the result that the first-best is not implemented, because the principal is not willing to pay wage bills exceeding the reward.

4.5.1 FULL LIABILITY

It is straightforward that, because of the players' risk neutrality and there is no limited liability constraint, the team can hire a principal to induce the first-best.

P R O P O S I T I O N 18: *When team members are fully liable the team can hire a principal to implement the first-best.*

The idea of the proof is that the team can set $\{w_{t,i}^S - w_{t,i}^F\}$ sufficiently high to incentivize all team members to invest effort. The principal receives zero expected profits when the levels of $\{w_{t,i}^S\}$ and $\{w_{t,i}^F\}$ are appropriately chosen. Because the first-best is implemented and the principal receives zero expected profits the team's expected payoff is maximized.

4.5.2 LIMITED LIABILITY

We next consider the case where the team members are protected by limited liability. We suppose that the reason for limited liability is that they have initially no wealth. Then they are unable to pay the principal a transfer before the reward is eventually realized. A necessary condition that the team can hire a principal is that the principal's expected profit is nonnegative; otherwise the principal would not participate.

In the first-best we have $e_t = n$ for all $t \in \{1, ..., T\}$. Because we seek to show that there is eventually no wage contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ for which the first-best is implemented and the principal's expected profit is nonnegative we set wages as low as possible, given the incentive and the limited liability constraints: $w_{t,i}^F = 0$ and $w_{t,i}^S = C_{t+1,i} + 1/p$.

By backward induction we yield that[23]

$$C_{T-x,i} = (x+1)(n-1), \; w_{T-x,i}^S = x(n-1) + 1/p. \tag{34}$$

The principal's expected profit at the beginning of the game is

$$\sum_{s=1}^{T} \left(e_s p \left[Z - \sum_{\mathcal{N}} w_{s,i}^S \right] \prod_{r=1}^{s-1} (1 - e_r p) \right). \tag{35}$$

[23] Also in Mason and Välimäki's (2008) model of dynamic moral hazard with a single agent the wage is decreasing over time.

For the case where the first-best is implemented and $w^S_{T-x,i} = x(n-1) + 1/p$ we get

$$\sum_{s=1}^{T} \left(np \left[Z - n((T-s)(n-1) + 1/p) \right] (1-np)^{s-1} \right). \tag{36}$$

Hence, when T is sufficiently large the principal's expected profit is negative.

PROPOSITION 19: *With limited liability, the first-best cannot be implemented with a principal when T is sufficiently large.*

Intuitively, when T is large the early wages $w^S_{t,i}$ have to be huge to convince players to invest effort. Then, in case of success, the principal has to pay wages which exceed the reward Z by a large extent. Because it is quite likely that the principal has to pay these huge wages, her expected profit is negative.

4.6 FURTHER RESULTS

In this section, we use our previous insights to obtain further results concerning (i) deadlines, (ii) discrimination and players' payoffs, and (iii) team size and the relevance of Assumption 1.

4.6.1 DEADLINES

We first consider the case where the team's reward is shared equally. Then with discounting, procrastination in teams has two negative effects on welfare. First, as without discounting, the total effort is too low because the team does not always invest, which is suboptimal. Second, the team invests too late. Holding the number of periods in which players invest effort constant, the team would be better off when players invest earlier. This can be achieved by a deadline, i.e., an artificially short time limit of $T^{deadline} < T$. Specifically, when δ is sufficiently close to one and $T > \bar{x} + 1$, then we know from Propositions 3 and 9 that players do not invest effort in the early periods $t < T - \bar{x}$. Then setting a deadline $T^{deadline} = \bar{x} + 1$ has the effect that players no longer procrastinate and invest for the same number of periods as without the deadline. Hence, the present value of the team's expected payoff increases. One may therefore conclude that deadlines are beneficial for a team. There are, however, two problems with this argument.

First, deadlines are not renegotiation-proof. When the team arrives at period $T^{deadline}$ and has no success, the expected payoff from obeying the deadline is zero, while ignoring the deadline and continuing yields a positive expected payoff.

Second, designing an appropriate contract is a better measure to alleviate procrastination than a deadline, at least when δ is sufficiently close to one. We next prove this claim. Denote the present value of the team's expected payoff, measured at the beginning of the game with the deadline, by $\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}}$. A necessary condition for $T^{deadline}$ to be optimal is that the team cannot improve by setting the deadline one period longer. When the team does this, the Bellman equation is

$$\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}+1} = e_1 pZ + (1 - e_1 p)\delta \sum_{\mathcal{N}} C_{2,i}^{T^{deadline}+1} - e_1. \tag{37}$$

Note that the team can at least implement $e_1 = n - 1$; this is implemented by setting $w_{1,i}^S$ sufficiently large for players $i \in \{2, ..., n\}$ and $w_{1,1}^S$ such that the budget is balanced. Moreover, the team can use the same contract as when the deadline would be $T^{deadline}$, with one period delay from period 2 on. All this implies that

$$\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}+1} \geq (n-1)pZ + (1-(n-1)p)\delta \sum_{\mathcal{N}} C_{1,i}^{T^{deadline}} - (n-1). \tag{38}$$

Therefore, when

$$(n-1)pZ + (1-(n-1)p)\delta \sum_{\mathcal{N}} C_{1,i}^{T^{deadline}} - (n-1) > \sum_{\mathcal{N}} C_{1,i}^{T^{deadline}}, \tag{39}$$

then the deadline $T^{deadline}$ cannot be optimal. Rewriting (39) yields

$$Z - 1/p > \frac{1-\delta}{p(n-1)} \sum_{\mathcal{N}} C_{1,i}^{T^{deadline}} + \delta \sum_{\mathcal{N}} C_{1,i}^{T^{deadline}}. \tag{40}$$

$\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}}$ is maximal when the team is able to implement the first-best. Hence, we get from (12) that

$$\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}} \leq \sum_{\mathcal{N}} \hat{C}_{1,i}^{T^{deadline}} = (npZ - n)\frac{1-(\delta(1-np))^{T^{deadline}}}{1-\delta(1-np)}. \tag{41}$$

Therefore, $\sum_{\mathcal{N}} C_{1,i}^{T^{deadline}} << Z - 1/p$. This implies that (40) is satisfied, at least for δ sufficiently close to 1. We conclude that a sufficient condition that players do not want to use a deadline—even if they could commit to it—is that δ is sufficiently close to 1.

4.6.2 DISCRIMINATION AND PLAYERS' PAYOFFS

One may presume that when a player i receives wages $(w^S_{t,i}, w^F_{t,i})$ which are higher than the ones of a player j, player i is in equilibrium better off than player j. To show that this is not true consider the following example. When $Z = 10$, $n = 2$, $p = 1/4$, $T = 2$, a handsome contract is

$$\boldsymbol{w^S} = \begin{pmatrix} 3+\varepsilon & 7-\varepsilon \\ 5 & 5 \end{pmatrix}, \ \boldsymbol{w^F} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{42}$$

where ε is positive and small. The players' expected per-period payoffs are the same in period 2, while in period 1, player 1 yields $(3+\varepsilon)/4$ and player 2 yields $(7-\varepsilon)/4-1 = (3-\varepsilon)/4$. Hence, although player 2 receives a higher wage than player 1, player 2 is worse off than player 1.

The intuition is that player 2 receives a higher wage which motivates her to invest effort, while the wage for player 1 is too low to motivate her to invest. Player 2's effort is also beneficial for player 1, due to the externalities of team production. Because only player 2 has to bear effort costs, she is worse off than player 1. This finding has the interesting implication that an outside party (e.g., a judge) is in general not able to infer that player 1 is disadvantaged compared to 2 when 1 receives lower wages than 2, unless the outside party knows both players' effort costs.

This result crucially depends on our assumption that there is team production. When there is no team production, so that each player works on her own, then a player who possess a higher remunerated contract than another player is necessarily better off, because she can mimic the latter's effort choice.

4.6.3 TEAM SIZE AND ASSUMPTION 1

One may be tempted to conclude that when one cannot design a wage contract where all players always invest effort, decreasing the size of the team need not reduce the team's expected payoff. This thought is wrong.

PROPOSITION 20: *For all $n' < n''$ and for all $Z' \leq Z''$ the team's expected payoff is lower with n' players than with n''.*

The idea is that when the team optimally designs the wage contract, then the implemented team effort e_t is weakly lower for all periods, and strictly lower for some periods with a small team than with a large team. This deteriorates the team's expected payoff. These arguments also imply that when the team designs the wage contract optimally, larger teams more likely solve the task. This result is in contrast to the one with an equal-sharing contract, see Proposition 5.

What happens when Assumption 1 is violated because $pz < 1$? Then with the equal-sharing contract no player ever invests. A team can implement that $n-1$ players invest effort in all periods by designing a discriminatory contract, where—just like in the early periods with a handsome contract—one player acts as a quasi-principal. This contract is beneficial for the team if $Zp > 1$. By arguments similar to the ones where Assumption 1 holds, this contract can be shown to be second-best and renegotiation-proof. Although a larger team never yields a lower expected team payoff than a smaller team (because it can mimic the effort profile of a smaller team), there are constellations where a larger team may yield the same expected payoff as a smaller team. A simple example is when $T = 1$, $n'/p \leq Z' = Z'' < (n'+1)/p$. Then with n' players $e_1 = n'$ is implementable, while with $n'' = n' + 1$ players one can at most implement $e_1 = n'' - 1 = n'$, too.

5 EXTENSIONS

In this section, we offer several extensions. Throughout we assume that the reward is shared equally. In Section 5.1, we suppose that players have social preferences. We explore how teams are optimally designed when players are heterogenous with respect to their social preferences and show that it is better to have teams with heterogenous players than to have homogenous teams. In Section 5.2, we explore the case when there is no time limit for solving the task. We characterize the symmetric Markov perfect equilibrium. In Section 5.3, we allow for very general success functions and consider the cases where the success function is superadditive or subadditive. In Section 5.4, we suppose that the reward which the team obtains for solving the task declines over time. We then show that there might be an artificial time limit and that players might not invest effort when the reward is high, but invest effort when it is rather low. In Section 5.5, we assume that there are several tasks which the team can solve. We show that when there are less tasks than periods in which the team can work on the tasks, the team might procrastinate.

5.1 SOCIAL PREFERENCES AND TEAM DESIGN

Numerous experiments show that many people are not purely self-interested. Motivated by these findings we assume in this section that players have social preferences.[24] Suppose that each player puts a weight of $\phi \in [0,1]$ on the other players' material payoffs.[25] The parameter ϕ captures how much a player cares about the other players' well-being, compared to her own well-being. A purely selfish player is characterized by $\phi = 0$, whereas for an altruistic player $\phi = 1$.

Simple calculations show that, taking the other players' well-being into account with weight ϕ, player i invests effort in period t if and only if

$$p\left(z + \phi(n-1)z - C_{t+1,i} - \phi \sum_{\mathcal{N}\setminus i} C_{t+1,j}\right) \geq 1. \tag{43}$$

Because $\phi(n-1)z - \phi \sum_{\mathcal{N}\setminus i} C_{t+1,j} > 0$, player i is, all else equal, more eager to invest effort the larger is ϕ.

We first explore how players, which have homogenous social preferences, invest effort. Then we allow the players to have heterogenous social preferences and study how teams are optimally designed. Throughout we assume that there are $n \geq 2$ players.

5.1.1 HOMOGENOUS PLAYERS

When players have homogenous social preferences we get the following result.

P R O P O S I T I O N 21: *For $\phi = 1$ players always invest effort. For $\phi < 1$ and when T is sufficiently large, players do not always invest effort. Compared to the*

[24]Kandel and Lazear (1992) consider a team model with moral hazard, where players are selfish, but care about peer pressure.

[25]See, for example, Charness and Rabin (2002) for a more general framework and an experimental investigation.

case where players are purely selfish ($\phi = 0$), social players ($\phi > 0$) begin to invest effort weakly earlier.

For $\phi = 1$ players are altruistic and always invest, because this maximizes welfare. For $\phi < 1$ and when the time limit T is sufficiently long we get again the result that the team procrastinates. But as the last part of Proposition 21 clarifies, procrastination is alleviated by social preferences.

5.1.2 HETEROGENOUS PLAYERS

We now suppose that players have heterogeneous social preferences. A classical issue of economics (e.g., Lazear 1989 and Jeon 1996) is how players should be grouped together, i.e., how teams should be designed. Suppose there are two players who are purely selfish ($\phi = 0$) and two social players with $\phi = \bar{\phi} \in (0, 1)$. How should one form two teams, each consisting of two players?

When a player decides to invest in period t, then the expected payoff must be at least as great as the one from not doing so. Moreover, we know from before that it is beneficial for a player's material well-being when another one invests. Both arguments imply that $C_{t,i} + \phi_i C_{t,-i} \geq C_{t+1,i} + \phi_i C_{t+1,-i}$ when some player invests in t. When no player invests then $C_{t,i} + \phi_i C_{t,-i} = C_{t+1,i} + \phi_i C_{t+1,-i}$. Together with (43) these findings imply that all players' strategies are simple threshold rules which specify when a player starts to invest. Denote the period in which player i starts to invest by $t(\phi_i, \phi_{-i})$, where ϕ_{-i} is the social preference parameter of the teammate. From before we know that a social player is more eager to invest effort than a selfish player. Hence, $t(0, \phi_{-i}) \geq t(\bar{\phi}, \phi_{-i})$. We get two further results.

LEMMA 3: *$t(0, 0) = t(0, \bar{\phi})$ and $t(\bar{\phi}, 0) \leq t(\bar{\phi}, \bar{\phi})$.*

That is, a selfish player invests rather late—due to $t(0, \phi_{-i}) \geq t(\bar{\phi}, \phi_{-i})$—and her behavior is not influenced whether she is matched with another selfish player or a social player. A social player i begins to invest earlier. When matched with a selfish player the term $C_{t+1,i} + \phi_i C_{t+1,-i}$ is rather low because the selfish player invests rather late. That is why a social player begins to invest earlier when matched with a selfish player than when matched with another social player.

We next explore whether it is better to have two homogenous teams (one with social and one with selfish players) or two heterogenous teams (each consisting of a social and a selfish player).

PROPOSITION 22: *The aggregated expected payoff is at least as high with heterogenous teams than with homogenous teams.*

From Lemma 3 we know that a social player begins to invest rather late when matched with another social player, compared to the case when matched with a selfish player. This effect can be exploited by having heterogenous teams. Then we have—on average—more investments than when teams are homogenous.

An Example.— Suppose that $z = 5$, $p = 1/4$, $T = 3$, and $\bar{\phi} = 1/4$. Consider first the case with a team consisting of two selfish players. They invest in $t = 3$ because $p(z - 0) \geq 1$. Hence, $C_{3,i} = 2pz - 1 = 3/2$. Given this, players do not invest in $t = 2$ because $p(z - C_{3,i}) = 7/8 < 1$ and for the same reason not in $t = 1$. Hence, the expected payoff of the team, measured at the beginning of the game, is $2C_{3,i} = 3$.

Next consider a team with two social players. Obviously, players invest in $t = 3$ and $C_{3,i} = 3/2$. In $t = 2$, players also invest because $p(z + \bar{\phi}z - C_{3,i} - \bar{\phi}C_{3,i}) = 35/32 \geq 1$. Hence, $C_{2,i} = 2pz - 1 + (1 - 2p)C_{3,i} = 9/4$. In $t = 1$, the players do not invest because $p(z + \bar{\phi}z - C_{2,i} - \bar{\phi}C_{2,i}) = 55/64 < 1$. Hence, the expected payoff of the team, measured at the beginning of the game, is $2C_{2,i} = 9/2$.

Finally, in a heterogenous team players obviously invest in $t = 3$ and hence $C_{3,i} = 3/2$. Denote the social player as 1 and the selfish player as 2. Player 2 does not invest in $t = 2$ because $p(z - C_{3,2}) = 7/8 < 1$. Player 1 invests in $t = 2$ because $p\left(z + \bar{\phi}z - C_{3,1} - \bar{\phi}C_{3,2}\right) = 35/32 \geq 1$. Hence, $C_{2,1} = pz - 1 + (1 - p)C_{3,1} = 11/8$ and $C_{2,2} = pz + (1 - p)C_{3,2} = 19/8$. In $t = 1$, player 2 does not invest because $p(z - C_{2,2}) = 21/32 < 1$. But player 1 invests because $p\left(z + \bar{\phi}z - C_{2,1} - \bar{\phi}C_{2,2}\right) = 137/128 \geq 1$. Hence, $C_{1,1} = pz - 1 + (1 - p)C_{2,1} = 41/32$ and $C_{1,2} = pz + (1 - p)C_{2,2} = 97/32$. Therefore, having two heterogenous teams leads to an aggregated expected payoff, measured at the beginning of the game, of $2(41/32 + 97/32) = 69/8$, whereas for two homogenous teams we only have $3 + 9/2 = 60/8$.

5.2 INFINITE HORIZON

Previously, we assumed that there is a finite time limit. We next suppose that there is no time limit. While in the model with a time limit the case with continuous effort is generally intractable (except when effort costs are linear, only numerical results can be obtained), this is not true when there is no time limit.

We measure effort in "probability units" of success: there is success in period t with probability $\sum_{\mathcal{N}} e_{t,i}$ and failure otherwise, where $e_{t,i} \in \mathbb{R}^{+}$. Effort causes private costs of $k(e_{t,i})$, with $k(0) = 0$, $k' > 0$, and $k'' > 0$.[26] To guarantee inner solutions we assume that $\lim_{e_{t,i}\to 0} k'(e_{t,i}) = 0$ and $\lim_{e_{t,i}\to 1/n} k(e_{t,i}) = \infty$.

The following Bellman equation holds:

$$C_{t,i} = \sum_{\mathcal{N}} e_{t,j} z + \left(1 - \sum_{\mathcal{N}} e_{t,j}\right) \delta C_{t+1,i} - k(e_{t,i}). \qquad \text{(BELLMAN)}$$

Player i chooses effort $e_{t,i}$ to maximize $C_{t,i}$. The resulting first-order condition or incentive constraint is

$$z - \delta C_{t+1,i} = k'(e_{t,i}). \qquad \text{(IC)}$$

We concentrate on symmetric Markov perfect equilibria (SMPE). We say that an equilibrium is symmetric if all players choose the same investment in each period (but the investment may depend on the period). A strategy is Markov if it does not depend on state variables that are functions of the history of the game, except the ones which affect payoffs. A Markov perfect equilibrium (MPE) is then a profile of Markov strategies that yields a Nash equilibrium in every proper subgame (cf. Fudenberg and Tirole 1991).

When the game has not ended in period t, then the team had no success until t. The history of the team is hence the series of failures it has experienced. Because prior failures do not affect current or future payoffs, a MPE specifies the same effort choice for a player in every period, given that the game has not yet ended.

In a SMPE a player chooses not only the same effort in every period, but all players choose the same effort. Formally, $e_{t,i} = e^{SMPE}$ for all $i \in \mathcal{N}$ and all $t \in \mathbb{N}$.

[26] Having convex effort costs is highly plausible and standard in agency theory. In most of our analysis nothing changes when effort costs are linear.

Because effort is constant in all periods and the same for all players we get from (BELLMAN) that $C_{t,i} = C^{SMPE}$ for all $i \in \mathcal{N}$ and all $t \in \mathbb{N}$. Hence, (BELLMAN) can be rewritten as

$$C^{SMPE} = \frac{ne^{SMPE}z - k(e^{SMPE})}{1 - \delta + \delta ne^{SMPE}} \tag{B SMPE}$$

and (IC) as

$$z - \delta C^{SMPE} = k'(e^{SMPE}). \tag{IC SMPE}$$

It is useful to determine the first-best benchmark.

LEMMA 4: *In the first-best,*

$$e^{FB} = argmax \frac{nez - k(e)}{1 - \delta + \delta ne} \tag{44}$$

and

$$C^{FB} = \frac{ne^{FB}z - k(e^{FB})}{1 - \delta + \delta ne^{FB}}. \tag{45}$$

PROPOSITION 23: *There always exists a SMPE. The SMPE is unique.* e^{SMPE} *and* C^{SMPE} *solve* (B SMPE) *and* (IC SMPE).

The next result shows that only when there is a single player the investments in effort are first-best. Otherwise, the players underinvest. See Figure 1 for an illustration.

PROPOSITION 24: *Keep z fixed. e^{SMPE} is decreasing in n, while e^{FB} is increasing in n. For $n = 1$, $e^{SMPE} = e^{FB}$ and $C^{SMPE} = C^{FB}$. Otherwise, $e^{SMPE} < e^{FB}$ and $C^{SMPE} < C^{FB}$. C^{SMPE} is increasing in n.*

The intuition why effort e^{SMPE} is decreasing in the number of players n is that the future externalities arising from team production are greater the larger the team is. In a larger team a player has therefore less incentives to invest effort. This holds despite that the per-capita reward z is kept fixed. The reason why the first-best effort e^{FB} is increasing in n is that in a larger team the team's total reward nz is increasing in n, keeping z fixed. Therefore, higher investments in effort are efficient.

We next explore the comparative statics with respect to the per-capita reward z. When one increases the size of a team it may be plausible that z decreases. We get that

$$\left.\frac{dC^{SMPE}}{dz}\right|_{\text{(IC SMPE)}} = \frac{1}{\delta} > \frac{ne}{1 - \delta + \delta ne} = \left.\frac{dC^{SMPE}}{dz}\right|_{\text{(B SMPE)}}. \tag{46}$$

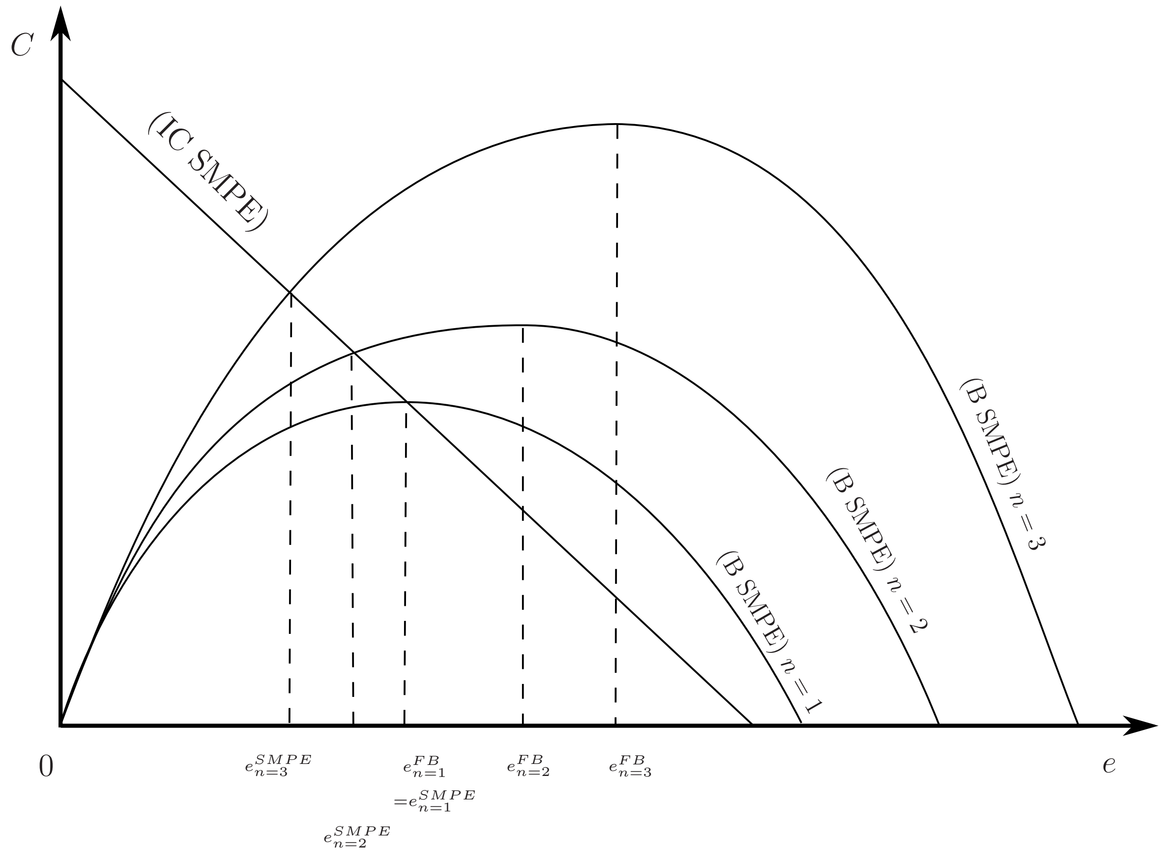

Figure 1: Only for $n = 1$ is the SMPE first-best.

Hence, lowering z, holding all else equal, decreases effort e^{SMPE} and expected payoff C^{SMPE}, which is intuitive; see Figure 2 for the case $n > 1$.

The comparative statics with respect to the discount factor δ are interesting. Because $\frac{dC}{d\delta}\Big|_{\text{(IC SMPE)}} < 0$ and $\frac{dC}{d\delta}\Big|_{\text{(B SMPE)}} > 0$ we get that $de^{SMPE}/d\delta < 0$. A single player always chooses first-best effort; $\frac{dC}{d\delta}\Big|_{\text{(B SMPE)}} > 0$ then implies that $dC^{SMPE}_{n=1}/d\delta > 0$. Put differently, a single player benefits from a higher δ. With several players this is not necessarily true. To see this, consider the case of quadratic effort costs $k(e) = e^2/2$. Using (B SMPE) and (IC SMPE) we get

$$e^{SMPE} = \frac{-(1-\delta) + \sqrt{(1-\delta)^2 + 2\delta(1-\delta)z(2n-1)}}{\delta(2n-1)}. \tag{47}$$

Table 1 shows an example with $n = 2$ and $z = 0.5$, where players may benefit from a lower δ: they are better off when $\delta = 0$ than with $\delta \in \{0.05, ..., 0.8\}$. The intuition is that although players are, all else equal, better off with a higher δ, see (B SMPE),

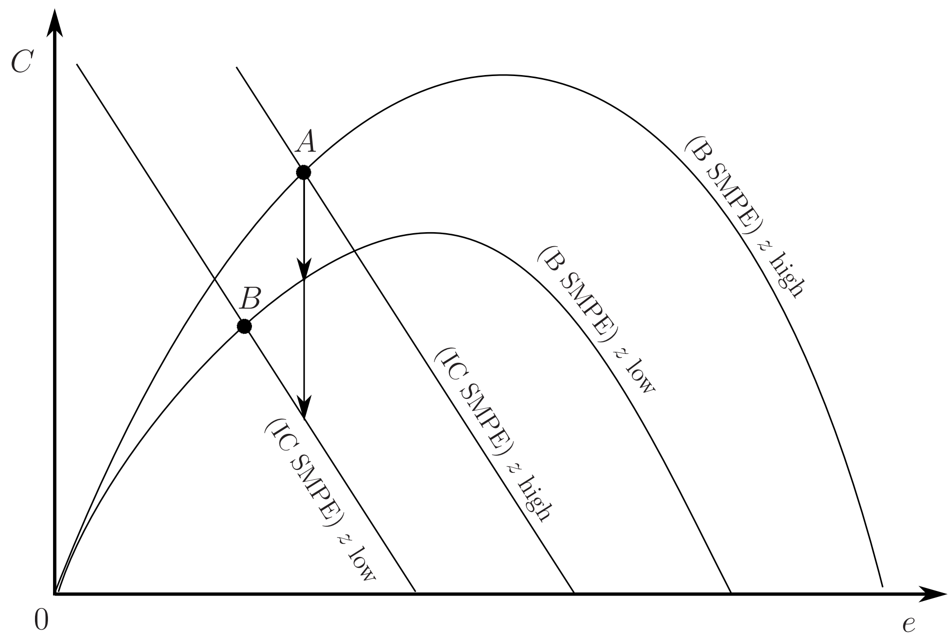

Figure 2: Point A is the initial SMPE, point B the SMPE for a lower z.

they may reduce effort e^{SMPE} so much that their expected payoff C^{SMPE} is impaired.

δ	0	0.05	0.1	0.2	0.3	0.4	0.5
e^{SMPE}	0.500	0.482	0.464	0.431	0.398	0.366	0.333
C^{SMPE}	0.375	0.366	0.359	0.347	0.340	0.335	0.333
δ	0.6	0.7	0.8	0.9	0.95	0.99	$\to 1$
e^{SMPE}	0.299	0.261	0.217	0.159	0.116	0.055	0.000
C^{SMPE}	0.335	0.341	0.354	0.379	0.404	0.450	0.500

Table 1: Example with $n = 2$, $z = 0.5$, and $k(e) = e^2/2$.

Interestingly, when there are several players, $n > 1$, making each player a residual claimant (each receives Z in case of success and 0 in case of failure) is not enough to motivate the players to invest first-best effort. To see this, adjust and combine (IC SMPE) and (B SMPE) to get

$$Z - \delta \frac{neZ - k(e)}{1 - \delta + \delta ne} = k'(e), \tag{48}$$

which is solved by $\ddot{e}$. In the first-best we $\max_e C_B(e,n) = \frac{nez-k(e)}{1-\delta+\delta ne}$. Hence, e^{FB} solves

$$Z - \delta \frac{neZ - nk(e)}{1-\delta+\delta ne} = k'(e). \tag{49}$$

Because $k'' > 0$, players underinvests when they are residual claimants: $\ddot{e} < e^{FB}$. The reason for this result is that a player does not take into account that solving the task now saves $n-1$ other players the future effort costs from investing.

Although the SMPE is appealing, there may also exists other equilibria. In Appendix B, we show, by means of two examples, that there can be symmetric non-Markov perfect equilibria as well as asymmetric Markov perfect equilibria. Interestingly, see Appendix B, players may be better off by playing a non-Markov perfect equilibrium than by playing a SMPE.

5.3 MORE GENERAL SUCCESS FUNCTIONS

Suppose that the task is solved in period t with probability $P(e_t)$. We assume that when no player invests effort, then there is success with probability zero, that success is more likely when more players invest effort, that success is also possible when all but one player of the team invest effort, and that success is not guaranteed even when all players invest.

A S S U M P T I O N 2: *Given* $n \geq 2$. $P(0) = 0$, $P'(e_t) > 0$, $P(n-1) > 0$, *and* $P(n) < 1$.

The continuation payoff, given that all players invest effort from t until T, is

$$\hat{C}_{t,i} = (P(n)z - 1)\frac{1-(1-P(n))^{T-t+1}}{P(n)}, \tag{50}$$

which is just a slightly adapted version of (3). For the limit case

$$\lim_{T\to\infty} \hat{C}_{t+1,i} = z - \frac{1}{P(n)}. \tag{51}$$

It is easy to show that, given that all other players invest in period t, it is optimal for a player i to invest effort as well if and only if

$$\left(P(n) - P(n-1)\right)\left(z - C_{t+1,i}\right) \geq 1. \tag{52}$$

For the limit case $T \to \infty$ and when all players always invest (52) gets

$$1 - \frac{P(n-1)}{P(n)} < 1. \tag{53}$$

Because $\hat{C}_{t,i}$ is continuous in T, we get the result that the team procrastinates when the time limit is sufficiently long.

PROPOSITION 25: *Given Assumption 2 and that $n \geq 2$. When the time limit T is sufficiently long, not all players always invest effort.*

Observe that when Assumption 2 is violated because $P(n-1) = 0$, the team's problem collapses to a coordination problem. Regardless of T, it is an equilibrium that all players always invest when $P(n)z \geq 1$; i.e., there is no procrastination. It is, however, also an equilibrium that nobody ever invests.

We next assume that the success function P is either super- or subadditive.

DEFINITION 3: *The function P is superadditive (subadditive) if $P(e' + e'') \geq (\leq)\ P(e') + P(e'')$ for all $e', e'' \geq 0$.*

Superadditivity captures the idea that success is more likely when two groups of players invest effort at the same time than when the groups invest at different times. Then it is optimal to pool forces. Note that one reason a team may form in the first place may be superadditivity.

From (52) we see that, given some $P(n)$, procrastination is more likely, in the sense that it occurs for a larger parameter set, the smaller is $(P(n) - P(n-1))$. Therefore, with subadditivity procrastination is more likely than with superadditivity. The intuition is that with subadditivity the player is inclined to invest no effort, given that all other players invest, because the probability of success is not much higher when she invests, namely $P(n)$, compared to the case when she does not invest, namely $P(n-1)$.

With subadditivity one may get asymmetric equilibria, where some players invest effort, while others do not. A simple example is

$$P(e_t) = \begin{cases} e_t \tilde{p} & , \quad \text{for } e_t < n, \\ (n-1)\tilde{p} + \varepsilon, & \text{for } e_t = n, \end{cases}$$

where $\tilde{p} \in (0, 1/(n-1))$ and ε is positive and small. Then it cannot be an equilibrium that all players invest for sure in some period. Additionally, there may also be equilibria in mixed strategies.

5.4 DECREASING REWARDS

We now suppose that the reward, which the team receives for solving the task, decreases with time. Formally, we assume that for all $t \in \{1, ..., T\}$, $Z_t > Z_{t+1}$. An example where a decreasing reward is highly plausible is when the team's task is to obtain an innovation; it is usually at least weakly better to have an innovation earlier.

Recognize that once $pz_t < 1$, where $z_t := Z_t/n$ is the per-capita reward, nobody will invest. Hence, when $pz_t < 1$ holds for a $t \leq T$, there is an artificial time limit $T^{art} = \max\{t \in \mathbb{N} | pz_t \geq 1\}$. Denote the minimum of the real and the artificial time limit (if there is one) by $T^{min} := \min\{T, T^{art}\}$.

We are interested whether the players always invest effort until T^{min}. This holds if and only if

$$p(z_t - \check{C}_{t+1,i}) \geq 1 \tag{54}$$

for all $t \leq T^{min}$, where $\check{C}_{t+1,i}$ is the continuation payoff when all players invest from $t+1$ until T^{min} and not afterwards. This continuation payoff evolves according to

$$\check{C}_{t,i} = npz_t + (1 - np)\check{C}_{t+1,i}. \tag{55}$$

Because $C_{T^{min}+1} = 0$ we know that all players invest in period T^{min}. When (54) fails for a single $t \leq T^{min}$ we know that players do not always invest until T^{min}. Then, because the reward is decreasing in time, we get the result that players do not only invest too seldom, but also at the wrong time. For example, when they do not invest in period $t' < T^{min}$ it would collectively be better for the players to invest in t' instead of T^{min}, because $z_{t'} > z_{T^{min}}$.

An Example.— Suppose that $n = 2$, $p = 1/4$, $T = 5$, $z_1 = 6.5$, $z_2 = 6$, $z_3 = 5$, $z_4 = 3.5$, and $z_5 = 3$. Because $pz_4 < 1$ and $pz_5 < 1$ we have an artificial time limit $T^{art} = 3$. In $t = 3$, both players invest because $1/4 \times (5 - 0) \geq 1$. This gives a continuation payoff $C_{3,i} = 1/2 \times 5 - 1 = 1.5$. In $t = 2$, players invest because $1/4 \times (6 - 1.5) \geq 1$, resulting in $C_{2,i} = 1/2 \times 6 + 1/2 \times 1.5 - 1 = 2.75$. In $t = 1$, players do not invest because $1/4 \times (6.5 - 2.75) < 1$.

5.5 MULTIPLE TASKS

We have assumed that the team has to perform one task. What happens when there are several tasks? Suppose that tasks are homogenous. When there are at least as many task as periods, then it is without loss of generality to assume that the players work on a different task in every period. Then there is no scope for procrastination, since current success or failure does not influence future investments and payoffs. Otherwise, the team might procrastinate because future externalities arising from team production make investing effort in the present unattractive. To see this, consider the following example.

An Example.— There are two tasks, $z \in [4, 6)$, $n = 2$, $p = 1/4$, and $T = 3$. Denote the number of tasks which are not solved at the beginning of period t by O_t. When $O_3 > 0$, then the players invest effort in $t = 3$, because this yields an additional expected payoff for a player of $1/4 \times z - 1 \geq 0$. In period 2, players invest effort if $O_2 = 2$: the game continues for sure until period 3 and the expected payoff of period 3 stays unaffected of what happens in period 2; therefore investing effort yields an additional expected payoff for a player of $1/4 \times z - 1 \geq 0$. For $O_2 = 1$ the continuation payoff of reaching period 3 with $O_3 = 1$ is too large to motivate players to invest effort in period 2; cf. Proposition 3. Given this, the continuation payoff of reaching period 2 depends on O_2: $C_{2,i}^{O_2=2} = 2\left(\frac{1}{2}z - 1\right)$, $C_{2,i}^{O_2=1} = \frac{1}{2}z - 1$. The additional expected payoff from investing effort in period 1 is

$$1/4 \times (z + C_{2,O_2=1} - C_{2,O_2=2}) - 1 = z/8 - 3/4, \tag{56}$$

which is negative because $z < 6$. To sum up, players only invest effort in periods 2 and 3. So we have again the result that the team procrastinates.

6 CONCLUSION

In this paper we show that when players are remunerated equally and face a sufficiently long time limit, they begin to invest effort rather late. Players procrastinate to benefit from future externalities arising from team production. This is socially inefficient, but individually rational. When players can design wage contracts they can mitigate procrastination. By remunerating players differently, one can motivate some players, though regularly not all of them, to invest effort also in early periods. Limited liability or the threat of sabotage can severely restrict the team's possibilities to motivate many players. Further results are that players who earn higher wages can be worse off than teammates with lower wages, that with social preferences teams are optimally designed heterogeneously, and that present-biased preferences mitigate procrastination.

We allowed players to have heterogenous social preferences. Players may also differ with respect to other characteristics, like effort costs or the effectiveness of their effort. Investigating these issues is an interesting topic for future research. When players are little heterogenous, we suspect that, generically, all of our results stay valid. The idea is that players are generically never indifferent, and therefore their decisions do not change when there is little heterogeneity instead of homogeneity.

APPENDIX A: PROOFS

PROOF OF PROPOSITION 1

From (57), see below, we see that $\sum_{\mathcal{N}} C_{1,i}$ is maximized when e_1 is chosen optimally and when $\sum_{\mathcal{N}} C_{2,i}$ is maximized. $\sum_{\mathcal{N}} C_{2,i}$ is in turn maximized when e_2 is chosen optimally and when $\sum_{\mathcal{N}} C_{3,i}$ is maximized. These arguments can be repeated and imply that we can solve the social planner's problem by backward induction.

For the special case where $n = 1$ and $pZ = 1$ we see from (2) that welfare $\sum_{\mathcal{N}} C_{1,i}$ is zero for all investment profiles. We now consider the case where this is not true. Assumption 1 implies that then $Z - 1/p > 0$. In period t, the social planner maximizes, cf. (1),

$$\sum_{\mathcal{N}} C_{t,i} = e_t pZ + (1 - e_t p) \sum_{\mathcal{N}} C_{t+1,i} - e_t. \tag{57}$$

The derivative is

$$\frac{d \sum_{\mathcal{N}} C_{t,i}}{de_t} = pZ - p \sum_{\mathcal{N}} C_{t+1,i} - 1. \tag{58}$$

A sufficient condition for this to be positive, and therefore for $e_t = n$ to be optimal, is that

$$\sum_{\mathcal{N}} C_{t+1,i} < Z - 1/p. \tag{59}$$

We start with period T. Because $C_{T+1,i} = 0$ and $Z - 1/p > 0$ holds, inequality (59) is satisfied and hence $e_T = n$.

From (57) we get

$$\sum_{\mathcal{N}} C_{t,i} = e_t p \left(Z - 1/p\right) + (1 - e_t p) \sum_{\mathcal{N}} C_{t+1,i}. \tag{60}$$

Next, consider period $t = T - 1$. From (60) we see that $\sum_{\mathcal{N}} C_{T,i}$ is equal to the weighted average of $Z - 1/p$ and $\sum_{\mathcal{N}} C_{T+1,i}$. Because $\sum_{\mathcal{N}} C_{T+1,i} = 0$, $\sum_{\mathcal{N}} C_{T,i}$ is lower than $Z - 1/p$. This implies that (59) is satisfied and the social planner sets $e_{T-1} = n$.

Next, consider period $t = T - 2$. From (60) we see that $\sum_{\mathcal{N}} C_{T-1,i}$ is equal to the weighted average of $Z - 1/p$ and $\sum_{\mathcal{N}} C_{T,i}$. Because $\sum_{\mathcal{N}} C_{T,i}$ is lower than $Z - 1/p$,

$\sum_{\mathcal{N}} C_{T-1,i}$ is lower than $Z - 1/p$, too. This implies that (59) is satisfied and the social planner sets $e_{T-2} = n$.

These arguments can be repeated to show that the social planner always sets $e_t = n$.

To prove the last part of the proposition note that because in the first-best all players always invest we have $\sum_{\mathcal{N}} C_{t+1,i} \leq \sum_{\mathcal{N}} \hat{C}_{t+1,i} < Z - 1/p$, where the last inequality follows from the arguments before. Hence, (58) is positive and implies that $\sum_{\mathcal{N}} C_{t,i}$ is increasing in e_t. From (57), $\sum_{\mathcal{N}} C_{t,i}$ is increasing in $\sum_{\mathcal{N}} C_{t+1,i}$, which is in turn increasing in $\sum_{\mathcal{N}} C_{t+2,i}$, and so on. Hence, $\sum_{\mathcal{N}} C_{t,i}$ increases with a higher level of effort e_s, where $s \geq t$. □

PROOF OF LEMMA 2

Part (i). In period T, all players face the same tradeoff: from Lemma 1 a player invest in T if and only if $p\,(z - C_{T+1,i}) \geq 1$. Therefore, in T all players choose the same investment. Because of this, $C_{T,i}$ is the same for all players. Hence, in period $T-1$ they again face the same tradeoff and therefore choose the same investment. These arguments can be repeated, which establishes that all players choose the same investment in every period.

Part (ii). When $e_t = 0$ it holds that $C_{t,i} = C_{t+1,i}$, see (1).

Part (iii). From Proposition 1 we know that $\sum_{\mathcal{N}} C_{t,i}$ increases with e_t. Parts (i) and (ii) imply that when players invest, $C_{t,i} > C_{t+1,i}$ must hold.

Part (iv). When players always invest from t on then, by definition, $C_{t,i} = \hat{C}_{t,i}$. Otherwise, Proposition 1 implies that $\sum_{\mathcal{N}} C_{t,i} < \sum_{\mathcal{N}} \hat{C}_{t,i}$. Part (i) then yields $C_{t,i} < \hat{C}_{t,i}$. □

PROOF OF PROPOSITION 3

From Lemma 1 it follows that players invest effort in period t if and only if the continuation payoff of $t+1$ is sufficiently low. From Lemma 2 we know that the continuation payoff is nonincreasing in t. Therefore, one of the following patterns has to hold: the players always invest effort or they do not invest effort until some period, and always invest effort from this period on.

Due to $p\left(z-\hat{C}_{T+1,i}\right) \geq 1$ and $p\left(z-\lim_{T\to\infty}\hat{C}_{t,i}\right) < 1$ we know that there must exist a $\grave{x}$ such that

$$p\left(z-\hat{C}_{T-\grave{x},i}\right) \stackrel{!}{=} 1, \tag{61}$$

where $t = T - \grave{x}$ is now treated as a continuous variable which is possibly below 1.

With help of (3) we can solve (61) for

$$\grave{x} = \frac{\ln(n-1) - \ln(npz-1)}{\ln(1-np)} - 1, \tag{62}$$

where ln is the natural logarithm.

It is useful to define

$$\bar{x} := \grave{x} + 1. \tag{63}$$

When $T \leq \bar{x}+1$, we claim that then $p\left(z - C_{t+1,i}\right) \geq 1$ for all $t \geq 1$. From Lemma 2 we know that $C_{t,i} \leq \hat{C}_{t,i}$ and that $\hat{C}_{t,i}$ is decreasing in t. So when we can show that

$$p\left(z-\hat{C}_{2,i}\right) \geq 1, \tag{64}$$

then the claim is true. Inequality (64) indeed holds because $\hat{C}_{2,i} \leq \hat{C}_{T-\grave{x},i}$ due to $2 \geq T - \grave{x} \iff T \leq \bar{x}+1$ and the finding that $\hat{C}_{t,i}$ is decreasing in t.

We finally prove part (ii), where $T > \bar{x}+1$. We first show that players invest effort in the periods $t \geq T-\bar{x}$. This requires that $p\left(z - C_{t+1,i}\right) \geq 1$ for all $t \geq T - \bar{x}$. This is indeed true because

$$C_{t+1,i} \leq \hat{C}_{t+1,i} \leq \hat{C}_{T-\grave{x},i}, \tag{65}$$

where the first inequality follows from Lemma 2 and the second from the finding that $\hat{C}_{t,i}$ is decreasing in t and the fact that $t+1 \geq T - \grave{x} \iff t \geq T - \bar{x}$.

We claimed that players do not invest effort in periods $t < T - \bar{x}$. Denote $t' = \min\{t | t \geq T - \bar{x}, t \in \mathbb{N}\}$. That is, t' is the lowest natural number for which $\hat{C}_{t'+1,i} \leq \hat{C}_{T-\grave{x},i}$. Hence, from Lemma 2,

$$\hat{C}_{t',i} > \hat{C}_{T-\grave{x},i}. \tag{66}$$

From before we know that players invest effort for all periods $t \geq t'$. Hence, $C_{t',i} = \hat{C}_{t',i}$. Together with (61) and (66) this implies that

$$p\left(z - C_{t',i}\right) < 1. \tag{67}$$

Hence, players do not invest in period $t' - 1$. From Lemma 2 we know that the continuation payoff is nonincreasing in t. Hence, players do not invest effort in periods $t < T - \bar{x}$. □

PROOF OF PROPOSITION 4

$$\frac{d\bar{x}}{dz} = \frac{\overbrace{-\frac{np}{npz-1}}^{<0}\overbrace{\ln(1-np)}^{<0}}{\underbrace{(\ln(1-np))^2}_{>0}} > 0. \tag{68}$$

Fix z.

$$\frac{d\bar{x}}{dn} = \overbrace{\frac{1}{(\ln(1-np))^2}}^{>0} \times \left[\underbrace{\left(\frac{1}{n-1} - \frac{pz}{npz-1}\right)}_{>0}\underbrace{\ln(1-np)}_{<0} + \underbrace{(\ln(n-1) - \ln(npz-1))}_{<0}\underbrace{\frac{p}{1-np}}_{>0}\right]. \tag{69}$$

Hence, $\frac{d\bar{x}}{dn} < 0$ when z is fixed.

We fix Z and assume that Assumption 1 remains valid. With fixed Z $d\bar{x}/dn < 0$ follows from $d\bar{x}/dn < 0$ when fixing z and that $d\bar{x}/-dz < 0$. □

PROOF OF PROPOSITION 5

From Proposition 3, players only invest in the last $\breve{T}$ period(s), where $\breve{T} := \#\{t | T \geq t \geq T - \bar{x}, t \geq 1\}$. Therefore, the likelihood that the task is solved during the whole game is

$$1 - (1-np)^{\breve{T}}. \tag{70}$$

This probability is bounded away from 1 even when T is large.

In contrast, a single player always invests, see Proposition 2. The likelihood that the task is solved during the whole game is

$$1 - (1-p)^{T}. \tag{71}$$

Therefore, when T is sufficiently large, the probability of solving the task sometimes is strictly higher for $n = 1$ than with $n \geq 2$; compare (70) and (71).

When T is sufficiently small, for example when $T = 1$, then $n \geq 2$ players more likely solve the task than a single player; compare again (70) and (71). □

PROOF OF PROPOSITION 6

We only have to slightly adjust the proof of Proposition 1 by replacing the equality in (60) by an inequality:

$$\sum_{\mathcal{N}} C_{t,i} = e_t pZ + (1-e_t p)\delta \sum_{\mathcal{N}} C_{t+1,i} - e_t < e_t p\,(Z - 1/p) + (1-e_t p) \sum_{\mathcal{N}} C_{t+1,i}. \quad (72)$$

The arguments concerning the weighted average must then be slightly adjusted to say that $\sum_{\mathcal{N}} C_{t,i}$ is lower than the weighted average of $Z - 1/p$ and $\sum_{\mathcal{N}} C_{t+1,i}$. □

PROOF OF PROPOSITION 7

The first part follows directly from the findings that with $\delta < 1$ instead of $\delta = 1$ (i) each player is more eager to invest when $C_{t+1,i}$ is stable or is lower when $\delta < 1$ instead of $\delta = 1$, see (13) and (ii) for a given effort profile $(e_1, ..., e_T)$, $C_{t+1,i}$ is lower for $\delta < 1$ than when $\delta = 1$, see (11).

The second part of the proposition follows from (13): when $\delta \to 0$, players always invest effort; for $\delta = 1$ this is not necessarily true, cf. Proposition 3. □

PROOF OF PROPOSITION 8

Because all players choose the same investment in every period, Proposition 6 implies that $C_{t+1,i} \leq \hat{C}_{t+1,i}$. From (12) we know that $\hat{C}_{t+1,i} < (npz - 1)\frac{1}{1-\delta(1-np)}$ for any finite T. Hence, $C_{t+1,i} < (npz - 1)\frac{1}{1-\delta(1-np)}$ for any finite T.

Plugging $C_{t+1,i} = (npz - 1)\frac{1}{1-\delta(1-np)}$ into (13) yields

$$p(z - (npz - 1)\frac{1}{1 - \delta(1 - np)}) \overset{?}{\geq} 1; \quad (73)$$

$\bar{\delta}$ solves this with equality.

For a given effort profile $(e_1, ..., e_T)$ the continuation payoff is lower when δ decreases, see (11), and for a lower δ and a lower continuation payoff (13) is more likely satisfied. Hence, when $\delta \leq \bar{\delta}$ then all players always invest effort. While for $\delta > \bar{\delta}$ and T sufficiently large it cannot hold that all players always invest because then, due to $\hat{C}_{t,i} \to (npz-1)\frac{1}{1-\delta(1-np)}$, (13) is violated. □

PROOF OF PROPOSITION 9

Whether a player invests or not is determined by (13). Suppose there is no discounting. Because all players choose the same investment in every period, there are only T formulas to consider (one formula in every period). Generically, no such formula holds with equality, i.e., generically a player is never indifferent. Suppose now that there is discounting. Formulas that hold with $>$ for $\delta = 1$ also hold with $>$ with δ close to 1. The same is true for formulas which hold with $<$. Hence, for $\delta \in (\grave{\delta}, 1)$ the players' investments are the same with discounting than without. □

PROOF OF PROPOSITION 10

Also with $\beta - \delta$ preferences the continuation payoffs are given by (11) and (12). What changes is player i's invest rule, see (16).

The term $C_{t+1,i}$ is maximized for $T \to \infty$ and when all players always invest. Then

$$\lim_{T\to\infty} \hat{C}_{t+1,i} = (npz-1)\frac{1}{1-\delta(1-np)}. \tag{74}$$

Plugging this into (16) yields

$$p\left(z - \beta\delta(npz-1)\frac{1}{1-\delta(1-np)}\right) \geq 1 \tag{75}$$

This is solved with equality for $\beta = \bar{\beta}$. From (16) we see that players always invest when $\beta \leq \bar{\beta}$.

While for $\beta > \bar{\beta}$ and T sufficiently large it cannot hold that all players always invest because then, due to $\hat{C}_{t,i} \to (npz-1)\frac{1}{1-\delta(1-np)}$, (16) is violated. □

PROOF OF PROPOSITION 11

From the proof of Proposition 3 we know that the equal-sharing contract $w^S_{t,i} = Z/n$ and $w^F_{t,i} = 0$ implements the first-best if and only if for all $t \in \{1, ..., T\}$

$$p(z - \hat{C}_{t+1,i}) \geq 1. \tag{76}$$

Suppose, contrary to our claim, that there exists a wage contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ which implements the first-best when it is not implementable with the equal-sharing contract. When all players invest in t, then (20) has to be satisfied for all players. Hence,

$$p\left(\sum_{\mathcal{N}} w^S_{t,i} - \sum_{\mathcal{N}} w^F_{t,i} - \sum_{\mathcal{N}} C_{t+1,i}\right) \geq n. \tag{77}$$

Using the budget balance conditions (22) and dividing by n yields

$$p\left(Z/n - \sum_{\mathcal{N}} C_{t+1,i}/n\right) \geq 1. \tag{78}$$

Because all players always invest and the budget is balanced,

$$\sum_{\mathcal{N}} C_{t+1,i}/n = \hat{C}_{t+1,i}. \tag{79}$$

Hence, (78) gets

$$p(z - \hat{C}_{t+1,i}) \geq 1. \tag{80}$$

Because (80) coincides with (76), (80) cannot hold for all $t \in \{1, ..., T\}$ when (76) does not. A contradiction.

The second part of the proposition directly follows from Proposition 3. □

PROOF OF PROPOSITION 13

Part 1. We claim that a necessary condition that another contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ is better than the handsome contract is that in more periods all players invest.

With the handsome contract $n-1$ players invest in the early periods and n in the late periods. That is, the effort profile is $(e_1 = n-1, ..., e_{\underline{t}-1} = n-1, e_{\underline{t}} = n, ..., e_T = n)$, with $\underline{t} = \min\{t \in \mathbb{N} : t \geq T - \bar{x}\}$.

Suppose $e'_\tau = n-1$ and $e'_{\tau+1} = n$ for an effort profile and $e''_\tau = n$ and $e''_{\tau+1} = n-1$ for an alternatively one. All other effort levels are the same: $e'_t = e''_t$ for all $t \neq \tau, \tau+1$. Hence, $\sum_{\mathcal{N}} C_{\tau+2,i}$ is the same for both effort profiles. Using (23) twice we get that

$$\sum_{\mathcal{N}} C'_{\tau,i} = (n-1)pZ - (n-1) + (1-(n-1)p)\left(npZ - n + (1-np)\sum_{\mathcal{N}} C'_{\tau+2,i}\right), \tag{81}$$

$$\sum_{\mathcal{N}} C''_{\tau,i} = npZ - n + (1-np)\left((n-1)pZ - (n-1) + (1-(n-1)p)\sum_{\mathcal{N}} C''_{\tau+2,i}\right). \tag{82}$$

Because $\sum_{\mathcal{N}} C'_{\tau+2,i} = \sum_{\mathcal{N}} C'''_{\tau+2,i}$, we have $\sum_{\mathcal{N}} C'_{\tau,i} = \sum_{\mathcal{N}} C'''_{\tau,i}$. Because before period τ $e'_t = e''_t$ we get that $\sum_{\mathcal{N}} C'_{1,i} = \sum_{\mathcal{N}} C'''_{1,i}$.

These arguments can be repeated for any type of permutation of the effort profile $(e_1 = n-1, ..., e_{\underline{t}-1} = n-1, e_{\underline{t}} = n, ..., e_T = n)$. Hence, a necessary condition that another contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ is better than the handsome contract is that in more periods all players invest.

Part 2. Denote by $\#_t$ the number of periods from t until T in which all players invest. Denote the number of periods in which all players invest with the equal-sharing contract from period 1 to T by $\#^{ES}$. For the contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ to be better than the handsome contract, we know by Part 1 that the contract must induce in more periods investments of all players than the handsome contract: $\#_1 > \#^{ES}$. Denote the period in which for the contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$ we have $\#_t = \#^{ES} + 1$ by $\tilde{t}$.

We claim that $\sum_{\mathcal{N}} \hat{C}_{T-\#^{ES}+1,i}$ is a lower bound for $\sum_{\mathcal{N}} C_{\tilde{t}+1,i}$. Because additional effort is beneficial for the team's expected payoff, see Proposition 1, we get the following insight. For any effort profile $(e_{\tilde{t}+1}, ..., e_s, ..., e_T)$ for which there exists an e_s which is $0 < e_s < n$, with $s \in [\tilde{t}+1, T]$, the team's expected payoff $\sum_{\mathcal{N}} C_{\tilde{t}+1,i}$ is higher than for the effort profile $(e_{\tilde{t}+1}, ..., e_s = 0, ..., e_T)$. When we want to minimize $\sum_{\mathcal{N}} C_{\tilde{t}+1,i}$, while holding $\#_{\tilde{t}+1}$ fixed, we set all $e_s \neq n$ to 0. Then

$$\sum_{\mathcal{N}} C_{\tilde{t}+1,i} = \sum_{\mathcal{N}} \hat{C}_{T-\#_{\tilde{t}+1}+1,i}, \tag{83}$$

where we have used that $\sum_{\mathcal{N}} C_{t,i} = \sum_{\mathcal{N}} C_{t+1,i}$ when no player invests in a period t, see (23). Plugging in $\#_{\tilde{t}+1} = \#^{ES}$ establishes that $\sum_{\mathcal{N}} \hat{C}_{T-\#^{ES}+1,i}$ is a lower bound for $\sum_{\mathcal{N}} C_{\tilde{t}+1,i}$.

Part 3. Because all players are required to invest in period $\tilde{t}$ with the contract $(\boldsymbol{w^S}, \boldsymbol{w^F})$, it most hold for all players $i \in \mathcal{N}$ that

$$p\left(w^S_{\tilde{t},i} - w^F_{\tilde{t},i} - C_{\tilde{t}+1,i}\right) \geq 1, \tag{84}$$

Summing over all players and using the budget balance constraints (22) yields

$$p(Z - \sum_{\mathcal{N}} C_{\tilde{t}+1,i}) \geq n. \tag{85}$$

From Part 2 we know that $\sum_{\mathcal{N}} \hat{C}_{T-\#^{ES}+1,i}$ is a lower bound for $\sum_{\mathcal{N}} C_{\tilde{t}+1,i}$. Hence, also

$$p(Z - \sum_{\mathcal{N}} \hat{C}_{T-\#^{ES}+1,i}) \geq n \tag{86}$$

must hold. Dividing (86) by n yields

$$p(z - \hat{C}_{T-\#^{ES}+1,i}) \geq 1, \tag{87}$$

where we have used that $\hat{C}_{t,i}$ is the same for all players, see (3).

By the construction of $\bar{x}$ we have that with the equal-sharing contract all players invest for $t \geq T - \bar{x}$ and players do not invest before. Hence, for all periods $t < T - \bar{x}$

$$p(z - \hat{C}_{t+1,i}) < 1. \tag{88}$$

With the equal-sharing contract all players invest from only $\underline{t}$ until T. So that the number of periods for which all players invest indeed coincides with $\#^{ES}$ we must have

$$T - \underline{t} + 1 = \#^{ES} \iff \underline{t} = T - \#^{ES} + 1. \tag{89}$$

Because players do not invest with the equal-sharing contract in period $t = \underline{t} - 1$, we must have that, using (88) and (89),

$$p(z - \hat{C}_{T-\#^{ES}+1,i}) < 1. \tag{90}$$

Hence, (87) cannot hold. A contradiction. □

PROOF OF PROPOSITION 14

Consider two different periods and two different time limits: (i) the team's expected payoff in period t' with a time limit T' is $\sum_{\mathcal{N}} C_{t',i}^{T=T'}$; (ii) with $t'' = t' - t' + 1 = 1$ and $T'' = T' - t' + 1$ it is $\sum_{\mathcal{N}} C_{1,i}^{T=T''}$. Suppose that in both cases the team uses a handsome contract. It is immediate that then $\sum_{\mathcal{N}} C_{t',i}^{T=T'} = \sum_{\mathcal{N}} C_{1,i}^{T=T''}$. By Proposition 13 the handsome contract is second-best and so $\sum_{\mathcal{N}} C_{1,i}^{T=T''}$ is maximized. Hence, when the team is in period t' and the time limit is T' it cannot renegotiate to yield a higher expected payoff for the team than with the handsome contract. This proves that the handsome contract is renegotiation-proof.

To prove the second part of the proposition, suppose, contrary to our claim, that there is another renegotiation-proof contract $(\boldsymbol{w}^{\boldsymbol{S}'}, \boldsymbol{w}^{\boldsymbol{F}'})$ which is second-best and implements another effort profile $\boldsymbol{e} = (e_1, ..., e_t, ..., e_T)$. Because the handsome contract is also second-best and it induces investment by all players in the late periods $t \geq T - \bar{x}$, the contract $(\boldsymbol{w^{S'}}, \boldsymbol{w^{F'}})$ must induce more investment in the early periods and less in the late periods. Otherwise, the contract $(\boldsymbol{w^{S'}}, \boldsymbol{w^{F'}})$ and the handsome contract cannot both be second-best or both contracts would induce the same effort profile. Given the contract $(\boldsymbol{w^{S'}}, \boldsymbol{w^{F'}})$, once the first period with $t \geq T - \bar{x}$ is reached, the team can renegotiate the wage contract to the handsome contract (which induces equal sharing in the remaining periods). Renegotiating the wage contract induces a higher expected payoff for the team, because more players invest effort is some period(s) $s \geq t$, cf. Proposition 1. Hence, the team can make transfers so that all players are better off due to renegotiation. This proves that the alternative contract $(\boldsymbol{w^{S'}}, \boldsymbol{w^{F'}})$ is not renegotiation-proof. □

PROOF OF PROPOSITION 16

Consider the following contract. For all periods and players $w^F_{t,i} = 0$. In the late periods $t \geq T - \bar{x}$, use the equal-sharing contract: $w^S_{t,i} = Z/2$. In the early periods $t < T - \bar{x}$, use the alternating contract $w^S_{t,1} = Z$ and $w^S_{t,2} = 0$ for t even and $w^S_{t,1} = 0$ and $w^S_{t,2} = Z$ for t uneven.

We now prove that this contract implements the second-best. In the late periods, all players invest because in this periods the equal-sharing contract is used as in the handsome contract, which implements the second-best; see Proposition 13.

Before we look at the early periods we have to restrict the continuation payoffs. It is useful to compare the two-player problem with the problem of a single player who faces the same parameter constellation. Observe that $C^{n=1}_{T,1} = pZ - 1 = 2pZ/2 - 1 = C^{n=2}_{T,i}$. If period $t < T$ is a late period we have for the single-player problem

$$C^{n=1}_{t,1} = pZ + (1-p)C^{n=1}_{t+1,1} - 1 \tag{91}$$

and for the two-player problem

$$C^{n=2}_{t,i} = 2pZ/2 + (1-2p)C^{n=2}_{t+1,i} - 1 = pZ + (1-2p)C^{n=2}_{t+1,i} - 1. \tag{92}$$

Because $C^{n=2}_{T,i} \leq C^{n=1}_{T,1}$, (91) and (92) imply that $C^{n=2}_{T-1,i} \leq C^{n=1}_{T-1,1}$. Repeating the arguments leads to the conclusion that in all late periods $C^{n=2}_{t,i} \leq C^{n=1}_{t,1}$.

We next look at the latest early period, called $\tilde{t}$. Player j is induced to invest effort while player $-j$ is not. Player j invests effort in this period if and only if, see (20) and recall that $w^S_{\tilde{t},j} = Z$ and $w^F_{\tilde{t},j} = 0$,

$$p\left(Z - C^{n=2}_{\tilde{t}+1,j}\right) \overset{?}{\geq} 1. \tag{93}$$

We know that the single player invests effort which implies that

$$p\left(Z - C^{n=1}_{\tilde{t}+1,1}\right) \geq 1. \tag{94}$$

Because we know from above that $C^{n=2}_{\tilde{t}+1,j} \leq C^{n=1}_{\tilde{t}+1,1}$, (93) has to be satisfied, too.

Consider next the period $\tilde{t}-1$, where player $-j$ should invest effort and $w^S_{\tilde{t}-1,-j} = Z$ and $w^F_{\tilde{t}-1,-j} = 0$. Because player j may have success in period $\tilde{t}$, it holds that $C^{n=2}_{\tilde{t},-j} < C^{n=2}_{\tilde{t}+1,j}$. Therefore, the arguments from above can be repeated to show that player $-j$ invests effort in period $\tilde{t}-1$. More generally, when player j invests effort in an early period t, player $-j$ also invests effort in period $t-1$.

Consider next the period $\tilde{t}-2$. It holds that player j's continuation payoff of reaching the next period $\tilde{t}-1$ is lower than the one of a single player:

$$\begin{aligned} C^{n=2}_{\tilde{t}-1,j} = (1-p)C^{n=2}_{\tilde{t},j} &= (1-p)\left(pZ - 1 + (1-p)C^{n=2}_{\tilde{t}+1,j}\right) \\ &\leq pZ - 1 + (1-p)\left(pZ - 1 + (1-p)C^{n=1}_{\tilde{t}+1,1}\right) \\ &= pZ - 1 + (1-p)C^{n=1}_{\tilde{t},1} = C^{n=1}_{\tilde{t}-1,1}, \end{aligned} \tag{95}$$

where the first equality follows from $w^S_{\tilde{t}-1,j} = w^F_{\tilde{t}-1,j} = 0$ and that only player $-j$ invests effort in period $\tilde{t}-1$. The second equality uses that $w^S_{\tilde{t},j} = Z$ and $w^F_{\tilde{t},j} = 0$ and that only player j invests effort in period $\tilde{t}$. The inequality follows from our above finding that $C^{n=2}_{\tilde{t}+1,j} \leq C^{n=1}_{\tilde{t}+1,1}$ and that $pZ \geq 1$.

Because the single player invests in period $\tilde{t}-2$, we have

$$p(Z - C^{n=1}_{\tilde{t}-1,1}) \geq 1. \tag{96}$$

Because $C^{n=2}_{\tilde{t}-1,j} \leq C^{n=1}_{\tilde{t}-1,1}$, also player j invests in period $\tilde{t}-2$:

$$p(Z - C^{n=2}_{\tilde{t}-1,j}) \geq 1. \tag{97}$$

The arguments can be repeated to show that j will invest in periods $\tilde{t}-4, \tilde{t}-6, \ldots$.

Trivially, a player i for whom $w^S_{t,i} = w^F_{t,i} = 0$ does not invest effort in period t. This proves the desired result. □

PROOF OF PROPOSITION 17

Part 1. We suppose that the second-best is implemented and let T be large. We seek to find a necessary condition on how large Z has to be. We consider the relaxed problem where $\sum_{\mathcal{N}} w^S_{t,i}$ is constant and where $\sum_{\mathcal{N}} w^S_{t,i} \leq Z$. We want to set wages $w^S_{t,i}$ as low as possible and obtain minimal continuation payoffs.

Consider some t which is far away from T. Then in the second-best $n-1$ players invest effort. For a player i who invests effort in t we set the lowest possible wage, given the incentive constraint (20),

$$w^S_{t,i} = \frac{1}{p} + C_{t+1,i}. \tag{98}$$

Player i's Bellman equation is

$$C_{t,i} = (n-1)pw^S_{t,i} + (1-(n-1)p)C_{t+1,i} - 1, \tag{99}$$

where we have used that $w^F_{t,i} = 0$. Plugging (98) into (99) we get

$$C_{t,i} = n - 2 + C_{t+1,i}. \tag{100}$$

There is also a player who does not invest effort in t. We call her player $j(t)$. For this player the Bellman equation is, setting $w^S_{t,j(t)}$ as low as possible, namely $w^S_{t,j(t)} = 0$,

$$C_{t,j(t)} = (1-(n-1)p)C_{t+1,j(t)}. \tag{101}$$

Summing over all players we get, using (100) and (101),

$$\sum_{\mathcal{N}} C_{t,i} = (n-1)(n-2) + \sum_{\mathcal{N}} C_{t+1,i} - (n-1)pC_{t+1,j(t)}. \tag{102}$$

Because T is large, relatively to t, and we assumed that the second-best is implemented we have $\sum_{\mathcal{N}} C_{t,i} \approx \sum_{\mathcal{N}} C_{t+1,i}$, where the failure gets arbitrarily small as T gets larger, see (24). Hence, (102) gets

$$C_{t+1,j(t)} = \frac{n-2}{p} + \varepsilon_1, \tag{103}$$

where ε_1 is arbitrarily small when T gets large.

Combining (101) and (103) yields

$$C_{t,j(t)} = \frac{n-2}{p} - (n-1)(n-2) + \varepsilon_2. \tag{104}$$

Part 2. When should which player not invest effort? It is useful to define the number of periods player i will invest effort uninterruptedly from period t on by $d_{t,i}$. For example, when $j(t) = i$ we have $d_{t,i} = 0$, when $j(t) \neq i$ and $j(t+1) = i$ then $d_{t,i} = 1$, and so on. Then we get from (100) that

$$C_{t,i} = d_{t,i}(n-2) + C_{t+d_{t,i},j(t+d_{t,i})=i}. \tag{105}$$

Note that due to limited liability continuation payoffs cannot be negative. Together with $\sum C_{t,i} < Z$ and (105) this implies that $d_{t,i}$ cannot be larger than $Z/(n-2)$; otherwise $C_{t,i} > Z$.

Consider the problem of minimizing $\sum C_{t,i}$ subject to (105) and that $d_{t,i} \leq Z/(n-2)$.

We claim that it is best to let in every period t until $t+n-1$ another player not invest effort. We prove by contradiction. When there is another schedule which is better, then there must be a player i' who does not invest in effort at least twice between t and $t+n-1$. Denote the period in which i' does not invest effort the second time by t'. There is also a player with the highest value of $d_{t,i}$ (from above we know that $d_{t,i} \leq Z/(n-2)$) which we call player i''. She invests effort from t until $t + d_{t,i''} - 1$, and does not invest in period $t + d_{t,i''}$. From (105) we get that

$$C_{t,i''} = d_{t,i''}(n-2) + C_{t+d_{t,i''},j(t+d_{t,i''})=i''}. \tag{106}$$

If we change the schedule and let player i'' not invest in period t' then we get

$$C_{t,i''} = (t'-t)(n-2) + C_{t',j(t')=i''}. \tag{107}$$

From (104) we see that $C_{t,j(t)}$ is approximately constant for periods t until $t + d_{t,i'}$. Hence,

$$(t'-t)(n-2) + C_{t',j(t')=i''} < d_{t,i''}(n-2) + C_{t+d_{t,i''},j(t+d_{t,i''})=i''}. \tag{108}$$

Because for all other players of (105) is left unchanged, $\sum_{\mathcal{N}} C_{t,i}$ is not minimized. A contradiction.

Part 3. If in every period t until $t+n-1$ another player does not invest effort then we get from (105) that

$$\sum_{\mathcal{N}} C_{t,i} = nC_{t,j(t)} + (n-2)\sum_{\mathcal{N}} d_{i,t} + \varepsilon_3$$
$$= nC_{t,j(t)} + (n-2)(0+1+...+(n-1)) + \varepsilon_3$$
$$= nC_{t,j(t)} + n(n-2)\frac{n-1}{2} + \varepsilon_3, \quad (109)$$

where ε_3 is arbitrarily small when T gets large.

Part 4. Plugging (104) into (109) yields

$$\sum_{\mathcal{N}} C_{t,i} = n(n-2)/p - n(n-1)(n-2)/2 + \varepsilon_4. \quad (110)$$

Proposition 1 and (3) imply that $\sum_{\mathcal{N}} C_{t,i} < Z - 1/p$ when $\sum_{\mathcal{N}} w^S_{t,i} = Z$, which also hold when $\sum_{\mathcal{N}} w^S_{t,i} \leq Z$. Hence, when

$$n(n-2)/p - n(n-1)(n-2)/2 + \varepsilon_4 \geq Z - 1/p \quad (111)$$

the second-best is not implementable when T is large. Rewriting (111), the second-best with limited liability is not implementable with limited liability if T is sufficiently large and

$$Z < \underline{Z} := n(n-2)/p + 1/p - n(n-1)(n-2)/2. \quad (112)$$

□

PROOF OF PROPOSITION 18

The proof is constructive. We construct a time-independent and nondiscriminatory wage contract: $w^S = w^S_{t,i}$ and $w^F = w^F_{t,i}$ for all $i \in \mathcal{N}$ and all $t \in \{1, ..., T\}$. When a player invests effort in t, her expected payoff in period t is zero when

$$npw^S + (1-np)w^F - 1 = 0. \quad (113)$$

When wages satisfy this for all periods, then $C_{t,i} = 0$ for all $t \in \{1, ..., T\}, i \in \mathcal{N}$. To induce always investment by all players we must have

$$p(w^S - w^F) \geq 1. \quad (114)$$

The wages w^S and w^F solve (113) and (114) and induce that all players always invest.

By choosing the appropriate transfer which the principal has to pay to the team, the team can make sure that principal's expected profit is zero. Alternatively, one can set the transfer equal to zero and adjust $w^S_{1,i}$ and $w^F_{1,i}$ accordingly. □

PROOF OF PROPOSITION 20

Consider first the case where $Z' = Z''$. With n' players we have $e_t^{n=n'} \leq n'$. With n'' players and when T is sufficiently short, it is optimal for the team to use the equal-sharing contract, see Proposition 11. Then $e_t = n''$ for all $t \in \{1, ..., T\}$. When T is longer, then the first-best cannot be implemented and the handsome contract is second-best, see Proposition 13. It implements $e_t^{n=n''} = n''$ for periods $t \geq T - \bar{x}$ and $e_t^{n=n''} = n'' - 1$ for periods $t < T - \bar{x}$.

From (24) we see that when $e_t^{n=n''} = e_t^{n=n'}$ then

$$\sum_{i=1}^{n''} C_{t,i}^{n=n''} = \sum_{i=1}^{n'} C_{t,i}^{n=n'}. \tag{115}$$

Proposition 1 says that the team's expected payoff $\sum_{i=1}^{n''} C_{t,i}$ is increasing in e_s for all $s \geq t$. From before we know that $e_s^{n=n''} \geq e_s^{n=n''}$ for all periods and $e_s^{n=n''} > e_t^{n=n''}$ for some periods $s \geq t$. Hence,

$$\sum_{i=1}^{n''} C_{t,i}^{n=n''} > \sum_{i=1}^{n'} C_{t,i}^{n=n'}. \tag{116}$$

From (24) we see that for a given effort profile $\boldsymbol{e} = (e_1, ..., e_t, ..., e_T)$ the team's expected payoff $\sum_{i=1}^{n''} C_{t,i}$ is increasing in Z. From Proposition 4 we know that $d\bar{x}/dz > 0$, which implies that for a higher Z an effort profile with weakly more team effort e_t in every period can be implemented. Both effects imply that a team's expected payoff is increasing in Z. Hence, for all $Z'' > Z'$ (116) holds. $\square$

PROOF OF PROPOSITION 21

Part 1. For $\phi = 1$, condition (43) gets

$$np\left(z - C_{t+1,i}\right) \geq 1, \tag{117}$$

where we have used that the continuation payoff is the same for all players. From Lemma 2 we know that $C_{t+1,i} \leq \hat{C}_{t+1,i}$. Moreover, for any finite T, $\hat{C}_{t+1,i}$ is lower than $\lim_{T\to\infty} \hat{C}_{t+1,i} = z - \frac{1}{np}$. Because

$$np\left(z - \left(z - \frac{1}{np}\right)\right) = 1, \tag{118}$$

(117) always holds. Hence, all players always invest effort.

Part 2. For $\phi < 1$, (43) gets

$$p(1+\phi(n-1))\,(z - C_{t+1,i}) \geq 1. \tag{119}$$

When all players always invest, $C_{t+1,i}$ is arbitrarily close to $\lim_{T\to\infty} \hat{C}_{t+1,i} = z - \frac{1}{np}$ when T is sufficiently large. Because

$$p(1+\phi(n-1))\left(z - \left(z - \frac{1}{np}\right)\right) = \frac{1+\phi(n-1)}{n} < 1, \tag{120}$$

it cannot hold that all players always invest when T is sufficiently large.

Part 3. Let $\grave{x}(\phi)$ solve (119):

$$p(1+\phi(n-1))\left(z - \hat{C}_{T-\grave{x}(\phi),i}\right) \stackrel{!}{=} 1. \tag{121}$$

Because $\hat{C}_{T-\grave{x}(\phi),i}$ is increasing in $\grave{x}(\phi)$, see (3), we get that $\grave{x}(\phi)$ is increasing in ϕ. Define $\bar{x}(\phi) := \grave{x}(\phi) + 1$. By the same arguments as in the proof of Proposition 3 we get that when the time limit is sufficiently short, namely $T \leq \bar{x}(\phi) + 1$, then players always invest effort and that when the time limit is sufficiently long, namely $T > \bar{x}(\phi) + 1$, then players do not invest effort in periods $t < T - \bar{x}(\phi)$ and invest effort in the periods $t \geq T - \bar{x}(\phi)$. $\square$

PROOF OF LEMMA 3

Start with the case $\phi_i, \phi_{-i} = 0$. $C_{t(0,0),i} > z - 1/p$; otherwise player i would invest in $t(0,0) - 1$. Moreover, $C_{t(0,0)+1,i} \leq z - 1/p$, otherwise player i would not invest in period $t(0,0)$. Because $t(0,\phi_{-i}) \geq t(\bar{\phi},\phi_{-i})$, all players invest from period $t(0,0)$ to T we get then with (43) and (3) that

$$\hat{C}_{t(0,0),i} = (npz-1)\frac{1-(1-np)^{T-t(0,0)+1}}{np} > z - 1/p \tag{122}$$

and

$$\hat{C}_{t(0,0)+1,i} = (npz-1)\frac{1-(1-np)^{T-t(0,0)}}{np} \leq z - 1/p. \tag{123}$$

Next consider the case $\phi_i = 0$ and $\phi_{-i} = \bar{\phi}$. Because $t(0,\phi_{-i}) \geq t(\bar{\phi},\phi_{-i})$, all players invest from period $t(0,\bar{\phi})$ to T. We must have

$$\hat{C}_{t(0,\bar{\phi}),i} = (npz-1)\frac{1-(1-np)^{T-t(0,\bar{\phi})+1}}{np} > z - 1/p. \tag{124}$$

and

$$\hat{C}_{t(0,\bar{\phi})+1,i} = (npz-1)\frac{1-(1-np)^{T-t(0,\bar{\phi})}}{np} \leq z - 1/p. \tag{125}$$

Comparing (122)-(125) yields that we must have $t(0,0) = t(0,\bar{\phi})$.

Finally, consider the remaining cases where $\phi_i = \bar{\phi}$. By (43) it must hold that

$$p\left(z + \bar{\phi}z - C_{t+1,i} - \bar{\phi}C_{t+1,-i}\right) \geq 1 \tag{126}$$

for all $t \geq t(\bar{\phi},\bar{\phi})$. Similarly,

$$p\left(z + \bar{\phi}z - C_{t+1,i} - \bar{\phi}C_{t+1,-i}\right) \geq 1 \tag{127}$$

for all $t \geq t(\bar{\phi}, 0)$.

For the case $\phi_i = \phi_{-i} = \bar{\phi}$ all players invest from period $t(\bar{\phi},\bar{\phi})$ until T. Hence,

$$C_{t(\bar{\phi},\bar{\phi})+1,i} + \bar{\phi}C_{t(\bar{\phi},\bar{\phi})+1,-i} = (1+\bar{\phi})\hat{C}_{t(\bar{\phi},\bar{\phi})+1,i}. \tag{128}$$

When $\phi_i = \bar{\phi}$ and $\phi_{-i} = 0$ we get that

$$\begin{aligned} &C_{t(\bar{\phi},0)+1,i} + \bar{\phi}C_{t(\bar{\phi},0)+1,-i} = (1-\bar{\phi})C_{t(\bar{\phi},0)+1,i} + \bar{\phi}\left(C_{t(\bar{\phi},0)+1,i} + C_{t(\bar{\phi},0)+1,-i}\right) \\ &\leq (1-\bar{\phi})C_{t(\bar{\phi},0)+1,i} + \bar{\phi}2\hat{C}_{t(\bar{\phi},0)+1,i} \leq (1-\bar{\phi})\hat{C}_{t(\bar{\phi},0)+1,i} + \bar{\phi}2\hat{C}_{t(\bar{\phi},0)+1,i} = (1+\bar{\phi})\hat{C}_{t(\bar{\phi},0)+1,i}, \end{aligned} \tag{129}$$

where the first inequality follows from the finding that in the first-best, i.e., where continuation payoffs are maximized, all players always invest effort. The second inequality follows from $t(0,\phi_{-i}) \geq t(\bar{\phi},\phi_{-i})$ and the fact that player i is better off when player $-i$ invests than when not (player i invests from period $t(\bar{\phi},0)$ on until T, while this is not necessarily true for player $-i$).

From (128) and (129) and the finding that $\hat{C}_{t+1,i}$ is decreasing in t it follows that whenever (126) holds for t, (127) has to hold, too. Hence, $t(\bar{\phi},0) \leq t(\bar{\phi},\bar{\phi})$. □

PROOF OF PROPOSITION 22

With homogenous teams their is one team with selfish players. This teams' aggregated expected payoff is, using (3),

$$2\hat{C}_{t(0,0),i} = 2(2pz-1)\frac{1-(1-2p)^{T-t(0,0)+1}}{2p}. \tag{130}$$

For the team with the social players we have

$$2\hat{C}_{t(\bar{\phi},\bar{\phi}),i} = 2(2pz-1)\frac{1-(1-2p)^{T-t(\bar{\phi},\bar{\phi})+1}}{2p}. \tag{131}$$

When there are heterogenous teams, the each team's expected payoff is, using (2),

$$(2pz-1)\frac{1-(1-p)^{t(0,0)-t(\bar{\phi},0)}}{p} + (1-p)^{t(0,0)-t(\bar{\phi},0)}2\hat{C}_{t(0,0),i}, \tag{132}$$

where the first part is the team's expected payoff from periods$t(\bar{\phi},\bar{\phi})$ until the period before $t(0,\bar{\phi}) = t(0,0)$. This part diminishes when $t(\bar{\phi},\bar{\phi}) = t(0,0)$. The second part is the teams' expected payoff from period $t(0,\bar{\phi}) = t(0,0)$ until T, where we have taken into account that this period is reached only with probability $(1-p)^{t(0,0)-t(\bar{\phi},0)}$.

We claim that having two heterogenous teams lead to a weakly higher aggregated expected payoff than having one team with selfish players and one with social players:

$$\begin{aligned}(4pz-2)&\frac{1-(1-p)^{t(0,0)-t(\bar{\phi},0)}}{p} + 2(1-p)^{t(0,0)-t(\bar{\phi},0)}(4pz-2)\frac{1-(1-2p)^{T-t(0,0)+1}}{2p}\\ &\geq (4pz-2)\frac{1-(1-2p)^{T-t(0,0)+1}}{2p} + (4pz-2)\frac{1-(1-2p)^{T-t(\bar{\phi},\bar{\phi})+1}}{2p}.\end{aligned} \tag{133}$$

Dividing by $(4pz-2)$ and multiplying by $2p$ yields

$$\begin{aligned}2\left(1-(1-p)^{t(0,0)-t(\bar{\phi},0)}\right) &+ 2(1-p)^{t(0,0)-t(\bar{\phi},0)}\left(1-(1-2p)^{T-t(0,0)+1}\right)\\ &\geq 1-(1-2p)^{T-t(0,0)+1} + 1-(1-2p)^{T-t(\bar{\phi},\bar{\phi})+1}.\end{aligned} \tag{134}$$

After simplifying and multiplying by -1 we get

$$2(1-p)^{t(0,0)-t(\bar{\phi},0)}(1-2p)^{T-t(0,0)+1} \leq (1-2p)^{T-t(0,0)+1} + (1-2p)^{T-t(\bar{\phi},\bar{\phi})+1}. \tag{135}$$

Dividing by $(1-2p)^{T-t(0,0)+1}$ yields

$$2(1-p)^{t(0,0)-t(\bar{\phi},0)} \leq 1 + (1-2p)^{t(0,0)-t(\bar{\phi},\bar{\phi})}. \tag{136}$$

For $t(0,0) - t(\bar{\phi},0) = 0$ or $t(0,0) - t(\bar{\phi},0) = 1$ this formula holds with equality. Because $t(0,0) - t(\bar{\phi},0) \geq 0$, it remains to check that it holds for $t(0,0) - t(\bar{\phi},0) \geq 2$. It is useful to define $g(p) = (1-p)^{\zeta}$. For $\zeta \geq 2$ this function is convex in p for $p \in [0,1]$. By convexity we have

$$g(p) \leq \frac{g(p+\nu) + g(p-\nu)}{2} \tag{137}$$

for all ν that satisfy $p+\nu, p-\nu \in [0,1]$. Hence, also

$$2g(p) \leq g(2p) + g(0). \tag{138}$$

Substituting $t(0,0) - t(\bar{\phi}, \bar{\phi})$ for ζ proves the desired result. □

PROOF OF LEMMA 4

The first-best maximizes $\sum_{\mathcal{N}} C_{t,j}$. We claim that in the first-best all players choose the same effort level in a period. Suppose not. Then there exist $e_{t,j} \neq e_{t,i}$. When both players i and j choose $\frac{e_{t,i}+e_{t,j}}{2}$, $\sum_{\mathcal{N}} C_{t,j}$ is improved because $2k\left(\frac{e_{t,i}+e_{t,j}}{2}\right) < k(e_{t,j}) + k(e_{t,i})$ due to $k'' > 0$.

Note that $C_{t+1,j} = C_{t+1}$ for all $j \in \mathcal{N}$ because all players choose the same investment in a period, and this is true for all periods. Iterative use of (BELLMAN) yields

$$C_t = \sum_{s=t}^{\infty} \left[(ne_s z - k(e_s)) \prod_{r=t}^{s-1} \delta(1 - ne_r) \right] \tag{139}$$

From (BELLMAN) we see that $\max_{e_t} C_t$ is increasing in C_{t+1}. Hence, to maximize C_t we also maximize C_{t+1}. In period t we maximize C_t, see (139), over $(e_t, e_{t+1}, ...)$, in period $t+1$ we maximize C_{t+1} over $(e_{t+1}, e_{t+2}, ...)$. So the problem is always the same and hence C_t is maximized when always the same effort e is chosen. That is, in the first-best all players choose e^{FB} at every time which yields $C_{i,t} = C^{FB}$. Rewriting (BELLMAN) yields that

$$C^{FB} = \frac{ne^{FB}z - k(e^{FB})}{1 - \delta + \delta ne^{FB}}, \tag{140}$$

where $e^{FB} = argmax \frac{nez - k(e)}{1-\delta+\delta ne}$. □

PROOF OF PROPOSITION 23

It is useful to write the C which satisfies (B SMPE) as a function $C_B(e,n)$, where B stands for the Bellman equation. Similarly, we write $C_{IC}(e)$ for the valuation C satisfying (IC SMPE).

Existence. We first proof existence. Essentially, we want to show that $C_B(e)$ and $C_{IC}(e)$ evolve like in Figure 3, where the functions $C_B(e,n)$ and $C_{IC}(e)$ cross at least once.

We first determine how $C_B()$ evolves. For all n, we get that

$$\frac{d^2 C_B(e,n)}{de^2} \propto -k''(e)\frac{(1+\delta-\delta ne)^2}{2\delta n} - z(n-1)(1-\delta) - (n-1)\delta c < 0. \tag{141}$$

Because $C_B(0,n) = 0$, $dC_B(0,n)/de > 0$, $C_B(e,n) \leq 0$ for e sufficiently large, and $dC_B(e,n)^2/de^2 < 0$, see (141), we know that $C_B(e,n)$ is inverted-U shaped in e and has a maximum at some $e > 0$, but no minimum. Moreover, $C_B(e,n) \geq 0$ if and only if $e \in [0, \bar{e}_B(n)]$, where $\bar{e}_B(n)$ is implicitly defined by the $e > 0$ which solves

$$n\bar{e}_B(n)z = k(\bar{e}_B(n)). \tag{142}$$

Next we look how $C_{IC}()$ evolves. We get that

$$\frac{dC_{IC}(e)}{de} = -\frac{k''(e)}{\delta} < 0. \tag{143}$$

Moreover, $C_{IC}(e) \geq 0$ if and only if $e \leq \bar{e}_{IC}$, where $\bar{e}_{IC}$ is implicitly defined by

$$z = k'(\bar{e}_{IC}). \tag{144}$$

We get from (142) and (144) that

$$\frac{k(\bar{e}_B(n))}{n\bar{e}_B(n)} = z = k'(\bar{e}_{IC}). \tag{145}$$

Because $k(e)/e < k'(e)$, $k(e)/e$ and $k'(e)$ are increasing in e, $n \geq 1$, we must have $\bar{e}_B(n) > \bar{e}_{IC}$.

Observe that $C_B(e,n)$ and $C_{IC}(e)$ are continuous in e and $C_B(0,n) = 0 < z/\delta = C_{IC}(0)$. Together with the insights from before it follows that $C_B(e)$ and $C_{IC}(e)$ cross at least once. That is, there exists a e such that $C_B(e,n) = C_{IC}(e)$.

We have to check that for an e which solves $C_B(e,n) = C_{IC}(e)$ the present value of the player's expected payoff is maximized not minimized. From (BELLMAN) we see that

$$\frac{d^2 C_{t,i}}{de_{t,i}^2} = -k''(e_{t,i}) < 0. \tag{146}$$

Hence, the second-order condition is satisfied. Therefore, when e solves $C_B(e,n) = C_{IC}(e)$ we have a maximum and no minimum. Hence, a MPE always exists.

Uniqueness. Next we prove uniqueness. We want to show that $C_{IC}(e)$ and $C_B(e,n)$ only intersect once, as in Figure 4.

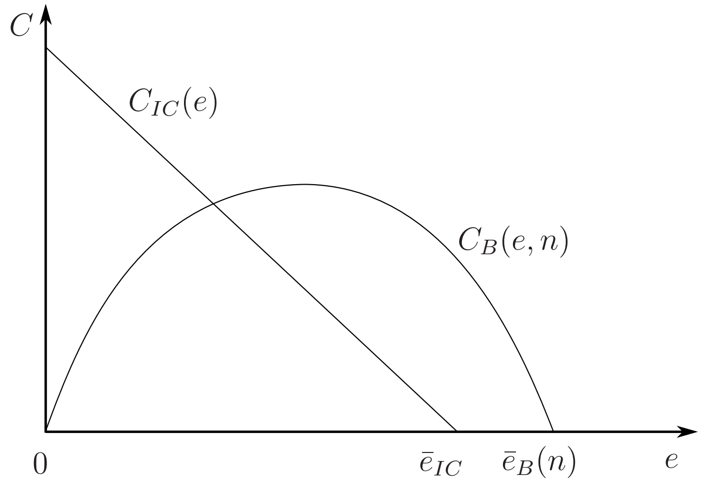

Figure 3: Existence.

For $n = 1$ the effort level e which solves $C_B(e,1) = C_{IC}(e)$ is the one that maximizes $C_B(e,1)$: $e^{FB}(1) = argmax_e C_B(e,1)$. Because $d^2C_B(e,n)/de^2 < 0$, it follows that $C_B(e,1)$ has no other extreme point. Consequently, $C_B(e,1) = C_{IC}(e)$ is only solved by $e^{FB}(1)$ and no other e. That is, the MPE is unique when $n = 1$.

For more players ($n > 1$) we get that, given some $e > 0$,

$$\frac{dC_B(e,n)}{dn} \propto z(1-\delta) + \delta k(e) > 0. \tag{147}$$

Note that $C_{IC}(e)$ does not change with n when z and all other parameters are kept fix.

Because

$$\frac{dC_B(e,n)}{de} \propto nz - \delta nz - k'(e)(1 - \delta + \delta ne) + k(e)\delta n, \tag{148}$$

$e^{FB}(n)$ is implicitly defined by

$$nz - \delta nz - k'(e^{FB}(n))(1 - \delta + \delta ne^{FB}(n)) + k(e^{FB}(n))\delta n = 0. \tag{149}$$

By differentiating implicitly we get that

$$\begin{aligned} \frac{de^{FB}(n)}{dn} &= -\frac{z - \delta z - k'(e^{FB}(n))\delta e^{FB}(n) + k(e^{FB}(n))\delta}{-k''(e^{FB}(n))(1 - \delta + \delta ne^{FB}(n))} \\ &\propto z(1-\delta)^2 + k(e^{FB}(n))\delta(1-\delta) > 0, \end{aligned}$$

where we have used the first-order condition $k'(e^{FB}(n)) = nz - n\delta C$.

We have that $dC_B(e,n)/de > 0$ for all $e < e^{FB}(n)$. Because $de^{FB}(n)/dn > 0$, for $n > 1$, we get that $dC_B(e,n)/de > 0$ for all $e \leq e^{FB}(1)$. Therefore, $C_B(n)$ and $C_{IC}(n)$ cross only once for $e \in [0, e^{FB}(1)]$. Because (i) $C_B(e,1)$ and $C_{IC}(e)$ do not cross for any $e > e^{FB}(1)$ and (ii) $C_B(e,n) > C_B(e,1)$ for all $n > 1$, also $C_B(e,n)$ and $C_{IC}(e)$ do not cross for any $e > e^{FB}(1)$. Hence, also for $n > 1$ the SMPE is unique. This is illustrated in Figure 4.

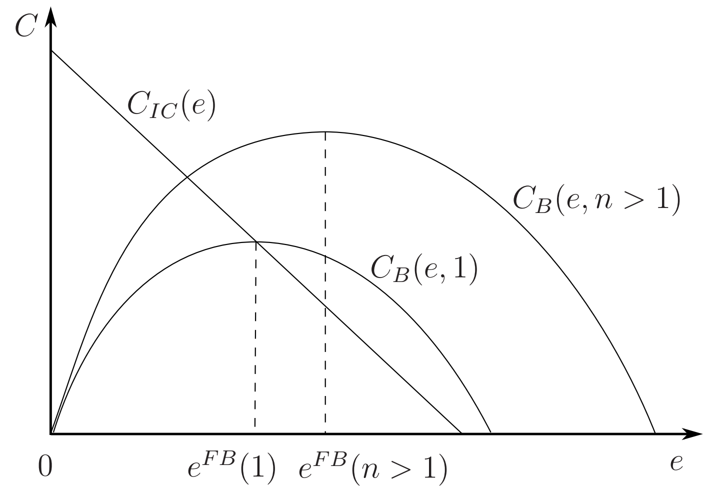

Figure 4: Uniqueness.

□

PROOF OF PROPOSITION 24

We again write $C_B(e,n)$ for the C which satisfies (B SMPE) and $C_{IC}(e)$ for the C satisfying (IC SMPE).

The results follow from, see especially the proof of Lemma 23, that (i) $C_B(e)$ is inverted-U shaped in e, (ii) $C_B(e)$ is increasing in n for all $e > 0$, (iii) $C_{IC}(e)$ is decreasing in e but constant in n, (iv) e^{FB} is the SMPE with $n = 1$, and (v) e^{FB} is increasing in n. This is illustrated in Figure 1, which is in the main part of the paper. □

APPENDIX B: SYMMETRIC NON-MPE AND ASYMMETRIC MPE

In our model, players are symmetric and play every period the same game. Nonetheless, there may exist non-Markov perfect equilibria where players choose symmetric investments as well as Markov perfect equilibria where players choose asymmetric investments. The following examples are instructive.

SYMMETRIC NON-MARKOV PERFECT EQUILIBRIA

Suppose $n = 2$, $\delta = 0.5$, and

$$k(e) = \begin{cases} (1-\varepsilon)e & , \quad \text{for } e \in [0, 0.1], \\ (1-\varepsilon) + (1+\varepsilon)e, & \quad \text{for } e \in (0.1, 0, 2], \\ \infty & , \quad \text{otherwise.} \end{cases}$$

We want to construct an equilibrium where players choose

$$e_t = \begin{cases} 0.2, & \text{for } t \text{ uneven}, \\ 0 \quad , & \text{for } t \text{ even}. \end{cases}$$

Suppose that this is an equilibrium. Then

$$C_{uneven} = 0.4 \times z + 0.6 \times 0.5 \times C_{even} - 0.2 \tag{150}$$

and

$$C_{even} = 0.5 \times C_{uneven}. \tag{151}$$

Inserting the latter in the former equation we get

$$C_{uneven} = \frac{0.4 \times z - 0.2}{0.85} \tag{152}$$

and

$$C_{even} = \frac{0.4 \times z - 0.2}{1.7}. \tag{153}$$

To prove that this is an equilibrium we have to check the incentive constraints. When t is uneven we need

$$z - 0.5 \times C_{even} \geq 1 + \varepsilon \iff z - \frac{0.4 \times z - 0.2}{3.4} \geq 1 + \varepsilon \tag{154}$$

and when t is even

$$z - 0.5 \times C_{uneven} \leq 1 - \varepsilon \iff z - \frac{0.4 \times z - 0.2}{1.7} \leq 1 - \varepsilon. \tag{155}$$

Hence, for $z \in (16/15, 15/13)$ and ε sufficiently small, both incentive constraints are satisfied. Intuitively, in uneven periods players do not invest because the continuation payoff of reaching the next period is high and the continuation payoff of even periods is high because players invest a lot of effort. The intuition for why there is high investment in even periods is vice versa.

Interestingly, players may be better off by playing a non-Markov perfect equilibrium than by playing a Markov perfect equilibrium. For example, when $z = 1.1$ and $\varepsilon = 0.02$ then $C_{uneven} \approx 0.282$ and $C_{even} \approx 0.141$. In the SMPE we have $e^{SMPE} = 0.1$ and $C^{SMPE} \approx 0.203$. So at the beginning of the first period players prefer to coordinate on playing the non-Markov perfect equilibrium than on playing the SMPE. This is intuitive: in period 1, players invest much more in the non-Markov perfect equilibrium than in the SMPE. Therefore, from the perspective of period 1, their expected payoff is higher in the non-Markov perfect equilibrium than in the SMPE.

ASYMMETRIC MARKOV PERFECT EQUILIBRIA

Suppose that $n = 2$, $\delta = 0.5$, and

$$k(e) = \begin{cases} e\ , & \text{for } e \leq 0.5, \\ \infty, & \text{otherwise.} \end{cases}$$

We want to check when $e_{t,1} = 0$ and $e_{t,2} = 0.5$ is a MPE. Then

$$C_{t,1} = \frac{0.5 \times z}{0.75} = \frac{2}{3}z \tag{156}$$

and

$$C_{t,2} = \frac{0.5 \times z - 0.5}{0.75} = \frac{2}{3}z - \frac{2}{3}. \tag{157}$$

It remains to check that the players' incentive constraints are satisfied. We need to have

$$z - 0.5 \times \frac{2}{3}z \leq 1 \iff z \leq 3/2 \tag{158}$$

and

$$z - 0.5 \times \left(\frac{2}{3}z - \frac{2}{3}\right) \geq 1 \iff z \geq 1. \tag{159}$$

Hence, for $z \in [1, 1.5]$ $e_{t,1} = 0$ and $e_{t,2} = 0.5$ is a MPE.

The intuition is that because player 1 invests no effort her continuation payoff $C_{t,1}$ is much higher than the one of player 2, $C_{t,2}$, who spends effort. Given this, player 1 has no incentive to invest in t or in subsequent periods, whereas player 2 has incentives to invest effort.

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