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strategic ambiguity

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Abstract

Most insurance companies publish few data on the occurrence and detection of insurance fraud. This stands in contrast to the previous literature on costly state verification, which has shown that it is optimal to commit to an auditing strategy. The credible announcement of thoroughly auditing claim reports is a powerful deterrent. Yet, we show that uncertainty about fraud detection can be an effective strategy to deter ambiguity-averse agents from reporting false insurance claims. If, in addition, the auditing costs of the insurers are heterogeneous, it can be optimal not to commit, because committing to a fraud-detection strategy eliminates the ambiguity. Thus, strategic ambiguity can be an equilibrium outcome in the market. Even competition does not force firms to provide the relevant information. This finding is also relevant in other auditing settings, like tax enforcement.

JEL classifications: D8, K4

Keywords: Fraud; Commitment; Ambiguity; Costly State Verification; Insurance; Audit

1 Introduction

Fraudulent claims on insurance policies are an important issue for insurers. The extent of insurance fraud varies widely from small overstatements of claims to deliberately pretending damages that never occurred or that were intentionally arranged. Due to the nature of fraud, estimating the losses for the insurance industry is not an easy task. Nevertheless, the Insurance Information Institute, for example, estimates that in both 2004 and 2005 insurance fraud amounted to \$30 billion in the US property and casualty insurance market.¹ This is consistent with the estimate of \$20 billion for 1994 by the National Insurance Crime Bureau as stated in Brockett et al. (1998). According to Caron and Dionne (1997), 10% of the insurance claims in the automobile insurance are fraudulent to some extent in the Canadian province of Quebec.

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¹See Rees and Wambach (2008), p. 135.

Therefore the strategies of insurers to deter insurance fraud do matter. Dionne et al. (2009, p. 69), for example, estimate that in their sample, companies could save up to 41% of the costs due to fraudulent claims by implementing the optimal auditing strategy. Such a strategy has to balance auditing costs and benefits, like exposed fraudulent claims. In the mass market and with small claims, it is too costly to audit each claim that is made. Consequently, claim reports are usually scanned for known patterns of fraud and only a certain fraction of these reports is verified in detail. Previous literature, like Picard (1996), who analyzes the canonical model of insurance fraud, suggests a commitment problem. Ex ante the insurers are interested in announcing a high level of auditing to deter insurance fraud. Given the announced level of auditing, the policyholders indeed report only few fraudulent claims. As auditing is costly, however, the insurer has an incentive to audit only very few claims ex post, rendering its ex-ante announcement not credible. Credible commitment to a certain level of auditing solves this dilemma. Thus, the absence of commitment implies a welfare loss. In contrast to this theoretical result, empirically it is very unusual for insurers to make their level of auditing publicly available. There are also no observable efforts to overcome the credibility issue by having an industry association scrutinize their level of auditing or using another third-party verification mechanism. Insurance firms not only announce no data on fraud detection and auditing, but even block access to it. Thus, there are very few empirical studies available.² This behavior indicates that conventional wisdom neglects some aspects of the setting.

Therefore we suggest that there is an additional issue. We depart from previous literature by assuming ambiguity-averse agents and uncertainty about the insurer's costs of an audit. We model the ambiguity on the type space, as the insured do not know which type of insurer they are facing. This leads to ambiguity about the probability of an audit. In our model, ambiguity-averse agents undertake less fraud due to this uncertainty. Yet commitment dissolves this ambiguity as it makes the level of auditing common information. We show that, even in a competitive market, it can be optimal for the insurers to maintain the ambiguity and forgo commitment.³ Thus, strategic ambiguity is an equilibrium outcome.⁴ First, we prove that holding insurers' behavior fixed, ambiguity makes fraud less appealing. Next, we endogenize the insurers' behavior. In the second step, we show that for a given contract, if the insurer abstains from commitment, ambiguity aversion either lowers the amount of fraud while holding the level of auditing fixed, or vice versa. Third, it will be shown that avoiding commitment is optimal if the auditing costs satisfy certain conditions discussed in the next paragraph. Finally, we also endogenize the contracts. It is shown that the utility-maximizing contracts that just break even under no commitment can be the unique equilibrium outcome.

The insurance companies have different reasons to forgo commitment. Insurance companies with high costs save on auditing costs, if they hide their type by abstaining from commitment, because the average auditing probability is higher than their own. Insurance companies with low costs also prefer the uncertainty to commitment, because a higher level of fraud due to the lower average auditing

²A notable exception is Dionne et al. (2009). In the context of tax enforcement, the Internal Revenue Service in the U.S. defended in several court cases its right to keep auditing procedures secret.

³Notice that this result requires uncertainty about primitives of the model, here the auditing costs. Uncertainty as a purification of mixed strategies, as proposed by Harsanyi (1973), is not sufficient.

⁴Strategic ambiguity denotes here the strategic choice to withhold information in order to maintain the ambiguity for the other contract party, not the choice of strategic uncertainty in the sense of ambiguous strategies. The notion is discussed at the end of this section.

makes their auditing even more profitable. This is caused by the improved ratio between their low costs and recovered indemnities and fines imposed on the uncovered fraudsters.

Risk aversion leads to different effects in the model than ambiguity aversion. If the degree of risk aversion increases, the deterrence of insurance fraud becomes easier both with and in the absence of commitment. Ambiguity aversion has only deterrence effects if there is no commitment. Therefore, only ambiguity aversion influences the balance between commitment and non-commitment. After all, it is the uncertainty that makes ambiguity-averse agents less inclined to engage in insurance fraud.⁵

In our model, the policyholders are ambiguity averse. Ambiguity denotes uncertainty about probabilities resulting from missing relevant information. We therefore distinguish ambiguity and risk.⁶ In the absence of ambiguity, there is a known probability distribution, while under ambiguity the exact probabilities are unknown. Savage (1954) and Schmeidler (1989) have developed two axiomatized approaches to this problem. The Subjective Expected Utility of Savage requires the decision maker to be ambiguity neutral. This approach has been criticized for various reasons. From a normative point of view, it seems appropriate to take into account the amount of information on which a decision is based. This point was first made by Ellsberg (1961). In addition, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), which suggest that the Subjective Expected Utility approach neglects the distinction between risk and ambiguity. Insurers, which face ambiguity, usually request higher premiums and reject to offer an insurance policy in more cases than in the absence of ambiguity. The model in our paper uses the representations of preferences with ambiguity aversion by Klibanoff et al. (2005) and Gilboa and Schmeidler (1989). In both representations, the decision maker judges situations with missing information more pessimistically than an ambiguity-neutral individual.

The problem of costly state verification considered here is not limited to insurance fraud, but also appears in different settings such as financing (Gale and Hellwig, 1985), accounting (Border and Sobel, 1987), principal-agent relationships (Strausz, 1997) or enforcement of TV license fees (Rincke and Traxler, 2011). The main point is that there is often asymmetric information between the parties of a contract. To avoid the exploitation of these asymmetries, the other side has to use costly state verification technologies, like ticket inspections in public transport. Townsend (1979) began this analysis of the trade-off between auditing costs and losses due to the remaining information asymmetries. Commitment is optimal in these models, as discussed in, e.g., Baron and Besanko (1984). Hence, there have been various proposals to make commitment feasible and credible. Melumad and Mookherjee (1989) introduce delegation as a commitment device and Picard (1996) proposes a common agency financed by lump-sum payments to subsidize auditing costs. This lowers the variable costs of auditing claims in order to solve the credibility problem. Yet we will argue in this paper that in some circumstances it is optimal for firms to avoid commitment to an auditing strategy, even if commitment were possible and costless.

Previous literature that combines costly state verification and uncertainty about auditing costs often uses a setting of tax evasion. Cronshaw and Alm (1995) analyze this case, but without ambiguity

⁵We were encouraged in this view when one insurance executive told us that besides being bad publicity, communicating detailed data on fighting insurance fraud, like the level of auditing, might induce more policyholders to give it a try. Moreover, according to Reinganum and Wilde (1988, p. 794), the IRS confirms that ‘one of the tools in the arsenal of the IRS which promotes voluntary compliance is the uncertainty in the minds of the taxpayers.’

⁶Unfortunately, the literature uses various notions. Sometimes ambiguity is called (Knightian) uncertainty or imprecision. The technical details of representations with ambiguity aversion are discussed in Appendix A.1.

aversion and the possibility of commitment. Therefore, in their model, uncertainty could be counter-productive. Snow and Warren (2005), on the other hand, model ambiguity aversion by a subjective weighting of probabilities. Their paper studies the behavior of taxpayers given this ambiguity, but there is no possibility of commitment. Thus, our paper is the first to consider the strategic decision of commitment versus uncertainty.

The notion of strategic ambiguity as the strategic choice to withhold information in order to maintain the uncertainty for the other contract party has been used by Bernheim and Whinston (1998) and Baliga and Sjöström (2008) in the context of ambiguity-neutral players. In Baliga and Sjöström (2008), a country in equilibrium withholds the information about its military arsenal instead of acquiring arms with certainty and uses strategic ambiguity as a substitute for arms acquisition. In Bernheim and Whinston (1998), on the other hand, strategic ambiguity denotes the choice of an incomplete contract. Bernheim and Whinston (1998, p. 920) show “that, when some aspects of behavior are observable but not verifiable, it may be optimal to write a contract that leaves other potentially contractible aspects of the relationship unspecified.” The assumption of observable, but unverifiable aspects, while common in this literature, does not apply here. Individual fraud is either unobservable and unverifiable without an audit or becomes verifiable after an audit. Aggregate fraud levels are unobservable for the policyholders and in reality for the insurers, too. The type of an insurer is unobservable for the insured, while the occurrence of an audit is verifiable. Therefore there is no scope for negotiations that could make incomplete contracts optimal. Instead it is one party, the insurer, who decides to withhold the information about its auditing probability at a later stage after the contracting. The optimality of incomplete contracts is confirmed by Mukerji (1998) for ambiguity-averse parties. In his paper, contractual incompleteness lessens the effects of ambiguity, because it leads to renegotiations that yield a proportional split of the surplus. This reduces the utility losses due to ambiguity, as it makes the considerations of both parties how to determine the worst distribution more similar.⁷ In our model, avoiding commitment enhances the effects of ambiguity.⁸

A second contribution of this paper is to scrutinize a model with ambiguity aversion in a game-theoretic framework. Although many papers deal with the effects of ambiguity aversion in decision making and finance, there are few papers on games with ambiguity-averse players.⁹ The reasons are problems with the equilibrium concepts, as addressed by Dow and Werlang (1994), Lo (1996, 1999), Eichberger and Kelsey (2000), Lo (2009), Bade (2011a), and Riedel and Sass (2011). We avoid these problems by modeling the ambiguity on the type space, i.e., the auditing costs of the insurers. This

⁷The reason is that the Choquet expectation is only additive for comonotonic acts. Thus, with ambiguity aversion the expected sum of the surpluses is larger than the sum of the expected surpluses, because the incentive compatibilities for the two parties require the transfers to be noncomonotonic. Therefore it is impossible to implement first-best effort. Contracts with comonotonic transfers, like incomplete contracts, cannot mitigate this, but avoid some of the ex-ante ambiguity premia and might be optimal.

⁸Another neoclassical explanation for the withholding of the auditing information and not using commitment might be the repeated structure of the interaction. Therefore the static contracts in use by the industry might be improved by leaving room for relational contracts. Yet again this requires some observability. Either the policyholders derive the level of auditing from, e.g., income statements or the competitors observe the amount of auditing implemented. As we argued before, firms try to withhold information about auditing levels. Therefore it is difficult to get this information. Moreover it seems implausible that policyholders choose their insurer according to past auditing strategies or stochastic information about it. If competitors were to use the repeated interaction to enforce joint auditing levels, that behavior might be illegal and, in addition, their incentives are unclear. Therefore we conclude that relational contracts do not explain the observed behavior.

⁹See Mukerji and Tallon (2004) and Gilboa and Marinacci (2011) for a survey of the literature.

approach is also used by Lo (1998), Levin and Ozdenoren (2004), Bose et al. (2006), and Bodoh-Creed (2012) to study auctions with ambiguity-averse bidders. Bade (2011b) uses this approach, too, in order to establish the existence of equilibria in games of multidimensional political competition. It allows the use of common equilibrium concepts, like perfect Bayesian equilibria.

The third contribution is to consider whether competition makes firms provide relevant information to consumers and educate them. The argument by, e.g., Laibson and Yariv (2007) has been that competitive pressure gives consumers all the relevant information, as a competitor could always reveal the information and win market share. In our model, this is not the case. There is a market equilibrium with perfect competition where firms do not announce their information about auditing levels and ambiguity prevails that allows mitigating the effects of insurance fraud. In this respect, our results are similar to Gabaix and Laibson (2006) and Heidhues et al. (2012), where in equilibrium firms shroud the prices of some add-ons to their products.

The remainder of the article is organized as follows. Section 2 sets up a stylized model to give an intuition as to how ambiguity about the level of auditing decreases insurance fraud. In addition, it explains the decision process of the ambiguity-averse policyholders. In Section 3, we take contracts as given and insurers decide on their auditing probabilities and whether or not to commit to their fraud detection strategy. We show that commitment can decrease profits and that insurers do not want to commit, even if they have the possibility to do so. In Section 4, insurers compete in contracts and decide on their auditing strategies. Even in this competitive market, firms in some cases want to forgo commitment. Then Section 5 compares the effects of ambiguity aversion with risk aversion. Finally, Section 6 exploits some extensions of the model and Section 7 contains the concluding remarks.

2 Ambiguity in Auditing

To strengthen the intuition of our results, we begin with a stylized model that shows how the ambiguity aversion of the policyholders makes them less inclined to engage in insurance fraud. The mechanism for the commitment decision of the insurers requires the full model which is set up in the next section. A risk-averse and ambiguity-averse agent takes out an insurance with a premium P and coverage q against a possible loss $L > 0$. Without loss of generality, we normalize the outside wealth of the agent to 0. The agent's preferences are represented by an increasing and strictly concave utility index u . A loss L occurs with probability δ and no loss with probability $1 - \delta$. Given this loss distribution, a policyholder who reports a loss smaller or higher than L is immediately recognized as a fraudster. If, however, no loss occurs, the policyholder can nevertheless claim a loss of L , because the occurrence of a loss is private information of the policyholder. As it is common in the literature on costly state verification, the policyholder faces no direct costs or disutility for this behavior.

The insurer cannot observe the loss directly. It just receives the report of the policyholder. If the insurer pays out the claim, the policyholder gets q and therefore in case of fraud ends up with a final wealth of $q - P$. The insurer, however, has a technology to audit a fraction p of the reports for their truth. This technology is deterministic. Thus, if the insurance company audits a report, it knows for sure whether the report is true or not.¹⁰ In case the insurance company detects a fraud, it pays no

¹⁰An alternative interpretation would be a stochastic technology in the sense that fraud is exposed only with a certain

indemnity and the policyholder has to pay a fine M that is determined by law. This is commonly known, but the fraction of audits p is private knowledge of the insurer. The policyholders only know that some reports will be verified. The insurer, however, may choose to disclose this fraction p to the policyholders. Without disclosure there is uncertainty about the level of auditing. We will show that the uncertainty lowers auditing costs, because it deters ambiguity-averse policyholders from fraud.

This uncertainty about probabilities due to the lack of relevant information is called ambiguity. In order to model ambiguity-averse agents, we use smooth ambiguity aversion by Klibanoff et al. (2005). A formal introduction to smooth ambiguity aversion is available in Appendix A.1. Yet the results of this paper do not depend on this specific representation of preferences. In Appendix A.2, we repeat the exercise with Maxmin Expected Utility. This confirms that additional uncertainty decreases the inclination of the policyholders to engage in fraud.¹¹

In the representation of smooth ambiguity aversion by Klibanoff et al. (2005), there is a set Π that contains the possible values for the first-order probability \hat{p} , here the probability of an audit. On the other hand, $\mu(\hat{p})$ denotes the second-order probability of \hat{p} being the correct first-order probability. We assume that Π and μ are such that the true value of p is contained in Π and equals the expected value, i.e., $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$. The ambiguity index ϕ is continuous, strictly increasing, and concave. Thus, without a loss, the policyholder's utility is $\phi(u(-P))$ if she makes no claim, and

$$\int_{\Pi} \phi((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)) d\mu(\hat{p})$$

for fraudulent claims. If the level of auditing is disclosed, the probabilities are known and become objective. Thus, there is no ambiguity and μ is degenerate. Therefore the policyholder overstates the loss if the probability p of an audit is smaller than

$$p^b = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}.$$

The following lemma compares this threshold to the case with ambiguity.

Lemma 1. *Suppose the level of auditing is fixed. If the insurer does not announce the level of auditing and the ambiguity-averse policyholders do not have all the relevant information to determine it exactly, there is less insurance fraud than in the case of available information about the auditing probability.*

The proof and all other proofs are given in Appendix A.3. Thus, not revealing the probability of an audit decreases the level of auditing that is necessary to deter the policyholders from committing fraud. This means that withholding information about the level of auditing from the policyholders reduces their inclination to engage in insurance fraud.

As the main model assumes heterogeneous policyholders with respect to the degree of ambiguity aversion, we next analyze comparative statics in the degree of ambiguity aversion. For this purpose, consider two policyholders with ambiguity index ϕ_1 and ϕ_2 . We call the second policyholder more

probability. Yet this does not change the analysis, because we can interpret p as the reduced probability of a claim being audited and being correctly identified if it was fraudulent.

¹¹Gollier (2011) finds that an increase in ambiguity aversion may actually increase the demand for an ambiguous asset, in contrast to our result. The intuition for his result is similar to Rothschild and Stiglitz (1971) who show that a higher riskiness does not necessarily lower the demand of risk-averse agents for the risky asset.

ambiguity-averse than the first policyholder if there is an increasing and strictly concave function g , such that $\phi_2 = g(\phi_1)$.

Lemma 2. *Suppose the level of auditing is fixed, but ambiguous. Then the more ambiguity-averse policyholders commit less insurance fraud.*

The next section sets up the main model in the framework of Picard (1996) in order to capture the commitment decision of the insurers.

3 The Main Model

There are $N > 3$ insurers facing a continuum of potential policyholders with mass one. The insurers make contract offers, then decide whether to commit to an auditing strategy. Finally, they choose their level of auditing. The policyholders select a contract and decide whether to make a claim.

The timing is summarized in Figure 1. First, the degree of ambiguity aversion is assigned to the potential policyholders. Then risk-neutral and ambiguity-neutral insurers make contract offers. Each insurer i provides a quote for coverage q_i and a premium P_i , such that $0 \leq P_i \leq q_i$. In the next stage, the insured choose a contract from the pool of contract offers. At $t = 3$, nature determines the costs c of an audit for the insurer from the set $\{c_L, c_H\}$ with $c_H > c_L > 0$. In the extension, we modify this timing by assuming that the insurance company knows its cost already before the contracting stage. The auditing costs are revealed only to the insurers. The policyholders only know the set of possible auditing costs, but not the distribution according to which nature is choosing. Therefore, the uncertainty is modeled, à la Harsanyi (1967), on the type space. The policyholders have no objective probabilities on the type space, but use subjective probabilities. Denote the subjective probabilities of facing a low-cost insurer by r , its non-degenerate distribution by $\mu(r)$, and the subjectively expected probability by $\bar{r} = \int r d\mu(r)$.¹²

After observing its auditing costs, every insurance company has the possibility to commit to some auditing level. The commitment could be implemented by delegation, as in Melumad and Mookherjee (1989), or by a common agency following Picard (1996). We abstract from this issue and assume that commitment is costless for the insurer to make our case as difficult as possible. If there are costs for communicating the auditing probability and making this announcement credible, it only strengthens our results. After that, at $t = 4$, the policyholders privately observe the occurrence of a loss L that occurs with probability δ . Then, at $t = 5$, they decide whether or not to file an insurance claim. At $t = 6$, the insurer chooses to what extent to audit the filed claims. The auditing technology works as before. Finally, the insurer pays the indemnity q or gets a part $m \leq M$ of the fine M a policyholder has to pay if an audited claim was fabricated. The remaining part is lost due, e.g., to litigation. As they are determined by law and legal process, M and m are exogenous in the model. This modeling choice is common in the costly state verification literature, like Picard (1996).

We restrict the analysis here to the case of smooth ambiguity aversion as proposed by Klibanoff et al. (2005).¹³ We assume a population of agents with different degrees of ambiguity aversion. Thus,

¹²This implies that auditing costs are realized independently for each insurer.

¹³The results of this paper are robust to other representations of preferences and, in particular, also hold with Maxmin and Choquet Expected Utility.

- At $t = 0$, the degrees of ambiguity aversion are realized and revealed to the insured.
- At $t = 1$, insurers make contract offers (q_i, P_i) .
- At $t = 2$, the insured choose contracts.
- At $t = 3$, auditing costs $c \in \{c_L, c_H\}$ are realized and revealed to the insurer; insurers can commit to an auditing probability p_i .
- At $t = 4$, losses L are realized.
- At $t = 5$, the policyholders make insurance claims.
- At $t = 6$, the insurers decide on the extent of auditing if no commitment was made.
- At $t = 7$, indemnities and fines are awarded after auditing the filed claims.

Figure 1: Timing of the Model

there is a family of strictly concave ambiguity functions $\phi_{\mathcal{A}}$ indexed by $\mathcal{A} \in [\underline{A}, \bar{A}]$. The higher \mathcal{A} , the more ambiguity-averse the agent is, as defined in Section 2 above. The degree of ambiguity aversion \mathcal{A} is distributed according to a distribution function F with a density $f > 0$. The insurers, who know this distribution, cannot observe the degree of ambiguity aversion of a policyholder. In this section, stages 1 and 2 are taken as given. Thus, only the stages 3 to 7 of the game are considered. Section 4 solves the full model. As a first step, we determine the equilibrium of the auditing game beginning after stage 4.

3.1 Solving the Auditing Game

There are two cases to consider. First, we consider the case in which the insurer commits itself to a certain level of auditing in stage 3. We solve the model backwards. If the insurer committed to a certain level of auditing p , in stage 6 it has to stick to that decision and conduct the audits accordingly. In the next step, we analyze the decision of the insured in stage 5 whether or not to report a claim in the absence of a loss. The level of auditing is known, so the policyholders do not care about the auditing costs of the insurer. Therefore their beliefs about the type of the insurer and the ambiguity aversion do not matter. As before, the critical value for the level of auditing is $p^b = \frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$. If more claims are audited, no fraud occurs. For lower levels of auditing, every policyholder makes a claim. In the third stage, the insurers choose p_i , depending on the costs of auditing c_i , to maximize their profits. The equilibrium in this game is the same as the one described in Proposition 1 of Picard (1996)¹⁴ and depends on the costs of auditing c_i . If the insurer's costs are above a threshold, i.e., $c_i > c' = \frac{(1-\delta)q}{\delta p^b}$, the insurer of type i does not audit any claims and all the policyholders claim a loss. If the costs of auditing are below the threshold, a fraction p^b of all claims is audited and no insurance fraud occurs.¹⁵

¹⁴Picard (1996) assumes an exogenously given fraction θ of opportunistic policyholders in an otherwise honest population. Setting $\theta = 1$ resembles our model with credible announcement.

¹⁵To make the equilibrium unique, the insured have to abstain from fraud if the level of auditing is p^b , although they are indifferent. This seems natural, as the insurer could audit a fraction $p^b + \epsilon$ of all insurance claims with an arbitrarily small ϵ to make this behavior of the policyholders a unique best response. On the other hand, the insurers are indifferent for $c = c'$. For uniqueness, it is assumed that insurers have a preference for less fraud if profits do not change.

We now turn to the case in which the insurer decides not to commit. Solving the model backwards, the analysis begins at $t = 6$. As no commitment was made, the insurer will choose the level of auditing p to maximize its profits, given that a fraction α of policyholders without a loss reported a false claim. A policyholder anticipates an auditing probability p_L of the low-cost insurer and p_H of the high-cost type. If the policyholder is ambiguity neutral, she expects an audit with probability $\bar{r}p_L + (1 - \bar{r})p_H$. Yet, the more ambiguity-averse she gets, the more averse she gets with respect to the risk of facing the low-cost insurer. Thus she reports truthfully if $\phi_{\mathcal{A}}(u(-P)) \geq$

$$\geq \int \phi_{\mathcal{A}} \left((r(1 - p_L) + (1 - r)(1 - p_H))u(-P + q) + (rp_L + (1 - r)p_H)u(-P - M) \right) d\mu(r). \quad (1)$$

Therefore, the following program determines the equilibrium, in which the insurers choose the auditing probabilities p_L and p_H , after the policyholders have decided whether to submit fraudulent claims.

$$\begin{aligned} & \max_{p_i \in [0,1]} P - q(\delta + \alpha(1 - \delta)(1 - p_i)) + m\alpha p_i(1 - \delta) - c_i(\delta + \alpha(1 - \delta))p_i, \quad \forall i \in \{L, H\} \\ \text{subject to } & \alpha = \int_A 1 dF(\mathcal{A}) \text{ with the set} \\ & A = \left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \mid \int \phi_{\mathcal{A}} \left((r(1 - p_L^*) + (1 - r)(1 - p_H^*))u(-P + q) + \right. \right. \\ & \quad \left. \left. + (rp_L^* + (1 - r)p_H^*)u(-P - M) \right) d\mu(r) > \phi_{\mathcal{A}}(u(-P)) \right\} \end{aligned}$$

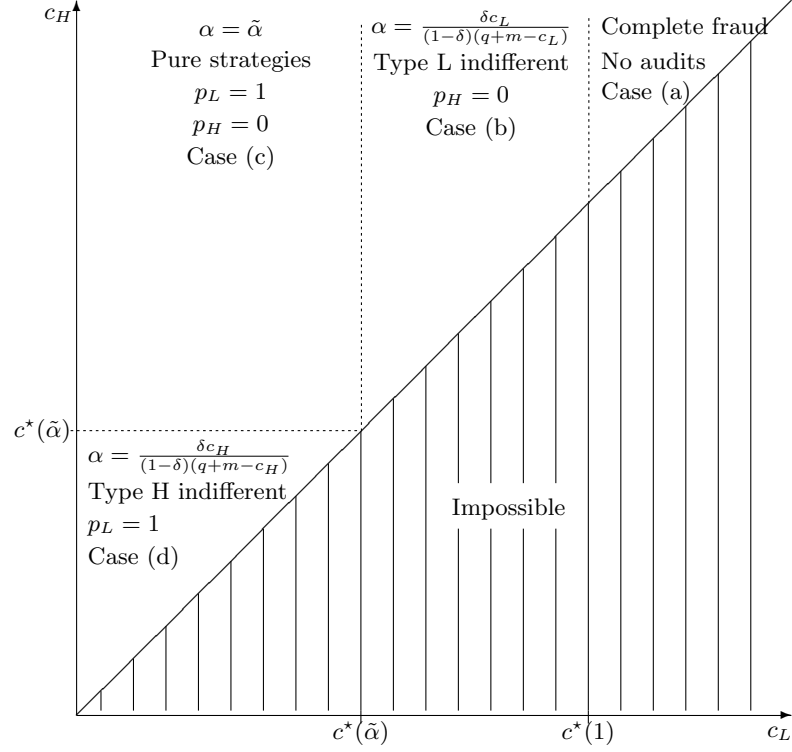
To calculate the optimal auditing probabilities, p_i^* , consider the reasoning of the insurer. The insurer acts after the insured reported their claims. Thus, the level of fraud α is taken as given. The insurer is indifferent between auditing or not, if the costs are at the threshold $c^*(\alpha)$, which depends on the amount of fraud.

$$c^*(\alpha) = \frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}(q + m) \quad \text{with} \quad \frac{\partial c^*(\alpha)}{\partial \alpha} > 0 \quad \forall \alpha \geq 0. \quad (2)$$

The fraction $\frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}$ is the insurer's belief after stage 5 about a claim to be false. Hence, at the threshold the costs of auditing equal the expected benefits of auditing, i.e., the claims q that need not to be paid and the fines m awarded to the insurer. This allows describing the unique perfect Bayesian equilibrium (modulo out-of-equilibrium beliefs and strategies) of the game after stage 4 given a contract with premium P and reimbursement q .¹⁶

The following proposition distinguishes four cases, which are illustrated in Figure 2. If the costs of both types are very high in case (a), there will be no auditing and complete fraud. For lower costs, there are two cases, (b) and (d), in which one type will be indifferent with respect to auditing. Finally, there remains the case (c) where every type of insurer plays a pure strategy as auditing is beneficial for the low-cost type, but not for the high-cost type.

¹⁶To have a unique equilibrium, firms have to prefer less auditing, ceteris paribus, in particular if it does not change the level of fraud. Moreover, while the insurer's type is unobservable, the policyholders nevertheless correctly anticipate the equilibrium strategy of each type of insurer. Hence the uncertainty only concerns the type space. Therefore the analysis does not require new equilibrium concepts, as discussed in the introduction.


 Figure 2: Auditing Equilibria in Proposition 1 for $0 < \tilde{\alpha} < 1$

Proposition 1. For given contracts, beliefs $\mu(r)$ and without commitment the equilibrium has the following form:

- (a) If the costs of both types are above the upper threshold, $c_L \geq c^*(1) = (1 - \delta)(q + m)$, there is complete fraud, $\alpha = 1$, and no audits, $p_H = p_L = 0$.
- (b) If the costs of the low-cost type are between the two thresholds, $c^*(\tilde{\alpha}) \leq c_L < c^*(1)$, there is a high level of fraud $\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)} \in (\tilde{\alpha}, 1)$, and a low level of audits $p_H = 0$ and $p_L = h(0, F^{-1}(\alpha))$.
- (c) If the costs of both types are separated by the lower threshold $c_L < c^*(\tilde{\alpha}) \leq c_H$, there is some fraud $\alpha = \tilde{\alpha}$ and partial audits of $p_H = 0$ and $p_L = 1$.
- (d) If the costs of both types are below the lower threshold, $c_H < c^*(\tilde{\alpha})$, there is a low level of fraud $\alpha = \frac{\delta c_H}{(1 - \delta)(q + m - c_H)} \in (0, \tilde{\alpha})$, and a high level of audits $p_H = h(1, F^{-1}(\alpha))$ and $p_L = 1$.

with $h(x, \mathcal{A}')$ a solution to

$$\phi_{\mathcal{A}'}(u(-P)) = \int \phi_{\mathcal{A}'} \left(u(-P + q) - (rx + (r + (1 - 2r)x)h(x, \mathcal{A}'))(u(-P + q) - u(-P - M)) \right) d\mu(r)$$

and the level of fraud $\tilde{\alpha} = F(\mathcal{A}^*)$ defined by case (c), such that

$$\mathcal{A}^* = \sup \left(\left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \mid \int \phi_{\mathcal{A}}((1 - r)u(-P + q) + ru(-P - M)) d\mu(r) > \phi_{\mathcal{A}}(u(-P)) \right\} \cup \{\underline{A}\} \right).^{17}$$

¹⁷As there is no continuity in \mathcal{A} , we have to consider the supremum of these values of \mathcal{A} for which fraud is optimal instead of using the indifference condition (1). \underline{A} captures corner solutions.

To sum up, in equilibrium smaller costs of auditing reduce the level of insurance fraud by increasing the auditing probabilities. Additionally, the level of auditing and the amount of insurance fraud depend negatively on each other. In general, insurers with high costs do not audit, except for the last case (d) of the proposition. In contrast to Picard (1996), it is possible to have auditing and the insurers employing pure strategies. Therefore the equilibrium differs if the auditing costs of both types are not too high; in particular case (c) is impossible without cost heterogeneity.

Introducing ambiguity aversion either lowers the amount of fraud, while holding the level of auditing fixed, or vice versa. Uncertainty about the level of auditing has an additional deterrence effect. The effect discussed in Section 2 causes this reduction.

The level of fraud $\tilde{\alpha}$ in case (c) is important for the structure of the proposition, because it determines the lower threshold for the costs $c^*(\tilde{\alpha})$. Thus, case (b) is only feasible if the low-cost type can induce some policyholders to behave honestly, $\tilde{\alpha} < 1$. This implies either a high expected probability \bar{r} for a low-cost insurer, a high amount of ambiguity in terms of the variance of μ or a high degree of ambiguity aversion in the population, or else that fraud is unattractive, i.e., p^b is low. On the other hand, if the low-cost type can induce all policyholders to behave honestly, $\tilde{\alpha} = 0$, the cases (c) and (d) do not arise at all.

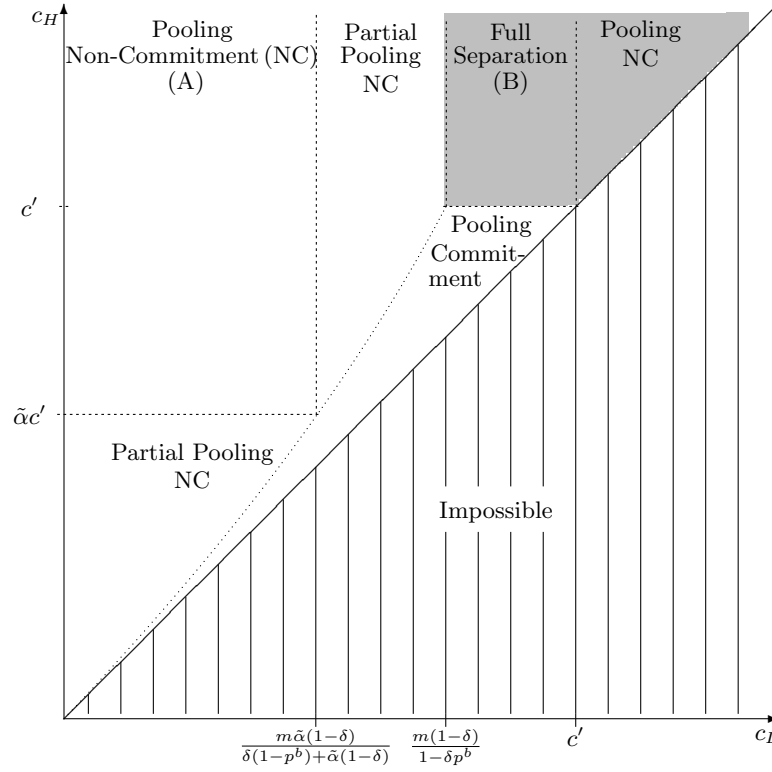
There remain two interesting implications of Proposition 1. First, the insurers' profits vary continuously, as the parameters change, even if the type of equilibrium changes. Second, commitment allows eliminating insurance fraud completely, which is impossible without commitment. Nevertheless, the next section shows that ambiguity aversion in some cases allows reducing the total costs of the insurers by forgoing commitment.

3.2 Comparing Commitment with Non-Commitment

In this section conditions are derived under which non-commitment may lower the insurers' total costs. If the insurers are of the high-cost type, they need to implement less audits than under commitment and can profit from the low fraud caused by the high average auditing probability. For insurers of the low-cost type auditing is cheap. Hence they profit from the higher fraud in the population compared to a situation with commitment due to the lower average auditing if the ratio of their costs to the fines is low enough. In order to show that non-commitment can be preferred, we compare the costs due to insurance fraud, $\alpha(1 - \delta)(1 - p_i)q$, and auditing, $(\delta + \alpha(1 - \delta))p_i c_i$, minus the recovered fines, $m\alpha p_i(1 - \delta)$, in the absence of commitment to the costs of auditing under commitment, $\delta p^b c_i$. Commitment implies a loss for the insurance firms if

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i c_i \leq \delta p^b c_i. \quad i \in \{L, H\} \quad (3)$$

The next proposition shows that this condition is feasible in the cases (a) and (c) of Proposition 1.


 Figure 3: Signaling Equilibria in Proposition 2 for $0 < \tilde{\alpha} < 1$

In area (A), all types avoid commitment. Adjacent to (A), there are two areas with partial pooling. Then, one type avoids commitment with a strictly positive probability, while the other type always abstains from commitment. Along the 45° degree line, there is pooling on commitment. Finally, in area (B) there is a fully separating equilibrium. Then, only the low-cost type uses commitment. Regions with multiple equilibria are shaded.

Proposition 2. *In the game beginning at stage 3, commitment has, in equilibrium, no advantage for the insurers if and only if*

- the costs of the low-cost type are low enough, while the costs of the high-cost type are sufficiently large, $\exists \alpha \in (0, 1]$

$$c_L \leq \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)} \quad \text{and} \quad c_H \geq \alpha c'. \quad (4)$$

If condition (4) holds for $\alpha = \tilde{\alpha}$ as defined in Proposition 1, there is pooling with respect to the commitment decision. For other values of α , there is partial pooling.

- the costs of auditing are high for both types, $c_L > c' = \frac{(1-\delta)q}{\delta p^b}$. In this case, the insurers do no auditing and therefore are indifferent with respect to the commitment decision.

Figure 3 summarizes the commitment decision. The most interesting case for the next section is pooling on non-commitment in the upper left corner (A). In this area, the costs of the types differ significantly and, as argued above, both types are better off by not committing. Partial pooling refers to a situation in which one type uses a mixed strategy with respect to the commitment decision. This allows adjusting the level of fraud in the absence of commitment to make non-commitment optimal for both types. For $\alpha < \tilde{\alpha}$, the high-cost insurer commits with some probability. For $\alpha > \tilde{\alpha}$, the low-cost insurer commits with some probability.

There is commitment to an auditing probability if the costs of an audit are close to each other for both types.¹⁸ Then auditing is sufficiently cheap for the high-cost type to prefer a positive auditing

¹⁸There is also an equilibrium with commitment for large costs, $c_L \geq \frac{m(1-\delta)}{1-\delta p^b}$, in particular if commitment does not deter fraud.

probability and commitment. Alternatively, auditing costs are sufficiently high for the low-cost type to prefer commitment and the corresponding reduction in auditing levels.

In area (B), it is optimal for the low-cost type to implement commitment and do some auditing. In the absence of commitment, there is a high level of fraud. Given the high costs of auditing, abstaining from commitment makes the low-cost type worse off. The high-cost type, on the other hand, implements no audits anyway and is therefore indifferent with respect to the commitment decision.

Finally, if the auditing costs are sufficiently high, both types abstain from auditing independent of their commitment decision. Hence, insurers are indifferent with respect to the commitment decision. Then there are multiple equilibria.

In summary, the policyholders do not know which type of insurer they face in the absence of commitment and there will be some, but not too much fraud. Both types of insurer could commit to a level of auditing p^b and completely deter the policyholders from filing fraudulent claims. In area (A) of Figure 3, however, they have an incentive not to do so and strictly prefer an equilibrium without commitment. We now consider the insurance market and characterize the equilibrium of the entire game starting at $t = 0$.

4 Market Equilibrium

So far we have analyzed the behavior of the policyholders and the insurers for given contracts. Now we endogenize these contracts according to the timing in Figure 1. The characterization of the equilibrium in the insurance market requires the definition of two benchmark contracts. These benchmark contracts serve the purpose to characterize the equilibrium. There are no restrictions on strategies. The first contract (q^{NC}, P^{NC}) is the utility-maximizing contract that just breaks even, if the insurers avoid commitment and only the low-cost insurer audits as in case (c) of Proposition 1. The second contract (q^C, P^C) is defined accordingly just for the case with commitment. Therefore define the contract (q^{NC}, P^{NC}) as an element of the following set

$$(q^{NC}, P^{NC}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P)$$

$$\text{with } P \geq \delta(q + \bar{r}c_L) + (1 - \delta)(\bar{r}(c_L - m) + (1 - \bar{r})q)\tilde{\alpha}[q, P] \quad (5)$$

and $\tilde{\alpha}[q, P]$ as defined in Proposition 1.¹⁹ Assume that this set is a singleton. The expected profits correspond to pooling in case (c) of Proposition 1. The contract (q^C, P^C) is defined analogously, but the budget constraint is this time

$$P \geq \delta(q + \bar{r}c_L p^b[q, P]) + (1 - \bar{r}) \min\{\delta c_H p^b[q, P], (1 - \delta)q\}.$$

The next proposition shows that in equilibrium firms will choose the non-commitment contract (q^{NC}, P^{NC}) and there is pooling with respect to the commitment decision.

¹⁹We write $\tilde{\alpha}[q, P]$ and $p^b[q, P]$ to make clear that both depend on the contract (q, P) .

Proposition 3. *Suppose the auditing costs of both types are not excessively high,*

$$c_H < \frac{(1 - \delta)q^{NC}}{\delta p^b[q^{NC}, P^{NC}]} \quad \text{and} \quad c_L < m(1 - \delta) \quad (6)$$

and condition (4) holds for $\alpha = \tilde{\alpha}[q^{NC}, P^{NC}]$ and $(q, P) \in \{(q^C, P^C), (q^{NC}, P^{NC})\}$. In any perfect Bayesian equilibrium, firms make zero profits and avoid commitment. Furthermore, (q^{NC}, P^{NC}) is the only contract accepted by policyholders in equilibrium.

In the appendix, we first show that there is a perfect Bayesian equilibrium as characterized by the proposition. By the definition of contract (q^{NC}, P^{NC}) insurers make zero expected profits. Proposition 2 showed that insurers are worse off with commitment given condition (4). Condition (6) ensures that there are no profitable deviations. The proof proceeds along the following lines. Fraud reduces the insurance contract to stochastic redistribution with efficiency losses. This cannot generate a positive surplus. Consequently, a deviation with a contract that only attracts policyholders anticipating fraudulent behavior is not profitable. Second, a deviation with a contract implementing commitment is unprofitable, because, given our assumptions, even the best contract with commitment, (q^C, P^C) , is less attractive than the contract (q^{NC}, P^{NC}) for the policyholders. Third, all the policyholders anticipating honest reporting behave homogeneously and therefore receive the same expected utility in equilibrium. In case of a deviation this means that the new contract attracts either all or no honest policyholders. This leaves only the policyholders anticipating fraudulent behavior in the previous contract and yields a change in the insurers' auditing strategy, because auditing becomes more beneficial as the probability of catching a fraudulent claim increases to $1 - \delta$. Since the remaining policyholders anticipate this behavior by the insurers, they also move contracts and cherry-picking by the deviating insurer becomes impossible. To show that these properties hold in any equilibrium, we prove that there is no market equilibrium in profitable contracts. Furthermore, it is impossible to offer a more attractive contract than (q^{NC}, P^{NC}) and avoid losses.

This concludes the analysis of the model, showing that even market pressures do not force insurers to implement commitment. They use the uncertainty created by missing commitment as a deterrence device that makes it possible to offer better contracts. The corollary summarizes this comparison.

Corollary 1. *If commitment is obligatory, insurers offer the contract (q^C, P^C) in equilibrium, which is in utility terms less attractive for the policyholders than the contract (q^{NC}, P^{NC}) without commitment given the conditions of Proposition 3. Therefore forgoing commitment implies an ex-ante Pareto improvement.*

5 The Model in the Absence of Ambiguity Aversion

In the absence of ambiguity aversion, it does not matter whether information about aggregate behavior is available. In particular, it does not matter whether policyholders expect an average auditing level or insurers also commit to this auditing level. In equilibrium, aggregate behavior is common information. Therefore announcing this information does not change agents' behavior. This irrelevance contrasts with the case where ambiguity aversion plays a relevant part, as then the availability of auditing data matters.

Ambiguity aversion has different implications from risk aversion for the commitment decision. This can be seen most clearly in inequality (3) summarizing the firm's commitment decision. The intuition is that ambiguity aversion changes the firm's profits in the absence of commitment, i.e., the left-hand side of inequality (3), without touching the profits with commitment, i.e., the right-hand side. Ambiguity aversion matters only in the case of non-commitment. Risk aversion, on the other hand, affects both cases and both sides of the inequality.

Formally, in the absence of ambiguity aversion, the ambiguity index ϕ is linear and can be neglected. According to Proposition 2, type uncertainty is a necessary condition to have insurers abstain from commitment in equilibrium. Additionally, insurers prefer to abstain from commitment only in the third case (c) of Proposition 1. Therefore, we focus on this case in the following. In the absence of ambiguity aversion, there is complete fraud.²⁰

Corollary 2. *Consider given contracts, beliefs $\mu(r)$, a linear ambiguity index ϕ , and no commitment in the game beginning in stage 4. If $\bar{r} < p^b$ and the costs of both types are separated by the threshold $c_L < c^*(1) \leq c_H$, there is complete fraud, $\alpha = 1$, and partial audits of $p_H = 0$ and $p_L = 1$.*

In contrast to Proposition 1, we can determine $\tilde{\alpha}$ explicitly by comparing the probabilities \bar{r} and p^b . If $\bar{r} \geq p^b$, the low-cost insurer, on its own, can completely deter fraud and $\tilde{\alpha} = 0$. This means that case (c) is impossible. On the other hand, for $\bar{r} < p^b$, ambiguity aversion changes behavior, because the low-cost insurer on its own cannot deter fraud and $\tilde{\alpha} = 1$. Then case (c) implies complete fraud, as $\tilde{\alpha} = 1$. Yet, in this case, Proposition 2 implies that a preference for non-commitment implies complete fraud, $\alpha = 1$. This reduces the insurance contract to stochastic redistribution – an undesirable feature.

As we allow for heterogeneity in ambiguity aversion, the counterpart might be heterogeneity in risk aversion, which we consider next. For this purpose, assume a family of strictly concave von-Neumann-Morgenstern utility indices $u_{\mathcal{R}}$ indexed by $\mathcal{R} \in [\underline{R}, \bar{R}]$. The higher \mathcal{R} , the more risk-averse the agent is. Policyholder 1 is more risk-averse than policyholder 2 if there is an increasing and strictly concave function g , such that $u_1 = g(u_2)$. The degree of risk aversion \mathcal{R} is distributed according to a distribution function F° with a density $f^\circ > 0$. With commitment, insurer $i \in \{L, H\}$ chooses the auditing level to maximize its profits according to

$$\sup_{R_i \in [\underline{R}, \bar{R}]} P - q(\delta + F^\circ(R_i)(1 - \delta)(1 - p^b(R_i))) + mF^\circ(R_i)p^b(R_i)(1 - \delta) - c_i(\delta + F^\circ(R_i)(1 - \delta))p^b(R_i)$$

In general, this yields a positive level of fraud $\alpha = F^\circ(R_i) > 0$. In the absence of commitment, heterogeneity of the risk preferences allows case (c) with an intermediate level of fraud, $0 < \alpha < 1$.

Corollary 3. *Consider given contracts, beliefs $\mu(r)$, a linear ambiguity index ϕ , no commitment, and heterogeneous risk aversion in the game beginning in stage 4. If $\bar{r} < p^b(\underline{R})$ and the costs of both types are separated by the threshold $c_L < c^*(\tilde{\alpha}) \leq c_H$, there is some fraud, $\alpha = \tilde{\alpha}$, and partial audits of $p_H = 0$ and $p_L = 1$. The level of fraud $\tilde{\alpha} = F^\circ(\mathcal{R}^*)$ is determined by*

$$\mathcal{R}^* = \sup \left(\{ \mathcal{R} \in [\underline{R}, \bar{R}] \mid (1 - \bar{r})u_{\mathcal{R}}(-P + q) + \bar{r}u_{\mathcal{R}}(-P - M) > u_{\mathcal{R}}(-P) \} \cup \{ \underline{R} \} \right).$$

²⁰In the non-generic case $\bar{r} = p^b$, there are multiple equilibria. Now, the low-cost insurer has to audit every claim to deter insurance fraud. Therefore the level of fraud is $1 \geq \alpha \geq \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$.

In this case, the low-cost insurer audits every claim and prefers non-commitment if its costs are low enough. Then the insurer has to pay few indemnities and earns some income from fine payments. Commitment reduces the auditing probability of the low-cost insurer. The corresponding savings on auditing costs do not compensate for the loss in indemnities and fines if the auditing costs of the low-cost type are sufficiently small. The advantage of not committing for the high-cost type is smaller than with ambiguity aversion. The insurer might still profit by reducing its auditing probability, if its costs are sufficiently high.

Yet, in a market equilibrium in which insurers set contracts, the heterogeneity in the risk aversion complicates the analysis of policyholders' behavior. Even if policyholders behave honestly, they have different valuations for a given policy. These differences in valuation make it possible to screen policyholders into different contracts. Then in each contract policyholders have a similar degree of risk aversion resulting in the setting of Corollary 2. Thus, whether a market equilibrium similar to Proposition 3 exists is an open question.

6 Extensions

As already mentioned in Footnote 5, the Internal Revenue Service in the U.S. stated on several occasions that it regards uncertainty about auditing procedures as a valuable method to increase tax compliance. Furthermore, it went to great lengths to defend this approach in several court cases brought under Freedom of Information Acts. If we assume that taxpayers are mobile to some extent and counties compete for tax revenues, the model in this paper can be modified accordingly. Instead of receiving insurance, agents have to pick one county where they pay taxes. Not declaring their income correctly would correspond to reporting a fraudulent claim. Then the mechanism in this paper might explain why counties stick to the IRS strategy of avoiding commitment. The deviation of attracting many taxpayers with low tax rates financed by committing to an auditing regime is not profitable in the equilibrium of our model.

The next extension goes back to the initial insurance model, but shifts the realization of the cost type after the commitment decision. Therefore, firms do not know which type they are, when they have the possibility to commit to a certain level of auditing. In this case, the considerations of the firms change. If the insurance company commits and the auditing costs are high, it has to bear the high auditing costs or the costs of fraud due to the low auditing probability. This threat is weighted against the usual advantages of commitment for the insurer with low costs. The decision about commitment depends on which effect dominates in equilibrium.

Another modification of the timing allows auditing costs to be realized before insurers make their contract offers. Figure 4 summarizes the changes. Now, insurers can signal the auditing costs by their contract offers and there is two-sided asymmetric information already at the contracting stage. Thus, at $t = 0$ nature determines the costs of an audit for the insurer, which are the same for all firms, but uncertain.²¹ After that, the game is the same as before. Therefore the analysis of Section 3 remains unchanged and there is again a perfect Bayesian equilibrium without commitment. In this

²¹See Jost (1996) for a model with heterogeneous costs. In the model of Jost (1996), however, the coverage q is conditional on the claim being audited, which is not a common feature of insurance contracts.

- At $t = 0$, auditing costs c are realized and revealed to the insurer; furthermore, the degrees of ambiguity aversion are realized and revealed to the insured
- At $t = 1$, insurers make contract offers (q_i, P_i)
- At $t = 2$, the insured choose contracts
- At $t = 3$, insurers can commit to an auditing probability p_i
- \vdots

Figure 4: Modified Timing of the Extended Model

equilibrium, a contract (\tilde{q}, \tilde{P}) is offered by both types, which is an element of the following set

$$(\tilde{q}, \tilde{P}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P)$$

$$\text{with } P \geq \delta q + (1 - \delta)q\tilde{\alpha}[q, P].$$

Similar to the last section, the existence of the equilibrium without commitment requires additional assumptions: we assume that condition (4) in Proposition 2 is satisfied for the contract (\tilde{q}, \tilde{P}) and for the contract $(\hat{q}, \hat{P}) \in \arg \max \delta u(-L + q - P) + (1 - \delta)u(-P)$ with $P - \delta q - \delta c_L p^b[q, P] \geq \tilde{P} - \delta \tilde{q} - \delta c_L p^b[\tilde{q}, \tilde{P}]$. Furthermore, $\tilde{\alpha}[\tilde{q}, \tilde{P}] < 1$ and the probability of a loss should be sufficiently high (respectively low), i.e.,

$$\delta \geq (\leq) \frac{(N - 1)\tilde{P} + \tilde{\alpha}[\tilde{q}, \tilde{P}](c_L - m)}{(N - 1)\tilde{q} + \tilde{\alpha}[\tilde{q}, \tilde{P}](c_L - m) + (N p^b[\tilde{q}, \tilde{P}] - 1)c_L} \quad (7)$$

depending on the sign of the denominator.²² This condition guarantees that the advantage of the uncertainty is sufficiently big to restrain the low-cost type from revealing itself and capturing the whole market. Intuitively, for a positive denominator there have to be enough losses to reduce the possible cases of fraud. Thus, the amount of claims to audit is quite high even with commitment, and commitment does not pay for the insurance company, because it loses the deterrence effect and the fine income. If, on the contrary, the denominator is negative, catching fraudulent claims is so attractive for the insurer that a low incidence of losses is necessary to stabilize the equilibrium.²³

Proposition 4. *Given the discussed conditions, there is an equilibrium with every insurer offering exclusively the contract (\tilde{q}, \tilde{P}) and avoiding commitment.*

The equilibrium has an interesting feature. When the insurers consider a deviation, both types want to mimic the other type. The high-cost type wants to deviate if the out-of-equilibrium beliefs are tilted towards the low-cost insurer, because there will be little fraud. If, on the other hand, the beliefs are tilted towards the high-cost type, the low-cost insurance company can by deviating increase its

²²It can easily be seen that the denominator is bigger than the numerator, such that the fraction is always smaller than one and the constraint set is therefore non-empty. If the denominator is positive, the fraction might be negative, and in this case, the constraint is trivially satisfied. If, on the other hand, the denominator is negative, the fraction is always positive and thus the probability of a loss can be lower than the threshold.

²³The equilibrium is not unique. There will usually be a continuum of the equilibria, like $(\tilde{q}, \tilde{P} + \epsilon)$, of the type described in Proposition 4, depending on the parameter values. Furthermore, there is a separating equilibrium with each cost type offering the best contract that just breaks even, if the type of the insurer is known. The change in the timing yields an informed principal problem that differs fundamentally from the model considered in the previous sections.

market share and profits due to the beliefs of the policyholders about a low auditing probability. Yet the competitors use the commitment decision to signal the type of the deviating firm. This is why the out-of-equilibrium beliefs depend on the type of the deviating firm. Hence, the deviation is no longer profitable, because once the type of the deviating firm is revealed, the insurer is worse off than before by the conditions of Proposition 4. This holds even though the insurer may serve the whole market after a deviation. Thus, the actions of the competitors make this equilibrium possible.

If the type is revealed before the contract stage, in the equilibrium with commitment, the insurance market can break down. This happens if the high-cost type is realized and $c_H \geq c'$. Then no agent has a utility higher than without an insurance. Ambiguity allows avoiding this fate by making contracts feasible that rely on the deterrence effect of the uncertainty in the absence of commitment. If there is sufficient ambiguity, the level of fraud is always smaller than 1.

7 Conclusion

In this article, we discuss a costly state verification model with ambiguity about auditing costs. For this purpose, we use an insurance fraud setting. We show that ambiguity aversion reduces the inclination to engage in insurance fraud at a given level of auditing. The insurers, on the other hand, can gain by not committing to an auditing probability and maintaining the uncertainty, even if this means abandoning the advantages of commitment. This is the main contribution of this paper, as we prove that uncertainty can be a feasible deterrence device.

The second contribution is to study a model with ambiguity aversion in a game-theoretic framework. Although ambiguity seems even more relevant in a strategic interaction than for a single player, the literature on ambiguity aversion has so far focused on decision theory and finance with notable exceptions discussed in the introduction. We provide a game-theoretic analysis of ambiguity-averse policyholders. Modeling the ambiguity on the type space, i.e., the auditing costs of the insurers, allows the use of common equilibrium concepts.

The third contribution of this article is to consider whether competition forces firms to educate consumers. According to a common line of argument, competitive pressure provides consumers with all relevant information, as competitors have an incentive to reveal the information in order to increase their market shares. In our model, uncertainty prevails and on the equilibrium path no firm has an incentive to make the auditing costs public. Therefore, there is a market equilibrium with perfect competition where firms do not grant access to their information about auditing probabilities and costs and the uncertainty allows mitigating the effects of insurance fraud.

Finally, we summarize the incentives of insurers to avoid commitment. Insurers benefit from the higher perceived probability of auditing and the resulting lower level of fraud if their costs of auditing are high enough. For low costs, however, the insurers gain from non-committing, as they catch more fraudsters, thus saving indemnities and earning fines at low costs. In some cases, these effects are so strong that the costs caused by fraud and its deterrence are lower than under credible commitment to an auditing level. Consequently, the insurers will opt to implement strategic ambiguity.

A Appendix

A.1 Decision Making with Ambiguity Aversion

There are several representations of preferences that allow for ambiguity aversion, like Schmeidler (1989) or Cerreia-Vioglio et al. (2011).²⁴ Formally, ambiguity aversion is defined to be the preference of a mixture of lotteries compared to the lotteries themselves if the agent is indifferent between the lotteries.²⁵ The paper mainly uses smooth ambiguity aversion proposed by Klibanoff et al. (2005), which goes back to Segal (1987).²⁶ The agent knows the first-order and second-order probability distributions, but does not compute the reduced lottery. The first-order probability distribution is a distribution for the states of the world, i.e., the state space. The second-order probability distribution, on the other hand, reflects the probability for a first-order distribution. In their interpretation, the first-order distribution characterizes risk and the second-order distribution ambiguity. This distinction corresponds to the assumption that the first-order and second-order probabilities are based on different information. The intuition is that the agents have some theories or models of the world, that assign probabilities to the states of the world. The trust in each model is denoted by its second-order probability. The agent's preferences are represented by

$$f \rightarrow \int_{\Pi} \phi \left(\int u \circ f dP \right) d\mu.$$

The function ϕ reveals the attitude of the agent towards ambiguity. Therefore we will call it the ambiguity index. An ambiguity-neutral subject with a linear ϕ simply takes the expectation and derives simple probabilities for each state of the world. With ambiguity aversion, ϕ is strictly concave. The concavity of this function corresponds to the degree of ambiguity aversion. The function u is a von-Neumann-Morgenstern utility index, which determines the attitude towards risk.²⁷ In addition, P is a probability measure on the state space and Π is a set of first-order probability measures P . μ is a probability measure that corresponds to the second-order distribution on Π . The preference functional may be interpreted as a double expectation. First, the expected utility for every first-order distribution P is calculated. Then the expected utility for every P is transformed by the function ϕ . Finally, the mean with respect to the second-order probabilities is calculated. Yet the results do not hinge on this choice of representation.

An alternative representation is Maxmin Expected Utility of Gilboa and Schmeidler (1989), which is equivalent to Choquet Expected Utility of Schmeidler (1989) in our setting. Again there is a (finite) set Π of first-order probability measures, which are considered relevant. In contrast to smooth ambiguity aversion, however, agents have no second-order probabilities available. Consequently, they behave *as if* the probability distribution that yields the lowest expected utility is correct. The preferences are represented by

$$f \rightarrow \min_{P \in \Pi} \int u \circ f dP$$

²⁴Gilboa and Marinacci (2011) provide an excellent survey of such representations.

²⁵See Ghirardato and Marinacci (2002) for alternative definitions.

²⁶Similar representations are Seo (2009), Ergin and Gul (2009), Chew and Sagi (2008) and Nau (2006).

²⁷Ambiguity aversion is independent of the attitude towards risk. An agent may be ambiguity-averse and at the same time risk-neutral, and conversely.

The axiomatisations of both representations are based on the common decision-theoretic axioms, except that the independence axiom is restricted to specific acts. This is less restrictive than independence for all acts.

A.2 The Model with Maxmin Expected Utility

This section shows that the results of Section 2 are valid also in Maxmin Expected Utility. We assume a set of relevant probability distributions Π such that the probability of an audit is in the interval $[(1 - \mathcal{A})p, (1 - \mathcal{A})p + \mathcal{A}]$ with a parameter $\mathcal{A} \in [0, 1]$. Accordingly, the policyholders know that the auditing probability is around p , but are unaware of the exact value.²⁸ For $\mathcal{A} = 0$, there is no ambiguity and agents simply take the subjective probability p of Section 2 into consideration. On the other hand, with ambiguity, $\mathcal{A} > 0$, policyholders are more cautious and allow for some margin of error. Consequently, they behave *as if* the probability of getting caught were higher.

Lemma A.1. *Suppose the level of auditing is fixed. If the insurer does not announce the level of auditing and the ambiguity-averse policyholders do not have all the relevant information to determine it exactly, there is less insurance fraud than with easily available information about the auditing probability.*

Proof: Without a loss, the Maxmin Expected Utility is

$$(1 - ((1 - \mathcal{A})p + \mathcal{A}))u(-P + q) + ((1 - \mathcal{A})p + \mathcal{A})u(-P - M)$$

for fraudulent claims and $u(-P)$ without a claim. First, suppose the level of auditing is disclosed. Thus, there is no ambiguity and $\mathcal{A} = 0$. Therefore the policyholder overstates the loss if the probability p of an audit is smaller than p^b , as before.

In the second case, the insurer does not reveal the probability of auditing a claim. Then there is ambiguity. With ambiguity aversion and ambiguity, $\mathcal{A} > 0$, the policyholder considers the worst probability distribution in her set Π . So an ambiguity-averse policyholder acts as if the probability of detection were $(1 - \mathcal{A})p + \mathcal{A}$. Once again there is a threshold p^* for honest reporting, with

$$p^* = \frac{(1 - \mathcal{A})u(-P + q) + \mathcal{A}u(-P - M) - u(-P)}{(1 - \mathcal{A})[u(-P + q) - u(-P - M)]} = p^b - \frac{\mathcal{A}}{1 - \mathcal{A}} \frac{u(-P) - u(-P - M)}{u(-P + q) - u(-P - M)} < p^b.$$

As the last fraction is positive, we can conclude that $p^* < p^b$ for $\mathcal{A} > 0$. \square

This confirms our earlier result of Lemma 1 and shows that it is robust to the way ambiguity aversion is modeled.

A.3 Additional Proofs

Proof of Lemma 1: First, suppose the level of auditing is disclosed. Then there is no ambiguity. Therefore the policyholder has an incentive to engage in fraud if the probability p of an audit is smaller than

$$p^b = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}.$$

²⁸In another approach, Gajdos et al. (2008) propose an axiomatic foundation for such a contraction representation.

If $p \geq p^b$, the policyholder will behave honestly and report only true losses.

In the second case, the insurer does not reveal the probability of auditing a claim. Thus, the policyholder lacks relevant information. The difference to the first case depends on the ambiguity aversion and the amount of ambiguity perceived by the policyholder. An ambiguity-neutral policyholder, i.e., with a linear ϕ , takes the same subjective probability into account and evaluates her possible actions as before. With ambiguity aversion ϕ is strictly concave. By Jensen's inequality it holds

$$\begin{aligned} \int_{\Pi} \phi((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M))d\mu(\hat{p}) &\leq \phi \left(\int_{\Pi} ((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M))d\mu(\hat{p}) \right) = \\ &= \phi((1 - p)u(-P + q) + pu(-P - M)). \end{aligned}$$

Thus, an ambiguity-averse policyholder acts as if the probability of detection were higher. Hence if the expected probability $p = \int_{\Pi} \hat{p}d\mu(\hat{p})$ is at least p^b , no insurance fraud occurs. If the second-order distribution is non-degenerate, this holds even for lower expected probabilities. \square

Proof of Lemma 2: Suppose the second-order distribution $\mu(\hat{p})$ is such that the first policyholder weakly prefers to abstain from fraud. Then

$$\int_{\Pi} \phi_1((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M))d\mu(\hat{p}) \leq \phi_1(u(-P)).$$

As the second policyholder is more ambiguity-averse than the first one, Jensen's inequality yields

$$\begin{aligned} \int_{\Pi} \phi_2((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M))d\mu(\hat{p}) &\leq g \left(\int_{\Pi} \phi_1((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M))d\mu(\hat{p}) \right) \leq \\ &\leq g(\phi_1(u(-P))) = \phi_2(u(-P)). \end{aligned}$$

If the second-order distribution is non-degenerate, the first inequality is strict. Hence, the more ambiguity-averse policyholders commit less insurance fraud. \square

Proof of Proposition 1: As the beliefs μ about r are considered as fixed in the Proposition, it does not consider any signaling or adverse selection effects. Lemma 2 ensures monotonicity of the fraud decision of the policyholders in \mathcal{A} . Now define $\tilde{\alpha}$ as the fraction of policyholders engaging in fraud if the low-cost insurer audits every claim, $p_L = 1$, and the high-cost insurer audits no claims, $p_H = 0$. Then $\alpha = F(\mathcal{A}^*)$ and

$$\mathcal{A}^* = \sup \left(\left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \mid \int \phi_{\mathcal{A}}((1 - r)u(-P + q) + ru(-P - M))d\mu(r) > \phi_{\mathcal{A}}(u(-P)) \right\} \cup \{\underline{A}\} \right).^{29}$$

Solving the equilibrium backwards, we consider the insurer setting the level of auditing. As the problem for the insurer is linear, at least one type has a corner solution and audits all or none of the claims made. If for a level of fraud α the costs of auditing are lower (resp. higher) than $c^*(\alpha)$, as defined in (2), all (none of the) claims are audited. Consequently, an ambiguity-neutral policyholder acts *as if* the expected probability of an audit is

²⁹See footnote 17 for an interpretation.

$$E(p) = \begin{cases} 1 & \text{if } c_H < c^*(\alpha) \\ p \in [\bar{r}, 1] & \text{if } c_H = c^*(\alpha) \\ \bar{r} & \text{if } c_L < c^*(\alpha) < c_H \\ p \in [0, \bar{r}] & \text{if } c_L = c^*(\alpha) \\ 0 & \text{if } c_L > c^*(\alpha) \end{cases}$$

depending on the auditing costs. Thus, we distinguish the following five cases: (a) no auditing $p = 0$, (b) low partial auditing $0 < p < \bar{r}$, (c) partial auditing $p = \bar{r}$, (d) high partial auditing $\bar{r} < p < 1$, and (e) complete auditing $p = 1$.

(a) If policyholders expect no audits, $p = 0$, every policyholder will report a claim, even if no loss occurred. Ex post it will still be optimal to abstain from auditing for the insurer if the costs of auditing c for both types of insurer are higher than the expected benefit of detecting a fraudster, $(1 - \delta)(q + m)$. This is the first case (a) of the proposition with $c_L \geq c^*(1)$. If the costs are lower, this is not an equilibrium as the insurers do some auditing.

(b) If the level of auditing is low, i.e., $0 < p < \bar{r}$, the low-cost insurer is exactly indifferent between auditing claim reports or not. Therefore the high-cost insurer will abstain from auditing any claims and we can solve the equilibrium backwards by calculating

$$\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$$

from the definition of $c^*(\alpha)$ in equation (2) to make the low-cost insurer indifferent. The level of fraud determines by equation (1) the necessary level of auditing as a solution p_L^* to

$$\phi_{\mathcal{A}'}(u(-P)) = \int \phi_{\mathcal{A}'} \left(u(-P + q) - r p_L (u(-P + q) - u(-P - M)) \right) d\mu(r)$$

with $\mathcal{A}' = F^{-1}(\alpha)$. The right-hand side of this equation is decreasing in p_L and is bigger than the left-hand side for $p_L = 0$. Therefore, $p_L^* > 0$. In addition, $p_L^* < 1$ and the low-cost insurer can on its own deter enough policyholders from filing false reports if $\tilde{\alpha} < 1$. This condition corresponds to a high expected probability \bar{r} for facing the low-cost insurer. Hence, the level of fraud is $\alpha \in (\tilde{\alpha}, 1)$ depending on the auditing costs c_L of the low-cost insurer. If these costs are lower than $c^*(\tilde{\alpha})$, the low-cost insurer has an incentive to audit as many claims as possible. Then it is impossible to make the low-cost insurer indifferent with respect to its auditing decision. If, on the other hand, these costs are higher than $c^*(1)$, it would not be worthwhile to audit any claims for the insurer. Consequently, the second part (b) of the proposition requires $c^*(\tilde{\alpha}) \leq c_L < c^*(1)$. Notice that $\tilde{\alpha} < 1$ if the condition is satisfied, because $c^*(\alpha)$ is increasing in α .

(c) In the next step, consider an intermediate level of auditing, $p = \bar{r}$. Then, the low-cost insurer audits every claim made and the high-cost insurer does not audit any claims. Therefore the costs have to be $c_L < c^*(\tilde{\alpha}) \leq c_H$. Otherwise one type of insurer has an incentive to deviate. The level of fraud is $\tilde{\alpha}$ by definition. $0 < c_L < c^*(\tilde{\alpha})$ implies that there will be some fraud and $\tilde{\alpha} > 0$, as $c^*(0) = 0$.

Part (c) of the proposition describes this equilibrium.

(d) More auditing is achieved if the low-cost insurer audits every claim and the high-cost insurer audits some claims, i.e., $p_L = 1$ and $p_H > 0$. The high-cost insurer has to be indifferent to find this level of auditing optimal. Therefore we solve equation (2) of the definition of the indifference costs for the corresponding level of fraud as in case (b)

$$\alpha = \frac{\delta c_H}{(1-\delta)(q+m-c_H)}.$$

α is smaller than one if and only if $c_H < (1-\delta)(q+m) = c^*(1)$. Equation (1) determines the level of auditing in equilibrium as a solution p_H^* to

$$\phi_{\mathcal{A}'}(u(-P)) = \int \phi_{\mathcal{A}'}\left(u(-P+q) - (r+(1-r)p_H)(u(-P+q) - u(-P-M))\right) d\mu(r)$$

with $\mathcal{A}' = F^{-1}(\alpha)$. The right-hand side of this equation is decreasing in p_H and is smaller than the left-hand side for $p_H = 1$. Therefore, $p_H^* < 1$. In addition, $p_H^* > 0$ if $\tilde{\alpha} > 0$ and the low-cost insurer cannot on its own deter all policyholders from filing false reports. This condition corresponds to a low subjective probability \bar{r} for facing the low-cost insurer. Hence, the level of fraud is $\alpha \in (0, \tilde{\alpha})$ depending on the auditing costs c_H of the high-cost insurer. If these costs are above $c^*(\tilde{\alpha})$, it would not be worthwhile to audit any claims for the insurer. Consequently, part (d) of the proposition requires $c_H < c^*(\tilde{\alpha})$. Notice that $\tilde{\alpha} > 0$ if the condition is satisfied.

(e) Finally, if every claim is believed to be audited, only true claims are reported. Then, however, the best strategy of the insurer ex post is not to audit any reports. Therefore, in the absence of commitment, some policyholders will always report false claims in equilibrium. \square

Lemma A.2 is required for the proof of Proposition 2.

Lemma A.2. *Assume there is no commitment and the insurer of type $i \in \{H, L\}$ is indifferent with respect to audits by the level of fraud, as $\alpha = \frac{\delta c_i}{(1-\delta)(q+m-c_i)}$. Then the insurer of type i prefers to commit to a level of auditing p^b independent of the policyholders' beliefs about its type.*

Proof: The costs with commitment are lower than in its absence if

$$\alpha(1-\delta)(1-p_i)q - m\alpha p_i(1-\delta) + (\delta + \alpha(1-\delta))p_i c_i \geq \delta p^b c_i.$$

Collecting the p_i terms we get

$$\alpha(1-\delta)q - p_i[\alpha(1-\delta)q + m\alpha(1-\delta) - (\delta + \alpha(1-\delta))c_i] \geq \delta p^b c_i.$$

Rearranging the terms in the square brackets gives

$$\alpha(1-\delta)q - p_i[\alpha(1-\delta)(q+m-c_i) - \delta c_i] \geq \delta p^b c_i.$$

As $\alpha = \frac{\delta c_i}{(1-\delta)(q+m-c_i)}$ the term in square brackets equals 0 and we get $\alpha(1-\delta)q \geq \delta p^b c_i$. This means that the auditing costs, $(\delta + \alpha(1-\delta))p_i c_i$, and cost savings due to exposed frauds, i.e., indemnities not paid out, $q\alpha p(1-\delta)$, in combination with fines received by the insurer, $m\alpha p(1-\delta)$, offset each other.

Consequently *only* the losses due to falsely stated claims, $\alpha(1-\delta)q$, matter. The indifference condition (2) of the insurer in equilibrium causes this effect. Inserting α yields $\frac{\delta c_i}{(1-\delta)(q+m-c_i)}(1-\delta)q \geq \delta p^b c_i$.

Multiplying the inequality by $q+m-c_i$ leads to $q \geq p^b(q+m-c_i)$. Finally, arranging the terms for c_i and dividing by p^b gives us

$$-\frac{q}{p^b} + q + m \leq c_i. \quad (8)$$

$m \leq M$ and $\epsilon = [u(-P) - u(-P-M)]q - [u(-P+q) - u(-P)]M > 0$, because the utility index u is strictly concave. Therefore,

$$q - p^b(q+m) = (1-p^b)q - p^b m \geq (1-p^b)q - p^b M = \epsilon [u(-P+q) - u(-P-M)]^{-1} > 0.$$

This ensures that the left-hand side of inequality (8) is negative. Then, the inequalities are satisfied and the respective insurer can make itself better off by committing to an auditing level p^b . \square

Proof of Proposition 2: We begin by considering pooling equilibria with both types avoiding commitment. Then the policyholders' beliefs remain unchanged at μ if no commitment is observed. In the case of commitment, the beliefs are irrelevant. In the following, we consider different cost ranges.

For high costs of auditing, the insurers abstain from auditing and this is common knowledge. Consequently, they are indifferent on the commitment issue and the beliefs do not matter. Due to Proposition 1, Lemma A.2 and the fact that $c' > c^*(1)$, this is the case for $c_L > c'$. Below c' , under commitment, audits become worthwhile and there is no insurance fraud. Without commitment, auditing is still too expensive. Therefore, commitment is necessary to avoid complete fraud and at least one type has an incentive to commit.³⁰ Yet, once the auditing costs of the low-cost type drop below $c^*(1)$, there is auditing even without commitment. We now distinguish the following cases according to Proposition 1.

In case (b) or (d), commitment is always preferable to no commitment, because the insurer which does partial auditing has an incentive to commit itself. The reason is the same as in Picard (1996). As the indifference of the insurer determines the level of fraud, the insurer's costs are independent of its level of auditing. Therefore replicating the auditing level p^b of the commitment case does not change profits. Without commitment, the insurer still faces fraud causing additional costs for indemnities and audits that are not balanced by income from fines. The details can be found in Lemma A.2. Consequently, there is an incentive to commit to an auditing level in these cases and no pooling equilibrium exists with both types avoiding commitment.

Now suppose case (c) of Proposition 1 with audits of $p_H = 0$ and $p_L = 1$. Then the low-cost type prefers not to commit if and only if equation (3) is valid for $p_L = 1$ or $-m\alpha(1-\delta) + [\delta + \alpha(1-\delta)]c_L \leq \delta p^b c_L$. Rearranging the terms yields

$$c_L \leq \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)}.$$

The fraction on the right-hand side is positive and does not depend on c_L . Furthermore the threshold

³⁰Due to the definition of p^b and the concavity of u it holds $c' > c^*(1)$. Formally, this is equivalent to $q/(\delta p^b) > q+m$. Consequently, it is enough to show that $q - p^b(q+m) > 0$. This is done in Lemma A.2.

is smaller than $c^*(\tilde{\alpha})$, because by Lemma A.2

$$\begin{aligned}
& q - p^b(q + m) > 0 \\
\Leftrightarrow & \alpha(1 - \delta)q + \delta[q - p^b(q + m)] > 0 \\
\Leftrightarrow & \alpha(1 - \delta)q[\delta(1 - p^b) + \alpha(1 - \delta)] - mp^b\delta\alpha(1 - \delta) > 0 \\
\Leftrightarrow & \alpha(1 - \delta)(q + m)[\delta(1 - p^b) + \alpha(1 - \delta)] > m(\delta + \alpha(1 - \delta))\alpha(1 - \delta) \\
\Leftrightarrow & \frac{m\alpha(1 - \delta)}{\delta(1 - p^b) + \alpha(1 - \delta)} < \frac{\alpha(1 - \delta)(q + m)}{\delta + \alpha(1 - \delta)} = c^*(\alpha).
\end{aligned}$$

Therefore condition (4) on c_L guarantees case (c). Consequently, for c_L small enough the low-cost type of insurer forgoes commitment. By equation (3), the high-cost type, on the other hand, avoids to commit if $\alpha(1 - \delta)q \leq \delta p^b c_H$, as $p_H = 0$. This leads to

$$c_H \geq \frac{\alpha(1 - \delta)q}{\delta p^b} = \alpha c'.$$

Moreover, this threshold is higher than the threshold for c_H in case (c) as seen by Lemma A.2 and

$$\begin{aligned}
& q - p^b(q + m) > 0 \\
\Leftrightarrow & \frac{q}{\delta p^b} > \frac{q + m}{\delta} > \frac{q + m}{\delta + \alpha(1 - \delta)} \\
\Leftrightarrow & \alpha c' = \alpha \frac{(1 - \delta)q}{\delta p^b} > \frac{\alpha(1 - \delta)(q + m)}{\delta + \alpha(1 - \delta)} = c^*(\alpha).
\end{aligned}$$

Thus, the high-cost insurer has no incentive to commit if its costs are high enough. In summary, we have found a range of parameters such that, in equilibrium, the insurers choose not to commit to an auditing level, even if they have the possibility to do so credibly and free of charge. Area (A) in Figure 3 on page 12 illustrates this range of parameters. So far, we have considered only complete pooling with respect to the commitment decision. Yet by including partial pooling, it is possible to increase the parameter range for c_L and c_H , because the line of argument does not depend on the specific level of fraud α . If condition (4) holds only for an $\alpha < \tilde{\alpha}$ with $\tilde{\alpha}$ as defined in Proposition 1, it is possible to choose α , such that the high-cost type is indifferent with respect to commitment. Thus, it plays a mixed strategy and commits to an auditing level with some probability σ_H . This changes equilibrium beliefs if no commitment was observed. Then, the probability of a low-cost insurer is $r + (1 - r)(1 - \sigma_H)$ with subjective probability $\mu(r)$. Hence, the equilibrium level of fraud decreases. As shown before, this behavior is sequentially rational. If, on the other hand, condition (4) holds only for $\alpha > \tilde{\alpha}$, choose α , such that the low-cost type plays a mixed strategy with respect to the commitment decision. This decreases equilibrium beliefs if no commitment was observed. Hence, the equilibrium level of fraud increases. Figure 3 depicts the bounds on the costs of the two types.

Now consider pooling equilibria with both types committing. For $c_L \geq c^*(1)$ this equilibrium exists independently of the out-of-equilibrium beliefs. For costs below this threshold, we distinguish three cases corresponding to cases (b), (c), and (d) in Proposition 1, depending on the out-of-equilibrium beliefs. Suppose the beliefs given that no commitment was observed are such that the insurer of type L is indifferent with respect to audits as in case (b). Then the high-cost insurer abstains from auditing,

$p_H = 0$, if it does not commit to an auditing level. By the same reasoning as before, commitment is only optimal in this case if $c_H \leq \frac{\alpha(1-\delta)q}{\delta p^b} = \frac{qc_L}{p^b(q+m-c_L)}$ and $c_L < c^*(1)$. If, on the other hand, the beliefs are such that the insurer of type H is indifferent as in case (d), the low-cost insurer will do complete auditing, $p_L = 1$, in the absence of commitment. Then commitment is only optimal for both types if $c_H \leq c^*(1)$ and $c_L \geq \frac{m\alpha(1-\delta)}{\delta(1-p^b)+\alpha(1-\delta)}$ with $\alpha = \frac{\delta c_H}{(1-\delta)(q+m-c_H)}$. Finally, the beliefs could be such that both insurers have a corner solution as in case (c) of Proposition 1. Analogously, this is sequentially optimal only if there exists an $\alpha \in (0, 1]$ such that

$$c^*(\alpha) < c_H \leq \frac{\alpha(1-\delta)q}{\delta p^b} \quad \text{and} \quad c^*(\alpha) > c_L \geq \frac{m\alpha(1-\delta)}{\delta(1-p^b)+\alpha(1-\delta)}.$$

In addition, there is a corner solution for $\alpha = 1$ with $c_H > c^*(1)$ and $c^*(1) > c_L \geq \frac{m(1-\delta)}{1-\delta p^b}$.

Finally, consider separating equilibria with type i committing and the other type j avoiding commitment. Then there is no ambiguity. Without ambiguity, however, commitment is at least weakly optimal. Policyholders' beliefs in the absence of commitment are degenerate at $r = 1$ or 0 , respectively. According to Proposition 1 with $\tilde{\alpha} = 0$ or 1 , respectively, this yields two cases for the non-committing insurer.

First, there might be some fraud, $\alpha = \frac{\delta c_j}{(1-\delta)(q+m-c_j)}$ for low costs, $c_j < c^*(1)$. In this case and in all other cases where the indifference of the insurer determines the level of fraud, Lemma A.2 shows that commitment is preferable to no commitment, as it decreases the insurer's costs.

Second, there is complete fraud, $\alpha = 1$, for high costs, $c_j \geq c^*(1)$. To ensure non-commitment is optimal for type j , it has to hold $c_j \geq c'$. Above this threshold, the non-committing insurer j is indifferent with respect to the commitment decision and avoiding commitment is sequentially optimal. Yet the committing insurer i might profit from a deviation to avoid commitment. This deviation is only profitable if the committing insurer i is the low-cost type. $-m(1-\delta) + c_L \geq \delta p^b c_L$ makes this deviation unprofitable, as the low-cost type will audit every claim. Rearranging the terms gives $c_L \geq \frac{m(1-\delta)}{1-p^b\delta}$, which is smaller than c' , as $\frac{q}{\delta p^b} > q + m$ by Lemma A.2. Together with $c_H \geq c'$, this allows for a fully separating equilibrium in area (B) of Figure 3. Moreover, we have shown that in every fully separating equilibrium at least one type is indifferent with respect to the commitment decision. \square

Proof of Proposition 3: First, we show that the strategy profile in the proposition is an equilibrium of the game. Given that the other insurers offer the contract (q^{NC}, P^{NC}) , each insurer makes zero expected profits in equilibrium, because by Proposition 1 and 2 in combination with condition (4) it is optimal to avoid commitment and have the low-cost type doing the auditing, i.e., $p_L = 1$ and $p_H = 0$. Therefore there is pooling with respect to the commitment decision. At the time of contracting, auditing costs have not been realized yet. Thus, signaling is impossible. Yet the commitment decision allows for signaling. The equilibrium beliefs are μ , as no commitment is observed. Off the equilibrium path, beliefs about types are given by the behavior characterized in Proposition 2 if they are on the equilibrium path of the continuation games beginning with the realization of insurer's types. Otherwise set them to μ .

Consider insurer j deviating by offering a less appealing contract, denoted by (\hat{q}, \hat{P}) , where the appeal or the attractiveness of a contract is given by $\delta u(-L + q - P) + (1-\delta)u(-P)$. By definition no

policyholder who behaves honestly with probability one will accept (\hat{q}, \hat{P}) independent of her beliefs. Thus, only policyholders who behave fraudulently might opt for the contract (\hat{q}, \hat{P}) . Yet, compared to the equilibrium contract, fraud implies stochastic redistribution financed by the policyholders themselves with efficiency losses due to the auditing costs and the difference $M - m$ in the fine payments. As agents are risk averse and ambiguity averse, any profitable contract with this property offers less utility than the contract (q^{NC}, P^{NC}) .³¹ Consequently, the contract (\hat{q}, \hat{P}) will either make a loss or attract no demand at all. Hence, this deviation is not profitable.

Now consider a deviation with a (weakly) more attractive contract (\bar{q}, \bar{P}) . In this case all consumers who are made weakly better off switch contracts. Then the insurer makes a loss with every policyholder unless the insurer succeeds in lowering its costs due to auditing and fraudulent claims by changing the level of fraud in this contract. The next two paragraphs show that it is impossible to do so.

First, assume that the new contract (\bar{q}, \bar{P}) implements commitment for some types of the insurer to reduce the costs related to fraudulent behavior. Given condition (4), however, commitment makes contracts more expensive for the insurer according to Proposition 2. Therefore even the best available contract (q^C, P^C) with commitment is less attractive than (q^{NC}, P^{NC}) . If the deviating insurer anticipates to use commitment independent of its type, the condition on c_L in (4) and $(1 - \delta)\tilde{\alpha}q < \min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\}$ by the condition on c_H result in

$$c_L < \frac{m\tilde{\alpha}(1 - \delta) + \frac{1 - \bar{r}}{\bar{r}}(\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\} - (1 - \delta)\tilde{\alpha}q)}{\delta(1 - p^b[q^C, P^C]) + \tilde{\alpha}(1 - \delta)}$$

$$\bar{r}\delta c_L + (1 - \delta)(\bar{r}(c_L - m) + (1 - \bar{r})q)\tilde{\alpha} < \bar{r}\delta c_L p^b[q^C, P^C] + (1 - \bar{r})\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\}. \quad (9)$$

The right-hand side of the inequality calculates the costs of fighting fraud with commitment. It is higher than the costs in the absence of commitment. If, on the other hand, the deviating insurer makes the commitment decision dependent on its type, the following cases are feasible by Proposition 2. The fully separating equilibrium for (\bar{q}, \bar{P}) is never profitable, because it implies complete fraud for the high-cost type and condition (9) ensures that the insurer is worse off. Moreover, it is impossible due to $c_L < m(1 - \delta)$. Now consider contracts that result in partial pooling, i.e., one type of insurer plays a mixed strategy with respect to the commitment decision. As the mixing type of insurer is indifferent between commitment and non-commitment, its profits are the same in both cases and the budget constraint (5) is still binding.³² By the definition of contract (q^{NC}, P^{NC}) the contract (\bar{q}, \bar{P}) cannot be profitable. Consequently, the contract (\bar{q}, \bar{P}) makes losses with commitment and the deviating insurer will not implement commitment.

Second, the deviating insurer engages in cherry-picking and the policyholders with a low degree of ambiguity aversion are attracted to the contract (q^{NC}, P^{NC}) offered by the remaining insurers. Thus, fraud will be low in contract (\bar{q}, \bar{P}) . This yields a change in the auditing regime in the contracts

³¹The condition $c_L < m(1 - \delta)$ ensures that a fully separating equilibrium is impossible, as $m(1 - \delta) < \frac{m(1 - \delta)}{1 - \delta p^b}$ for all $p^b > 0$. If $\tilde{\alpha}[q^{NC}, P^{NC}] + p^b[q^{NC}, P^{NC}] < 1$, condition (4) already implies $c_L < m(1 - \delta)$.

³²The change in the level of fraud caused by the partial pooling makes it more difficult to satisfy the budget constraint (5). If the low-cost insurer is using partial commitment, the level of fraud increases in the case without commitment compared to both types not committing. Yet partial pooling is only implemented if in the contract (\bar{q}, \bar{P}) the level of fraud with pooling is lower than in a corresponding contract where complete pooling is optimal. Therefore fraud is still lower than in the pooling contract. If such a contract is more profitable than (q^{NC}, P^{NC}) , this contradicts the definition of (q^{NC}, P^{NC}) . The argumentation is analogous if the high-cost insurer uses partial commitment.

(q^{NC}, P^{NC}) . Due to the assumptions on c_L and c_H , complete fraud is never optimal in contract (q^{NC}, P^{NC}) , as the insurers adapt their auditing strategies accordingly. Thus, some policyholders will report honestly, although they have chosen the contract (q^{NC}, P^{NC}) , which is a contradiction. Hence, this deviation is not profitable. Together with Propositions 1 and 2, this completes the first part of the proof and shows that offering the contract (q^{NC}, P^{NC}) without commitment and the low-cost type doing the auditing, i.e., $p_L = 1$ and $p_H = 0$, is a perfect Bayesian equilibrium of the game.

In the second part of the proof, we show that any equilibrium satisfies the properties stated in the proposition. For this purpose, assume to the contrary that there is an equilibrium with different contracts accepted by the policyholders. If in expectation insurers make profits on their contracts in this alternative equilibrium, we show a contradiction in the next three steps. First, assume to the contrary that there are at least two profitable contracts with complete fraud. If $c_L \geq c'$ in the corresponding contract and the contract is profitable, no one will accept the contract. Therefore the only remaining case is pooling on non-commitment with $\tilde{\alpha} = 1$. Then it is a profitable deviation to propose a contract that does not attract any honest policyholders, but is preferred by the fraudsters from the first two contracts. This is always feasible and decreases profits per policyholder, but increases total profits due to the gain in market share. Therefore there is at most one contract with complete fraud.

Second, assume to the contrary that in equilibrium there are at least two profitable contracts with commitment and some policyholders who always behave honestly. This implies partial pooling or complete pooling on commitment. Now reduce the premium P by a small $\epsilon > 0$ and implement commitment as in the initial contract. The honest policyholders now choose the new contract and increase the market share of the insurer, making this deviation profitable. Therefore there is at most one profitable contract with commitment and honest policyholders in equilibrium.³³

Third, take one of the profitable contracts with honest policyholders and no commitment, (q^1, P^1) . By the previous steps, there exists at least two of them, as full separation is impossible by $c_L < m(1-\delta)$. Moreover, by Proposition 2, the auditing regime corresponds to pooling on case (c) of Proposition 1. In these contracts, beliefs about types and auditing probabilities for each type of insurer are identical at the contracting stage. This allows for a profitable deviation by offering a contract that is slightly more attractive than (q^1, P^1) instead of (q^1, P^1) . The modified contract attracts all the policyholders from the contracts in this class.

Therefore, in equilibrium, some insurers make zero profits on their contracts. If these contracts are less attractive than the contract (q^{NC}, P^{NC}) , an insurer may deviate by offering the contract $(q^{NC}, P^{NC} + \epsilon)$ with $\epsilon > 0$, such that the contract is still more attractive than the equilibrium contracts. Then all policyholders, who before anticipated behaving honestly independently of the commitment decision or were in a contract with complete fraud, opt for the new contract, because it increases their utility. Other policyholders follow suit, as they anticipate that auditing regimes are changing due to the different distribution of ambiguity aversion in the previous contracts. This

³³In equilibrium it is infeasible to have partial pooling with respect to commitment and every policyholder filing a claim in the absence of commitment. The reason is that policyholders get a higher utility the higher the probability is of facing a high-cost type in the absence of commitment. This can be achieved by changing the probability of commitment of the types. The level of fraud remains unchanged, as $\alpha = 1$. Furthermore, profits remain unchanged due to the indifference condition of the mixing type of insurer. Consequently, the policyholders would be willing to enter a more profitable contract. This is a profitable deviation and shows why such an auditing regime is impossible.

guarantees positive profits for the deviating insurer.

Assume to the contrary that the equilibrium contracts are (weakly) more attractive than contract (q^{NC}, P^{NC}) . As shown in the first part of the proof, the contracts make losses if they use commitment or if there is complete fraud. Therefore, by Proposition 2, the only remaining auditing regime is pooling in case (c) of Proposition 1 and all equilibrium contracts offer the same expected utility for an honest policyholder. By the definition of contract (q^{NC}, P^{NC}) , insurers make a loss if the policyholders are split up equally between insurers. Now assume the distribution of policyholders into contracts is heterogeneous, so that the amount of fraud differs between contracts. In some contracts, it is above $\tilde{\alpha}[q, P]$, while in others it is below $\tilde{\alpha}[q, P]$. Yet it is impossible to reduce the costs due to auditing and fraudulent claims and screen the policyholders according to their ambiguity aversion. The reason is the following. If $\bar{r}/(1 - \bar{r}) = q/(m - c_L)$, profits do not change in the amount of fraud and any contract except (q^{NC}, P^{NC}) that is (weakly) more attractive than (q^{NC}, P^{NC}) makes a loss. If $\bar{r}/(1 - \bar{r}) < q/(m - c_L)$, profits are decreasing in the level of fraud and contracts with fraud above $\tilde{\alpha}[q, P]$ make losses. Given the high indemnity, however, these contracts attract the policyholders anticipating fraudulent behavior, as the auditing regime remains unchanged. If, on the other hand, $\bar{r}/(1 - \bar{r}) > q/(m - c_L)$, the contracts with fraud below $\tilde{\alpha}[q, P]$ make losses. Again, the low indemnity deters the fraudsters from those contracts generating a loss for the insurer. Yet, in equilibrium there are no insurers with loss-making contracts. Consequently, (q^{NC}, P^{NC}) is the only accepted contract in any equilibrium of the game. \square

Proof of Proposition 4: The beliefs of the insured about the type of insurer are μ if they observe the contract (\tilde{q}, \tilde{P}) . If, on the other hand, they observe a different contract and at least $N - 2$ of the insurers commit, they update their beliefs to $\mu(1) = 1$. Otherwise beliefs remain at μ . If a deviation at the contracting stage occurs, firms with low costs c_L commit at $t = 3$. The beliefs of the insurer about the ambiguity aversion of its policyholders are according to the distribution F .

The low-cost type makes positive profits with the contract (\tilde{q}, \tilde{P}) , because according to Proposition 2 in combination with condition (4) auditing is profitable and the premium is set, such that no auditing gives zero profits and auditing is profitable for the low-cost type. If a firm j of the low-cost type tries to capture the whole market by offering a more attractive contract (\hat{q}, \hat{P}) , due to the out-of-equilibrium beliefs agents know its type, since the behavior of the competitors reveals it. Consequently, insurer j always wants to commit to an auditing level in its contract (\hat{q}, \hat{P}) . No matter whether the insured go to the deviating insurer or stay with the equilibrium contract, we show that the deviation is not profitable.³⁴ The profits with the new contract (\hat{q}, \hat{P}) are lower, because by assumption $\hat{P} - \delta\hat{q} - \delta p^b[\hat{q}, \hat{P}]c_L \leq \tilde{P} - \delta\tilde{q} - \delta p^b[\tilde{q}, \tilde{P}]c_L$ and condition (7) yields

$$\begin{aligned} & \tilde{P} - \delta\tilde{q} - \delta p^b c_L \leq \frac{1}{N} \left[\tilde{P} - \delta\tilde{q} + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \right] \\ \Leftrightarrow & (\tilde{P} - \delta\tilde{q})(N - 1) - N\delta p^b c_L \leq m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \\ \Leftrightarrow & (N - 1)\tilde{P} + \alpha(c_L - m) \leq \delta \left[\tilde{q}(N - 1) + Np^b c_L - m\alpha - (1 - \alpha)c_L \right] \end{aligned}$$

³⁴Indeed, for $c_H \leq \frac{(1-\delta)\tilde{q}}{\delta p^b[\tilde{q}, \tilde{P}]}$, all insured opt for the new contract (\hat{q}, \hat{P}) , because the new contract is more attractive and the insured behave honestly. If c_H is higher, some policyholders may stay with the old contract.

$$\Leftrightarrow \quad \delta \geq (\leq) \frac{(N-1)\tilde{P} + \alpha(c_L - m)}{(N-1)\tilde{q} + \alpha(c_L - m) + (Np^b - 1)c_L}.^{35}$$

The direction of the inequality in the last line depends on the sign of the denominator, as discussed before. The strategy of the other insurers is sequentially optimal, as commitment is optimal for an insurer offering contract (\tilde{q}, \tilde{P}) given the beliefs $\Pr(c_L) = 1$. Therefore it is a best response for the low-cost insurer to offer (\tilde{q}, \tilde{P}) in this equilibrium.

The high-cost type, on the other hand, has no incentive to deviate either, because by offering the contract (\tilde{q}, \tilde{P}) with commitment, the insurer would make a loss according to Proposition 2. Similarly, the insurer would incur a loss if it offered a more attractive contract by the definition of contract (\tilde{q}, \tilde{P}) . Given the beliefs μ , the other insurers have no incentive to commit. Therefore no profitable deviation is possible. In equilibrium, both types of insurers decide to avoid commitment and every firm offers the contract (\tilde{q}, \tilde{P}) . \square

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³⁵Here we suppress the dependency of α and p^b on $[\tilde{q}, \tilde{P}]$ for notational convenience.

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