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An incomplete contracts perspective on the provision and pricing of excludable public goods*

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Abstract

We study whether a firm that produces and sells access to an excludable public good should face a self-financing requirement, or, alternatively, receive subsidies that help to cover the cost of public-goods provision. The main result is that the desirability of a self-financing requirement is shaped by an equity-efficiency trade-off: While first-best efficiency is out of reach with such a requirement, its imposition limits the firm's ability of rent extraction. Hence, consumer surplus may be higher if the firm has no access to public funds.

Keywords: Incomplete Contracts, Excludable Public Goods, Regulation

JEL: D82, D86, H41, L51

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1 Introduction

This paper studies whether a firm who produces and prices an excludable public good should face a self-financing requirement, or whether tax revenues should be used to cover parts of the costs. If we think of the excludable public good as a bridge, the question that is addressed in this paper can be framed as follows: Should the financing of the bridge rely exclusively on tolls, or is there a role for a head tax that every inhabitant of the relevant city has to pay, irrespectively of whether he will cross the bridge frequently or only rarely. Alternatively, if the excludable public good is a network for telecommunications, the question is whether this infrastructure investment must be financed exclusively out of the revenues that can be generated by selling telecom services, or whether there is a role for taxes.

The disadvantage of self-financing requirements is that they induce distortions. Consumers are excluded from the consumption of the public good, even though, because of non-rivalry, admitting them involves no cost. If the firm has no access to public funds these inefficiencies cannot be avoided. By contrast, if tax revenues are available, we can get rid of these inefficiencies.

This observation is a challenge for the literature on public-sector pricing problems in the tradition of Ramsey (1927) and Boiteux (1956) which is based on the premise that a self-financing requirement is in place. It is subject to a similar critique as the one voiced by Atkinson and Stiglitz (1976) who question the relevance of Ramsey models of taxation in economies with a representative agent. In these models, lump-sum taxes are a first-best solution to the policy problem at hand. Therefore, the Ramsey tax problem is interesting only if lump-sum taxes are assumed unavailable. This makes the characterization of an optimal tax system a somewhat contrived exercise. Analogously, finding an optimal mechanism for the provision of an excludable public good in the presence of a self-financing requirement appears to be an artificial problem.

This paper provides a justification for self-financing requirements. As a main result, it is shown that their imposition may be the optimal reaction to an equity-efficiency trade-off. This trade-off involves, on the one hand, a comparison of the total surplus from public-goods production which is higher if there is no such requirement, and an assessment of the distribution of the surplus between consumers and the producer of the public good, on the other. In particular, the imposition of a self-financing requirement may lead to a higher level of consumer surplus.

This result is derived in a model in which a profit-maximizing firm proposes a mechanism for the provision and financing of the public good. The firm is, in turn, supervised by a policy maker who has to approve the firm's proposal. The policy maker remains ignorant about the state of demand (as shaped by the distribution of public-goods preferences) and supply (as shaped by production costs). However, she has probabilistic beliefs about demand and supply and can therefore assess the expected performance of a mechanism that the firm proposes. Under these premises, the policy maker formulates an optimal rule for approving the firm's proposals. We compare two alternatives for formulating such a rule: Rule A, a minimal level of expected consumer surplus, so that a mechanism is approved only if it exceeds this threshold; Rule B, a reservation utility level, so that every consumer's expected utility has to be larger than this reservation utility level.

We will show that these alternatives can be interpreted as follows: Rule A is equivalent to a model where the firm has to deliver an upfront payment in order to become the provider of the public good, and then has to propose a mechanism subject to the requirement of a non-negative level of consumer surplus. In particular, this mechanism may involve lump-sum contributions, i.e., payments which do not depend on a consumer's demand of the public good. Rule B is equivalent to a model where, again, the firm has to deliver an upfront payment and then faces a self-financing requirement when producing the public good. This makes it impossible to acquire payments from consumers with no demand for the public good.

In both models, the upfront payment can be interpreted as a tax on the firm's expected profits.[1] Under B, setting this tax is all that the policy maker has to do. Rule B therefore gives rise to a situation where a firm is subject to a tax on profits and then chooses a mechanism subject to a budget constraint that does not include tax revenue as a source of income. This setup is akin to the public sector pricing models in the tradition of Ramsey (1927) and Boiteux (1956). Under A, by contrast, the policy maker grants access to tax revenues provided that the firm delivers a minimal level of consumer surplus. This is akin to a model of procurement or regulation where a firm receives subsidies in exchange for a commitment to meet a certain performance standard. We say that a self-financing requirement is desirable if the expected level of consumer surplus that can be obtained under Rule B exceeds the expected level of consumer surplus that can be obtained under Rule A.

A comparison of these rules yields the following trade-off. Under Rule A, any mechanism generates a first best level of total surplus. However, the expected level of consumer surplus under this mechanism is zero. The only source of consumer surplus is therefore the redistribution of profit income via the tax system. Under Rule B, by contrast, consumers not only receive their share of profit income, but are also guaranteed an information rent. The combination of information rents and tax revenues that is possible with Rule B may, from the consumer's perspective, be more attractive than the reliance just on tax revenues if Rule A is used. At the same time, however, using Rule B implies that first-best outcomes can not be reached.

We will show that the imposition of a self-financing requirement is desirable if there

[1] Such taxes have been considered before in the literature on regulation; see Loeb and Magat (1979).

is a substantial uncertainty about the firm's production costs. Being uncertain about the firm's profitability, a policy maker may shy away from profit taxation because he fears that otherwise the firm may go out of business, so that there would be no public-goods provision at all. This may leave substantial profits to the firm. In this case, the use of Rule B, which limits the firm's capability of rent extraction, is more attractive from the consumers' perspective.

This model is based on an incomplete contracts approach in the sense that the policy maker's interaction with the firm is not derived from an optimal plan that is responsive to all conceivable state and demand contingencies. Instead, the firm proposes a mechanism and the policy maker decides whether or not to approve the firm's proposal. This decision is based on a rule that is optimal, conditional on the assumption that the policy maker is uninformed about the current state of demand and supply.[2]

This approach admits two different interpretations: First, in some cases the rules analyzed in this paper may be viewed as descriptive of real-world institutional settings. For instance, if we think of TV channels as being excludable public goods, a firm may produce and sell access to this public good without any substantial government interference, and in particular without receiving tax revenues. An alternative, however, would be a national TV channel that is financed by lump-sum contributions and which is subject to stricter performance standards.[3] Other examples are streets, highways, or railroads. Such infrastructure can be financed by relying on user fees, or on tax revenue, or a mixture of the two. This paper sheds light on the question which of these alternatives is preferable, under the assumption that the institution in charge of organizing its provision is self-interested and can be monitored only in an incomplete way.[4]

Second, the legal framework for the interaction between a firm and a policy maker may require the use of general rules, as opposed to micro-management by the policy maker.

[2]The term "incomplete contracts" is used in different ways by different authors. This paper's approach is in line with Hart (1995) who views absence of complete contingent planning as the source of incompleteness. Alternatively, Bolton and Dewatripont (2005) summarize contributions to the hold-up problem under the heading "incomplete contracts". That said, having incompleteness in one way or another is necessary for the result that a self-financing requirement à la Ramsey (1927) and Boiteux (1956) may be desirable. Trivially, the mechanism proposed by a benevolent policy maker with full commitment power yields more consumer surplus if it is possible to violate such a requirement.

[3]A further example has recently been provided by the Swiss Poste, a national monopolist, that reported plans to finance mail services not only by charging the senders of mail but also the receivers by means of a lump-sum contribution requested from every household with a mailbox (Baseler Zeitung, 5 December 2009). I am grateful to Jos Jansen for bringing this example to my attention.

[4]Often this infrastructure is run by the state, as opposed to a private firm. The framework developed in this paper would still be applicable under the assumption that the responsible politicians or bureaucrats may try to extract some of the surplus for themselves. This argument is developed more fully in a companion paper, Bierbrauer (2009), which relates the desirability of a self financing requirement to a policy maker's degree of benevolence.

As an example, think once more of a privately-run TV channel. In this context, a policy intervention would have to be based on a legal rule that applies equally to all firms in this business and which can not be tailored to specific demand and supply conditions.[5] This implies that any intervention (e.g., a regulation of admissible content, or a regulation of time available for commercials) has to be based on a rule that is incomplete in the same way as Rules A and B are, namely that it cannot be made fully contingent on all conceivable states of the environment (e.g., whether or not there is a major sports event). Hence, this paper's approach to model government interventions as rules that cannot be made fully contingent on all conceivable states of the economy has a wider scope, and is not limited to situations where the only policy choice is whether or not a firm should receive subsidies.

This can also be seen if we think of the excludable public good as being a regulated natural monopoly – for instance, a telecommunications network – and assume that the policy maker is a regulatory agency. Admittedly, regulatory agencies often engage in micro-management in the sense that try to make their interventions contingent on current demand and supply conditions. If, say, the owner of the telecommunications network sells access to providers of telecommunication services, the regulatory agency may have to approve the access pricing schedule; and for this purpose it may use information about costs and demand. However, an institutional framework for regulation typically works such that a regulated firm makes a proposal and that the regulatory agency then reacts to this proposal. It seems reasonable to assume that the regulator evaluates this proposal without having access to all the pieces of information that the firm has used. (If the regulator had all the relevant information, she could just prescribe the use of the optimal mechanism, and it would make little sense to let the firm propose something.) But then, the best the regulator can do is to behave optimally, conditional on being imperfectly informed. This does again imply that an incomplete contracts perspective, similar to the one developed in this paper, would be warranted.

The provision of an excludable public good by a regulated firm is a common practice in reality, e.g., in the areas of public transportation, telecommunications, electricity generation or other network industries. Moreover, for the financing of such infrastructures tax revenues and user fees are alternatives which are both used in practice. This paper shows that a complete reliance on user fees may be desirable in order to limit the firm's capability of rent extraction.

The remainder of the paper is organized as follows. The next section gives a more

[5]To give a specific example for such a legal rule, article 19 in Germany's constitutional law posits that a law may restrict basic rights, one of which being economic freedom, only if it takes the form of a general rule: "Insofar as, under this Constitutional Law, a basic right may be restricted by or pursuant to a law, such law must apply generally and not merely to a single case."

detailed literature overview. Section 3 specifies the economic environment. Section 4 contains benchmark results under the assumption that there is a benevolent mechanism designer. Section 5 contains the incomplete contracts approach and the derivation of the main result. The last section concludes.

2 Related Literature

The paper draws on a literature that uses a mechanism design approach to characterize an optimal allocation of excludable public goods under the constraint that participation in the system is voluntary; see Schmitz (1997), Hellwig (2003), and Norman (2004). This literature is related because, as we will show, a mechanism design problem with participation constraints has many similarities with a problem of public sector pricing subject to a self-financing requirement. This observation has previously been made by Hellwig (2007), albeit in a somewhat different model. Moreover, Hellwig (2007) does not address the question whether the imposition of participation constraints, or, equivalently, of self-financing requirements is desirable.

The work by Schmitz (1997), Hellwig (2003), and Norman (2004) contains characterizations both of welfare and of profit-maximizing mechanism. Some of the characterizations in this paper are similar. However, there are also some differences. Most notably, Schmitz (1997), Hellwig (2003), and Norman (2004) assume that there is a commonly known technology for the production of the public good.[6] Here, by contrast, there is a firm with private information about costs, which is an assumption that is typically made in the literature on regulation; see Baron and Myerson (1982) and Laffont and Tirole (1993). This is important for our main result. As will be shown below, with a commonly known technology, we cannot justify the requirement of self-financing.

Finally, this paper uses ideas from the literature on incomplete contracts.[7] In particular, it compares a setting where a benevolent mechanism designer engages in complete contingent planning of final outcomes to a setting where a policy maker delegates this task to a self-interested firm and therefore can remain ignorant with respect to information about preferences and technologies. This approach, which is meant to capture some aspects of real-world institutions, provides a justification for the incorporation of participation constraints into models of mechanism design. By its nature, mechanism design theory – viewed as an institution-free characterization of incentive-feasible outcomes

[6]More subtle differences are the following: Schmitz (1997), Hellwig (2003), and Norman (2004) focus on limit outcomes as the number of consumers goes to ∞. Our analysis, by contrast, is based on an arbitrary number of consumers. In addition, Schmitz (1997), Hellwig (2003), and Norman (2004) assume that the consumers preferences are derived from an arbitrary atomless distributions, while, here, we work with an arbitrary discrete distribution. This last assumption considerably simplifies the analysis.

[7]For an overview, see Hellwig (1996) or Tirole (1999). A survey of the implications of incomplete contracting for public-goods provision is provided by Martimort et al. (2005).

under conditions of incomplete information – cannot itself provide such a foundation.

Typically, the notion of an equity-efficiency trade-off is, in the tradition of Mirrlees (1971), associated with problems of redistributive income taxation or social insurance. In these models the state's power of coercion is taken as given so that participation constraints are not included in the analysis. A major insight of this paper is that a similar consideration may justify the imposition of participation constraints in models of mechanism design which do not focus on redistribution, but on the aggregation of preferences, e.g., in order to determine how much of a public good should be provided.

3 The Environment

Consumers. The set of consumers is denoted by $I = \{1, \ldots, n\}$. Consumer i's preferences are given by $u_i = \theta_i q_i - t_i$, where q_i is i's consumption of an excludable public good, t_i is a monetary payment and θ_i is a taste parameter that belongs to a finite ordered set $\Theta = \{\theta^0, \theta^1, \ldots, \theta^m\}$ of possible taste parameters. We assume that $\theta^0 = 0$, $\theta^1 = 1$, etc.

Consumer i privately observes θ_i. From the perspective of anyone else, θ_i is a random variable with support Θ and probability distribution $(p^0, \ldots, p^m)$. The taste parameters of different consumers are assumed to be independent random variables. We write $\theta = (\theta_1, \ldots, \theta_n)$ for the vector of all taste parameters and θ_{-i} for a vector that lists all taste parameters except θ_i.

We impose a monotone hazard rate assumption: let $p(\theta_i)$ and $P(\theta_i)$ be a random variables that take, respectively, the values p^l and $\sum_{k=0}^{l} p^k$ if θ_i takes the value θ^l, and define $h(\theta_i) := \frac{1-P(\theta_i)}{p(\theta_i)}$. We assume that h is a non-increasing function.

The Firm. There is a firm that produces the excludable public good. The firm's profits are given by $\pi = \sum_{i=1}^{n} t_i - \beta k(q)$, where $q := \max_{i \in I} q_i$, and k is an increasing and convex cost function satisfying $k(0) = 0$, $\lim_{x \to 0} k'(x) = 0$, and $\lim_{x \to \infty} k'(x) = \infty$. The cost parameter β is privately observed by the firm. For anyone else, β is a random variable with support $\{\beta^1, \ldots, \beta^r\}$ and probability distribution $(f^1, \ldots, f^r)$. We assume that $\beta^1 = r$, $\beta^2 = r - 1$, etc., so that firms with a higher index have a superior technology.

We impose another monotone hazard rate assumption: let $f(\beta)$ and $F(\beta)$ be a random variables that take, respectively, the values f^j and $\sum_{k=1}^{j} f^k$ if β takes the value β^j, and define $g(\beta) := \frac{1-F(\beta)}{f(\beta)}$. We assume that g is a non-decreasing function.

The consumers' preference parameters and the firm's cost parameter are assumed to be stochastically independent. Whenever we use the expectations operator E in the following, this indicates that expectations are taken with respect to the joint probability distribution of θ and β.

Mechanisms. We use a mechanism design approach to characterize the provision and pricing of the excludable public good. (Proposition 1 below provides a reinterpretation of these mechanisms in terms of price schedules that involve self-financing requirements.)

We appeal to the revelation principle, and limit attention to direct mechanisms so that a truthful revelation of public-goods preferences by consumers and of production costs by the firm is a Bayes-Nash equilibrium. A direct mechanism is a collection of functions $(q_i, t_i)_{i=1}^n$, where $t_i : \Theta^n \times \{\beta^1, \dots, \beta^r\} \to \mathbb{R}$ specifies i's payment as a function of the vector of preference parameters and the firm's cost parameter; analogously the function $q_i : \Theta^n \times \{\beta^1, \dots, \beta^r\} \to \mathbb{R}_+$ determines i's consumption of the excludable public good.

Truth-telling of consumer i is a best response if, for all l and k,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^k) - T_i(\theta^k), \tag{1}$$

where $Q_i(\theta^l) := E[q_i(\theta_{-i}, \hat{\theta}_i, \beta) \mid \hat{\theta}_i = \theta^l]$ and $T_i(\theta^l) := E[t_i(\theta_{-i}, \hat{\theta}_i, \beta) \mid \hat{\theta}_i = \theta^l]$ are i's expected consumption and payment, respectively, in case of reporting a preference parameter of θ^l.[8]

Likewise, truth-telling of the firm is a best response if, for all l and k,

$$R(\beta^l) - \beta^l K(\beta^l) \geq R(\beta^k) - \beta^l K(\beta^k), \tag{2}$$

where $R(\beta^l) := E[\sum_{i=1}^n t_i(\theta, \hat{\beta}) \mid \hat{\beta} = \beta^l]$ and $K(\beta^l) := E[k(q(\theta, \hat{\beta})) \mid \hat{\beta} = \beta^l]$.

We require that expected revenues $R(\beta)$ suffice to cover the firm's expected production cost $\beta K(\beta)$: For each l,

$$R(\beta^l) - \beta^l K(\beta^l) \geq 0 \; . \tag{3}$$

Pricing mechanisms, lump-sum taxes and participation constraints

For the purposes of this paper, there is no loss of generality in limiting attention to symmetric mechanisms, i.e., to mechanisms such that, for any pair of consumers i and j, any θ and any β, $\theta_i = \theta_j$ implies $q_i(\theta, \beta) = q_j(\theta, \beta)$ and $t_i(\theta, \beta) = t_j(\theta, \beta)$.[9] We will now demonstrate that symmetric mechanisms can be implemented by means of non-linear pricing schedules, possibly in combination with a lump-sum tax; i.e., these mechanisms can be implemented in a way that looks empirically more plausible than, say, a communication game in which each agent reports her privately held information to a mechanism designer.

Definition 1 *A symmetric mechanism $(q_i, t_i)_{i=1}^n$ is called a pricing mechanism if there is a non-decreasing schedule $s : \mathbb{R}_+ \to \mathbb{R}_+$ with $s(0) = 0$ and a number τ so that, for each i and l,*

$$Q_i(\theta^l) \;\in\; \mathit{argmax}_{x \in \mathbb{R}_+} \;\; \{\theta^l x - \tau - s(x)\} \quad \textit{and} \quad T_i(\theta^l) = \tau + s(Q_i(\theta^l)) \; . \tag{4}$$

[8]The assumptions that the taste parameters of different consumers are stochastically independent and also independent of the firm's cost parameter implies that all types of consumer i have the same beliefs on θ_{-i} and β. Hence, we can view i's expected consumption level and payment solely as a function of i's announcement, $\hat{\theta}_i$.

[9]All mechanisms that are characterized in the remainder are symmetric.

For $\tau \leq 0/\ \tau > 0$, we say that the mechanism is a pricing mechanism without/ with lump-sum taxes.

Proposition 1

i) A symmetric mechanism satisfies the consumers' incentive compatibility constraints in (1) if and only if it is a pricing mechanism.

ii) A symmetric mechanism satisfies the consumers' incentive compatibility constraints in (1) and the consumers' participation constraints

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq 0\ , \tag{5}$$

for all i and l, if and only if it is a pricing mechanism without lump-sum taxes.

A proof of the Proposition is in part A of the Appendix. The Proposition 1 is a generalization of a result, sometimes referred to as the taxation principle, which states that the set of outcomes that can be reached with incentive-compatible direct mechanisms and the outcomes that can be reached with non-linear pricing or tax schedules coincide; see, e.g., Mussa and Rosen (1978) or Guesnerie (1995). Typically, models of non-linear pricing are based on the assumption of a commonly known cross-section distribution of preferences. Our approach differs in that this distribution is a random quantity; e.g., the fraction of consumers having a θ^k-preference is not a priori known.

If lump-sum taxes are positive, then even consumers who choose not to consume the public good, contribute to the cost of producing it. Obviously, this makes them worse off in comparison to a status quo with no public-goods provision at all. In fact, Proposition 1 shows that the possibility to use lump-sum taxes is equivalent to the possibility to violate the consumers' participation constraints.

Intuitively, a pricing mechanism works as follows: Consumers buy a lottery that is, due to risk neutrality, completely characterized by an expected level of public goods consumption x. The pricing schedule s determines the price $s(x)$ which they have to pay in order to get this expected consumption level. Possibly, consumers also have to pay a lump-sum tax τ. The designer of the pricing mechanism observes the consumers' choices which are informative about their preferences, i.e., upon observing i's choice of x he deduces θ_i. Consequently, observing every consumer's choice of a lottery reveals θ. Based on this observation, the mechanism designer then interacts with the firm in order to learn β and determines the final outcome $(q_i(\theta,\beta), t_i(\theta,\beta))_{i=1}^n$.

The reason that consumers can only buy an expected – as opposed to a deterministic – level of public goods consumption is that, in the given environment, any mechanism serves two purposes at the same time. On the other hand, there is a problem of cost sharing: For a given production level, it has to be determined who should contribute how much to the cost of provision. On the other hand, there is a problem of information aggregation because how much of the public good is produced depends on the vector of preferences θ. This latter aspect implies that consumer i's consumption depends on the preferences of all other consumers, which are random from i's perspective.

4 Optimal Mechanism Design

Before we introduce a model of incomplete contracts, we characterize as a benchmark case the mechanism that maximizes expected consumer surplus S

$$S := E[s(\theta,\beta)] \quad \text{where} \quad s(\theta,\beta) := \textstyle\sum_{i=1}^{n}(\theta_i q_i(\theta,\beta) - t_i(\theta,\beta)) \ ,$$

i.e., the ideal mechanism that would be chosen by a benevolent mechanism designer.[10] In particular, we compare the outcome that is obtained if lump-sum taxes are assumed unavailable to the outcome that is obtained otherwise. As will become clear, with a benevolent mechanism designer, access to lump sum taxes is clearly desirable. Hence, in this framework, a justification for a self-financing requirement can not be found.

The following Proposition characterizes the optimal mechanism given that lump sum taxes are available.

Proposition 2 *Any mechanism $(q_i^*, t_i^*)_{i=1}^n$ that maximizes S subject to the constraints in (1), (2), and (3) has the following properties:*

i) There is no exclusion. For all θ and β, and for all i, $q_i^*(\theta,\beta) = q^*(\theta,\beta)$, where $q^*(\theta,\beta) := \max_{j\in I} q_j(\theta,\beta)$.

ii) There are rents left to the firm:

$$E\left[\sum_{i=1}^{n} t_i^*(\theta,\beta)\right] = E\left[(\beta + g(\beta))\, k(q^*(\theta,\beta))\right] > E[\beta k(q^*(\theta,\beta)] \ . \tag{6}$$

iii) Output is distorted downwards. For any θ and β, the provision level $q^*(\theta,\beta)$ satisfies the first order condition,

$$\sum_{i=1}^{n} \theta_i = (\beta + g(\beta))\, k'(q^*(\theta,\beta)) \ . \tag{7}$$

[10] The literature on regulation focusses on consumer surplus as the efficiency criterion, see Baron and Myerson (1982) and Laffont and Tirole (1993). At the very least it is assumed that the consumer surplus receives more weight in the policy maker's objective function. Absent this assumption, it would no longer be desirable to limit the firm's rents and the problem would become uninteresting.

A proof of Proposition 2 is in part B of the Appendix. The Proposition adapts some well-known results from the mechanism design literature to the given setting. In particular, the fact that the firm has private information about costs and that its expected profits have to be non-negative implies that the firm is able to extract an information rent. This gives rise to a second-best analysis which follows the same logic as a first-best analysis, except that the cost function of the first-best analysis is replaced by the virtual cost function;

$$(\beta + g(\beta))\, k(q) \ ,$$

i.e., the costs are inflated by the presence of the hazard rate $g(\beta)$. These results have been established by Baron and Myerson (1982) for a model of private goods provision by a regulated monopolist, with no private information on the preferences of consumers. A model with private information on preferences and a commonly known cost function has been studied by d'Aspremont and Gérard-Varet (1979). Given the virtual cost function, Proposition 1 reproduces their result that first-best allocations of public goods can be attained if consumers have private information on their preferences.[11]

The optimal mechanism in Proposition 2 relies on the availability of taxes for the financing of the public good. The following Proposition shows what the second-best mechanism looks like if these taxes cannot be used, or, equivalently, if the consumers' participation constraints have to be respected.

Proposition 3 *A mechanism $(q_i^*, t_i^*)_{i=1}^n$ that maximizes S subject to the constraints in (1), (2), and (3) satisfies the consumers' participation constraints in (5) if and only if*

$$E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q^*(\theta,\beta)\right] \geq E[(\beta + g(\beta))k(q^*(\theta,\beta))] \ . \tag{8}$$

If condition (8) is violated, then the mechanism $(q_i^{**}, t_i^{**})_{i=1}^n$ that maximizes S subject to the constraints in (1), (2), (3) and (5) has the following properties:

i) There is exclusion. There is a critical index $l > 0$, so that

$$q_i^{**}(\theta,\beta) = \begin{cases} 0, & \text{if} \ \ \theta_i < \theta^l \ , \\ q^{**}(\theta,\beta), & \text{if} \ \ \theta_i \geq \theta^l \ . \end{cases} \tag{9}$$

where $q^{**}(\theta,\beta) := \max_{j\in I} q_j^{**}(\theta,\beta)$.

ii) There are rents left to the firm:

$$E\left[\sum_{i=1}^{n} t_i^{**}(\theta,\beta)\right] = E\left[(\beta + g(\beta))\, k(q^{**}(\theta,\beta))\right] \ . \tag{10}$$

[11] Their analysis is based on a pure public good, as opposed to a non-excludable public good. This difference is, however, inconsequential because the conditions characterizing a first-best allocation of pure public goods are equivalent to those for excludable public goods.

iii) Output is distorted downwards, even in comparison to Proposition 2. For any θ and β, the provision level $q^{**}(\theta, \beta)$ satisfies the first order condition,

$$\sum_{\{i|q_i^{**}(\theta,\beta)>0\}} \left(\theta_i - \frac{\lambda}{1+\lambda} h(\theta_i)\right) = (\beta + g(\beta))\, k'(q^{**}(\theta, \beta)) \ , \tag{11}$$

for some number $\lambda > 0$.

A proof of Proposition 3 is in part B of the Appendix. The Proposition clarifies the conditions under which the optimal mechanism in Proposition 2 violates the consumers' participation constraints. The maximal expected revenue that can be extracted from consumers in the presence of participation constraints is given by

$$E\left[\sum_{i=1}^{n} (\theta_i - h(\theta_i)) q_i(\theta, \beta)\right] \ .$$

The interpretation is that, due to the interplay of incentive and participation constraints, consumers can now also reap an informational rent. Consequently, one can make them pay for their consumption of the excludable public good only up to their virtual valuation of the public good which is given by $(\theta_i - h(\theta_i))q_i$. Given that the consumers' expected payments are limited, the optimal mechanism in Proposition 2 satisfies the consumers' participation constraints if and only if, given that $(q_i)_{i=1}^n = (q_i^*)_{i=1}^n$, the sum of the virtual valuations exceeds the virtual cost of providing the non-excludable public good.

It can be shown that, if the number of consumers is sufficiently large, then condition (8) is always violated.[12] The reason is that, with many consumers, a single consumer's impact on the provision level q^* is close to zero. Hence, if there is no threat of exclusion and, due to participation constraints, consumers can drive their contribution to the costs down to zero, then the only way to achieve incentive compatibility is to have a zero contribution for everybody in the first place. But this implies that there is insufficient revenue to finance a positive supply of public goods.

Finally, Proposition 3 states the properties of the optimal mechanism that satisfies the consumers' participation constraints, if condition (8) is violated. In particular, now the possibility of exclusion is used. The threat of exclusion makes it possible to raise more funds to finance public goods provision and therefore becomes a valuable tool in the hands of the mechanism designer. Also, since consumers will not pay more than their virtual valuation, there is a further downward distortion of output – relative to the one already identified in Proposition 2.

[12] Related results have been established by Mailath and Postlewaite (1990) and Hellwig (2003). In these papers, the density p is assumed to be atomless, which implies that condition (8) is violated irrespective of the number of consumers. Here, condition (8) may be satisfied with finitely many consumers, but is certainly violated if the number of consumers is sufficiently large. A proof is available upon request.

On the desirability of lump-sum taxation

The results above have documented that the absence of lump sum taxes, or, equivalently, the imposition of participation constraints generates distortions. Specifically, consumers are excluded from the consumption of a public good that they would otherwise enjoy and there is underprovision of the public good. Hence, maximizing expected consumer surplus subject to participation constraints is bound to lead to a smaller level of consumer surplus. This suggests that the imposition of participation constraints, or equivalently, the exclusion of lump-sum taxation for public goods finance cannot be justified, and therefore questions the relevance of the theory of public sector pricing in the tradition of Ramsey (1927) and Boiteux (1956) which is based on the assumption that lump-sum taxes are unavailable.

This criticism cannot be overcome in a pure mechanism design framework. As we have just seen, to every mechanism that satisfies participation constraints/ excludes lump-sum taxes, there is a superior mechanism that violates participation constraints/ relies on lump-sum taxation; unless condition (8) is satisfied, in which case the two approaches yield equivalent results. In the following section, we will therefore change the perspective and look at the provision of public goods with an incomplete contracts approach. It will be shown that, in this framework, we can derive conditions under which the imposition of a self-financing requirement is desirable.

5 An incomplete contracts perspective

The benevolent mechanism designer of the preceding section engages in complete contingent-planning. For every possible configuration of the consumers' preferences $\theta \in \Theta^n$ and every possible technology of the firm β, she specifies, for each consumer i, a payment $t_i(\theta, \beta)$ and a consumption level $q_i(\theta, \beta)$.

In the following, we will instead assume that the task of mechanism design is delegated to the firm, i.e., the firm is in charge of adjusting its production level and pricing policy to the details of demand and supply conditions. A policy maker has to approve the mechanism that has been designed by the firm. This relation between the policy maker and the firm is incomplete since the former is neither involved in the design, nor in the execution of the mechanism. She only formulates a rule for approving the firm's proposal. This rule is chosen optimally under the assumption that she remains uninformed about the current state of preferences and technologies.

We compare two versions of such a rule. The first version, Rule A, specifies a minimal level of consumer surplus that the firm has to deliver. The second version, Rule B, specifies a reservation utility level so that each consumer's expected utility must exceed this reservation utility level. We show that these models can alternatively be described in a way that looks empirically more plausible: The policy maker sets a tax on the firm's expected profits and redistributes the proceeds to consumers. The firm is entitled to produce the public good only if it is willing to pay this tax. Under Rule A, the policy maker approves the mechanism proposed by the firm if it generates a non-negative level of consumer surplus. In particular, this includes the possibility to use lump-sum taxes. Under Rule B the firm's mechanism has to satisfy the consumers' participation constraints. Put differently, the firm has to use a pricing mechanism with a self-financing requirement. If the approach in B turns out to be superior from the policy maker's perspective, we say that a self-financing requirement is desirable.

The main insight of the analysis below is that the desirability of self-financing is shaped by the following trade-off: If self-financing is required the consumers participation constraints have to be respected. This has the advantage that consumers are guaranteed an information rent so that there is a minimal level of consumer surplus. The disadvantage, however, is that first-best outcomes are out of reach so that there will be exclusion and a downward distortion of production. How these two forces play out will be shown to depend on the fraction of the surplus from public-goods provision that the firm can extract.

5.1 Approval rules and the taxation of monopoly profits

In the following, a firm with private information on costs proposes a mechanism. We will represent such a mechanism as a collection of functions $(q_i, t_i)_{i=1}^n$, where $q_i : \theta \mapsto q_i(\theta)$ gives i's consumption and $t_i : \theta \mapsto t_i(\theta)$ gives i's payment as a function of the vector of preference parameters. The expectations operator E henceforth refers to expectations taken with respect to θ.

Rule A: A minimal level of consumer surplus

We think of the policy maker as choosing a number τ_A so that a mechanism $(t_i, q_i)_{i=1}^n$ proposed by the firm is approved only if the resulting level of expected consumer surplus satisfies

$$E\left[\sum_{i=1}^{n}(\theta_i q_i(\theta) - t_i(\theta))\right] \geq \tau_A \ . \tag{12}$$

The firm's problem hence is as follows: Produce the public good if and only if the mechanism $(t_i, q_i)_{i=1}^n$ that maximizes expected profits, $E\left[\sum_{i=1}^n t_i(\theta) \ - \beta k(q(\theta))\right]$, subject to the consumers' incentive constraints in (1) and the policy maker's approval rule in (12), yields a non-negative level of expected profits.

If we let, for each i and θ, $t_i'(\theta) = t_i(\theta) + \tau_A$, this problem can equivalently be written as follows: Produce the public good if and only if the mechanism $(t_i', q_i)_{i=1}^n$ that maximizes

$E\left[\sum_{i=1}^{n} t_i'(\theta) \ - \beta k(q(\theta))\right]$ subject to the consumers' incentive constraints in (1) and the approval rule

$$E\left[\sum_{i=1}^{n}(\theta_i q_i(\theta) - t_i'(\theta))\right] \geq 0 \tag{13}$$

is such that

$$E\left[\sum_{i=1}^{n} t_i'(\theta) \ - \beta k(q(\theta))\right] \geq \tau_A \ . \tag{14}$$

Consequently, we can think of the policy maker as setting tax of τ_A which the firm has to pay in order to be entitled to produce the public good, with the understanding that only those mechanisms will be approved which generate a non-negative level of consumer surplus. The firm will therefore enter only if after-tax profits under this constraint exceed τ_A. Otherwise, staying out would be optimal from the firm's perspective.

Rule B: A reservation utility level

Alternatively, we may think of the policy maker as choosing a number τ_B so that a mechanism $(t_i, q_i)_{i=1}^n$ is approved only if every consumer's expected utility exceeds τ_B, i.e., for all i, and l it has to be true that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \tau_B \ . \tag{15}$$

Upon defining $t_i'(\theta) := t_i(\theta)+\tau_B$, the firm's problem can equivalently be written as follows: Produce the public good if and only if the mechanism $(t_i', q_i)_{i=1}^n$ that maximizes after-tax profits $E\left[\sum_{i=1}^{n} t_i'(\theta) \ - \beta k(q(\theta))\right]$ subject to the consumers' incentive constraints in (1) and the approval rule

$$\theta^l Q_i(\theta^l) - T_i'(\theta^l) \geq 0 \ , \quad \text{where} \quad T_i'(\theta^l) := T_i(\theta^l) + \tau_B \ , \tag{16}$$

for all i, and l, is such that

$$E\left[\sum_{i=1}^{n} t_i'(\theta) \ - \beta k(q(\theta))\right] \geq \tau_B \ . \tag{17}$$

Hence, we can once more think of the policy maker as setting a tax, denoted by τ_B, which the firm has to pay upon entry. The firm then has to propose a mechanism which satisfies the participation constraints in (16), i.e., the firm's mechanism must rely on self-financing. The optimal entry decision is as follows: Enter if and only if the maximal level of profits that can be attained under this self-financing requirement exceeds the tax payment τ_B.

Note that under both regimes, a tax on profits is distortionary since it affects the firm's entry decision. In particular, if the tax is set in such a way that not every type of firm enters, this leads to an inefficient outcome since our assumptions about the cost function imply that, even with a bad technology, it is desirable to have a strictly positive public-goods supply. This possibility is eliminated if entry is deterred.

As we will see, the policy maker may be willing to accept such inefficiencies if this makes it possible to generate more tax revenue. To see why an inefficient exclusion of some firms may be helpful in this respect, suppose that the worst type of firm makes hardly any profit. This firm enters only if the entry fee is literally zero. But this implies that all firms with better technology can retain all of their monopoly profits for themselves, i.e., the possibility to channel parts of these profits back to consumers is lost. By contrast, a higher tax inevitably leads to exclusion but makes it possible to have a significant redistribution of monopoly profits. The optimal tax will therefore be shaped by this equity-efficiency trade-off.

5.2 The firm's mechanism design problem

Before we discuss whether Rule A or Rule B can generate a higher level of expected consumer surplus, we first characterize the solution to the firm's mechanism design problem in either model.

Rule A: Profit Maximization if lump-sum taxes are available

The mechanism that maximizes the expected profits of a firm with cost parameter β^l, subject to the consumers' incentive constraints in (1) and the condition that the expected consumer surplus is non-negative, (13), is in the following denoted by $(q_i^{l*}, t_i^{l*})_{i=1}^n$.

Proposition 4 *The mechanism (q_i^{l*}, t_i^{l*}) has the following properties:*

i) There is no exclusion. For all θ and for all i, $q_i^{l*}(\theta) = q^{l*}(\theta)$, where $q^{l*}(\theta) := \max_{j \in I} q_j^{l*}(\theta, \beta)$.

ii) The expected consumer surplus under the mechanism is equal to zero, i.e., condition (13) holds as an equality.

iii) Output is undistorted. For every θ, the provision level $q^{l*}(\theta)$ satisfies the Samuelson rule, i.e.,

$$\sum_{i=1}^n \theta_i = \beta^l k'(q^{l*}(\theta)) \ . \tag{18}$$

iv) The firm's expected profits are given by

$$\Pi_A(\beta^l) := E\left[\sum_{i=1}^n \theta_i q^{l*}(\theta) - \beta^l k(q^{l*}(\theta))\right] \ . \tag{19}$$

A proof of the Proposition is in part B of the Appendix. It is an adaptation of the arguments that were used in the proof of Proposition 2, which characterized the mechanism that maximizes expected consumer surplus subject to the firm's incentive and non-negative profits constraints. Proposition 4 looks at the dual problem of maximizing expected profits subject to a minimal level of expected consumer surplus.

The Proposition shows that the removal of participation constraints and the delegation of mechanism design to a profit maximizing firms eliminates all distortions. As in Proposition 2, there is no exclusion. In addition, the downward distortion of output that was induced by the optimal mechanism in Proposition 2 disappears. Public-goods production satisfies the Samuelson rule. However, this comes at cost for consumers. Their expected surplus is zero. Hence, the consumer's only source of utility are the revenues that can be generated by the taxation of profits.

Rule B: Profit Maximization subject to self-financing

Analogously, we now study the mechanism designed by a firm with cost parameter β^l which has payed the entry fee and does face a self-financing requirement, or, equivalently, has to respect the consumers' participation constraints. The mechanism that maximizes the expected profits of a firm with cost parameter β^l, subject to the consumers' incentive constraints (1) and the participation constraints in (16) is in the following denoted by $(q_i^{l**}, t_i^{l**})_{i=1}^n$.

Proposition 5 *The mechanism $(q_i^{l**}, t_i^{l**})_{i=1}^n$ has the following properties:*

i) There is exclusion:

$$q_i^{l**}(\theta) = \begin{cases} 0, & \textit{if} \;\; \theta_i - h(\theta_i) < 0 \; , \\ q^{l**}(\theta), & \textit{if} \;\; \theta_i - h(\theta_i) \geq 0 \; . \end{cases} \tag{20}$$

where $q^{l**}(\theta, \beta) := \max_{j \in I} q_j^{l**}(\theta)$.

ii) The expected consumer surplus from the mechanism is strictly positive and given by

$$S_B(\beta^l) := E\left[\sum_{i=1}^n h(\theta_i) q_i^{l**}(\theta)\right] . \tag{21}$$

iii) Output is distorted downwards. For every θ, the provision level $q^{l**}(\theta)$ satisfies the first order condition

$$\sum_{\{i|\theta_i - h(\theta_i) \geq 0\}} \theta_i - h(\theta_i) = \beta^l k'(q^{l**}(\theta)) \; . \tag{22}$$

iv) The firm's expected profits are given by

$$\Pi_B(\beta^l) := E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q_i^{l**}(\theta) - \beta^l k(q^{l**}(\theta))\right] . \tag{23}$$

A proof of the Proposition is in part B of the Appendix. Similarly as in Proposition 3, if we introduce participation constraints, this implies that the possibility of exclusion will be used. However, in contrast to Proposition 3, this does not depend on whether or not first-best outcomes can be reached with participation constraints. The monopolistic firm always uses exclusion because this allows to extract larger payments from consumers. A further observation is that the consumers' information rents once more lead to a downward distortion of output. However, the information rents also generate a significant level of consumer welfare. Hence, if there are participation constraints, the consumers benefit not only from the taxation of profits, but also from the mechanism proposed by the firm.

5.3 The Main Result

In the following, we refer to the sum of tax revenues and of the expected consumer surplus generated by the firm's mechanism as *total* expected consumer surplus. We denote by τ_A^* the tax that maximizes total expected consumer surplus if lump sum taxes are available and participation constraints can be ignored. Analogously, let τ_B^* be the tax that maximizes total expected consumer surplus if there is a self-financing requirement and participation constraints have to be respected.

With this terminology, we can now state the main result of the paper. It provides a set conditions so that total expected consumer surplus is higher if there are no participation constraints, and another set of conditions so that total consumer surplus is higher with participation constraints.

Proposition 6

i) If the probability f^r that the firm has the best possible technology is sufficiently high, then τ_A^* and τ_B^* are both such that only the firm with the best technology, $\beta = \beta^r$, enters. In this case, total expected consumer surplus is higher under rule A, i.e., if there are no participation constraints.

ii) If the probability f^1 that the firm has the worst possible technology is sufficiently high, then τ_A^* and τ_B^* are both such that even the firm with the worst technology, $\beta = \beta^1$, enters. In this case, total expected consumer surplus is higher under rule B provided that

$$\Pi_A(\beta^1) < \Pi_B(\beta^1) + \sum_{k=1}^{r} f^k S_B(\beta^k) . \tag{24}$$

The proof of Proposition 6 is in part A of the Appendix. The Proposition does not provide a full characterization of optimal profit taxes. Instead it states a sufficient condition for self-financing requirements to be desirable and a sufficient condition for them to be undesirable. The advantage of this approach is that it makes it possible to highlight the main trade-off without having to go through all conceivable parameter constellations of the model.[13]

The logic of the proof of Proposition 6 is as follows: Suppose that there is a critical cost parameter β^k, so that all firms with superior technology enter and produce the public good, i.e., the profit tax τ_c is just equal to the profits of the critical firm, $\Pi_c(\beta^k)$, for $c \in \{A, B\}$. The revenue from this entry fee is higher if lump-sum taxes are available because the firm's profits are higher in this case. However, these tax revenues are the only source of consumer surplus. By contrast, with a requirement of self-financing consumers get the revenue from profit taxation and, in addition, an information rent whenever the firm has a cost parameter smaller than the critical value β^k.

The Proposition shows that there are cases where the larger revenue from profit taxation is the dominant concern so that self-financing is undesirable and other cases where information rents are more significant so that self-financing should be required. To explain the logic of this trade-off, suppose first that the probability f^r that the firm has the best technology is very close to 1. This implies that even a tax as high as $\Pi_c(\beta^r)$ distorts the firm's entry decision with a negligible probability, and we have $\tau_A^* = \Pi_A(\beta^r)$ and $\tau_B^* = \Pi_B(\beta^r)$. It follows from Proposition 4 that $\Pi_A(\beta^r)$ is equal to the first-best surplus from public-goods provision,

$$\Pi_A(\beta^r) = E\left[\sum_{i=1}^{n} \theta_i q^{r*}(\theta) - k(q^{r*}(\theta))\right] ,$$

It follows from Proposition 5 that total consumer surplus with self-financing,

$$\Pi_B(\beta^r) + S_B(\beta^r) = E\left[\sum_{i=1}^{n} \theta_i q_i^{r**}(\theta) - k(q^{r**}(\theta))\right] ,$$

is equal to the second-best surplus since q_i^{r**} is distorted downwards, and hence falls short of the first-best surplus. This implies that the requirement of self-financing is not attractive. By contrast, if the probability that the firm has a bad technology is sufficiently high (i.e., if f^1 is close to one), then the distortions implied by a tax on profits are drastic and it becomes optimal to have $\tau_A^* = \Pi_A(\beta^1)$ and $\tau_B^* = \Pi_B(\beta^1)$ so that the firm enters even if it has the worst possible technology. The fact that the technology is bad implies in particular that $\Pi_A(\beta^1)$ and $\Pi_B(\beta^1)$ are close to zero so that there is hardly any revenue

[13] Proposition 7 below imposes additional assumptions so that a sharper characterization becomes available. We will see, in particular, that participation constraints are desirable if r is a large number, f is a uniform distribution, and k is a quadratic cost function.

generated by the taxation of profits. In this case, the consumers' information rents become the dominating force so that it is desirable to rely on self-financing.

This shows, in particular, that we can find a role for self-financing requirements only if we assume that there is a firm with private information about its technology. If there was a commonly known technology, or, equivalently, only one type of firm, then case i) in Proposition 6 applies trivially. We should let the firm maximize profits without being impeded by self-financing and, simultaneously, use the tax system to redistribute these profits from the firm to the consumers. As follows from Proposition 4 this would make it possible to reach a first-best level of consumer surplus. Hence, a requirement of self-financing can be justified only if we assume that the firm has private information about costs.

The following Proposition, which is proven in part A of the Appendix, introduces some further assumptions so that a full characterization of the optimal taxes on monopoly profits is possible. In particular, it provides conditions so that, the characterization in case *ii*) in Proposition 6 is relevant, i.e., taxes should be set such that there is public-good provision with probability 1, and self-financing is desirable.

Proposition 7 *Suppose that f is a uniform distribution and that k is a quadratic cost function, $k(q) = \frac{1}{2}q^2$. If r is sufficiently large, the following is true:*

i) The optimal profit tax with participation constraints, τ_B^*, is such that there is public-goods production with probability 1; i.e., the possibility to exclude some types of the firm in order to generate more tax revenue should not be used.

ii) The optimal profit tax if there are no participation constraints, τ_A^*, is indeterminate; i.e., the total expected consumer surplus does not depend on which types of the firm are encouraged to enter.

iii) Total expected consumer surplus is higher if there are participation constraints.

6 Concluding Remarks

This paper has provided a microfoundation for models of mechanism design in which the presence of participation constraints, or, equivalently, of self-financing requirements makes it impossible to reach efficient outcomes. Looking at the case of a regulated monopolist who produces and sells access to an excludable public good, it has been shown that the requirement of self-financing may be desirable as a way to guarantee a minimal level of consumer surplus.

The analysis has established a link between the desirability of self-financing and the desirability of a tax on the monopolist's expected profits. Such a tax on profits is distortionary as it affects the firm's entry decision, i.e., the decision whether to produce the excludable public good at all. Such a tax may still be attractive if it raises substantial revenue from those firms who do enter. Given that it is attractive, self-financing requirements reduce the tax revenue because they reduce the firm's profits. Hence, the more attractive the taxation of profits is, the less attractive is the imposition of a self-financing requirement. Conversely, if profit taxation is unattractive, then self-financing should be required. Otherwise consumers who buy access to the excludable public good would not benefit at all from its provision.

The analysis has completely abstracted from commitment problems. Such problems would likely strengthen the case for self-financing requirements. The logic is as follows. Suppose that the policy maker has chosen a significant tax on profits and then observes that the firm refuses to produce the public good. But then she knows that if she lowers the tax, eventually the firm will be ready to enter and that this will create a strictly positive consumer surplus. Lack of commitment means that the policy maker is unable to resist this temptation. Anticipating this behavior, the firm will be willing to produce the public good only if the tax has been reduced to a level such that each type of firm, even the one with the worst technology, would be willing to produce the public good. According to our main result, such a limited taxation of profits makes the imposition of a self-financing requirement desirable.[14]

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[14]It is beyond the scope of this paper to provide a rigorous game-theoretic analysis of the relationship between the policy maker and the firm in the absence of commitment. In the context of a buyer-seller relationship, these considerations have been formalized by Hart and Tirole (1988).

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A Proofs of Propositions 1, 6 and 7

A.1 Proof of Proposition 1

Part i) *Step 1.* We first show that every pricing mechanism is incentive-compatible. Suppose, to the contrary, that a pricing mechanism is not incentive-compatible. Then there exist, i, l, and k so that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) < \theta^l Q_i(\theta^l) - T_i(\theta^k) \; .$$

Using that, for each $x \in \{Q_i(\theta^0), \ldots, Q(\theta^m)\}$,

$$T_i(x) = \tau + s(x) \; ,$$

this implies that

$$\theta^l Q_i(\theta^l) - \tau - s(Q_i(\theta^l)) < \theta^l Q_i(\theta^l) - \tau - s(Q_i(\theta^k)) \; .$$

But this contradicts the assumption that

$$Q_i(\theta^l) \quad \in \quad \text{argmax}_{x \in \mathbb{R}_+} \quad \{\theta^l x - \tau - s(x)\} \; .$$

Hence, the assumption that a pricing mechanism is not incentive-compatible has led to a contradiction and must be false.

Step 2. We now show that to every incentive-compatible mechanism there is a pricing mechanism, i.e., a mechanism satisfying the properties in (4).

Note first that the incentive compatibility constraints imply that the provision rule for the public good satisfies the following monotonicity conditions, $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$ and $T_i(\theta^l) \leq T_i(\theta^{l+1})$, for all i and l.[15]

Given an incentive-compatible mechanism, we construct a pricing mechanism as follows: The lump-sum component τ is chosen such that

$$\tau = T_i(Q_i(\theta^0)) \; . \tag{25}$$

For $x \in [0, Q_i(\theta^0)]$, we choose

$$s(x) = 0 \; . \tag{26}$$

The price of units $x \in \{Q_i(\theta^1), \ldots, Q_i(\theta^m)\}$ is chosen such that

$$s(x) = T_i(x) - \tau \;\; . \tag{27}$$

Further, whenever $Q_i(\theta^k) < Q_i(\theta^{k+1})$, the price for $x \in]Q_i(\theta^k), Q_i(\theta^{k+1})[$ is chosen such that

$$s(x) = \max\{s(Q_i(\theta^{k+1})) - \theta^{k+1}(Q_i(\theta^{k+1}) - x), s(Q_i(\theta^k)) + \theta^l(x - Q_i(\theta^l))\} \; . \tag{28}$$

[15] To see this, just add the following two incentive compatibility constraints: $\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l+1}) - T_i(\theta^{l+1})$, and $\theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) \geq \theta^{l+1} Q_i(\theta^l) - T_i(\theta^l)$.

Finally, if $x > Q(\theta^m)$, then

$$s(x) = \infty \ . \tag{29}$$

Step 2.1 We note that, given an incentive-compatible mechanism, this schedule is non-decreasing. Obviously, s is non-decreasing if attention is restricted to

$$x \in [0, Q_i(\theta^0)] \cup \{Q_i(\theta^1), \ldots, Q_i(\theta^m)\} \ .$$

In addition, (28) implies that s is non-decreasing over the range $]Q_i(\theta^k), Q_i(\theta^{k+1})[$, and that $s(Q_i(\theta^{k+1})) \geq s(x) \geq s(Q_i(\theta^k))$, for every $x \in]Q_i(\theta^k), Q_i(\theta^{k+1})[$.

Step 2.2 We have to show that $Q_i(\theta^l) \in \operatorname{argmax}_{x \in \mathbb{R}_+} \{\theta^l x - \tau - s(x)\}$, or, equivalently, that, for each i, l and $x \in \mathbb{R}$

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l x - \tau - s(x) \ .$$

This is trivially true if x is such that $s(x) = \infty$. Now suppose that $x \in \{Q_i(\theta^0), \ldots, Q_i(\theta^m)\}$. Then this condition becomes

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^k) - T_i(\theta^k) \ ,$$

for some $k \in \{0, \ldots, m\}$. This is implied by the incentive compatibility of the mechanism.

We also have to show that choosing $x = Q_i(\theta^l)$ is preferred over choosing $x \in [0, Q_i(\theta^0)]$, i.e.,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l x - \tau - s(0) = \theta^l x - T_i(\theta^0) \ .$$

This is true, since incentive compatibility and $x \leq Q_i(\theta^0)$ imply that,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^0) - T_i(\theta^0) \geq \theta^l x - T_i(\theta^0) \ .$$

Finally, let $x \in]Q_i(\theta^k), Q_i(\theta^{k+1})[$. Suppose first that $\theta^l \geq \theta^{k+1}$. Since incentive compatibility implies that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{k+1}) - T_i(\theta^{k+1}) \ ,$$

it suffices to show that

$$\theta^l Q_i(\theta^{k+1}) - T_i(\theta^{k+1}) = \theta^l Q_i(\theta^{k+1}) - s(Q_i(\theta^{k+1})) - \tau \geq \theta^l x - s(x) - \tau \ .$$

This is implied by (28), for all $l \geq k+1$. A similar argument applies if $\theta^l \leq \theta^k$. Then it suffices to show that

$$\theta^l Q_i(\theta^k) - T_i(\theta^k) = \theta^l Q_i(\theta^k) - s(Q_i(\theta^k)) - \tau \geq \theta^l x - s(x) - \tau \ ,$$

which is implied by (28), for all $l \leq k$.

Part ii) We first show that if a mechanism is incentive-compatible and satisfies participation constraints, then the associated pricing mechanism must not involve lump-sum payments.

By part i), if a mechanism is incentive-compatible, then there exists a pricing mechanism so that

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) = -T_i(\theta^0) = -s(Q_i(\theta^0)) - \tau \ .$$

The participation constraints require, in particular, that

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) = -T_i(\theta^0) \geq 0 \ .$$

Hence, it must be that

$$-s(Q_i(\theta^0)) - \tau \geq 0 \ .$$

Since $s(0) = 0$ and s is non-decreasing, this implies that we must have $\tau \leq 0$.

We now show that if a pricing mechanism has no lump-sum taxes, then this implies that the participation constraints of all consumers are satisfied.

With a pricing mechanism every consumer can choose $x = 0$ which yields a payoff of $-\tau$. Hence, it must be the case that consumer choices under the pricing mechanism at least yield a payoff of $-\tau$: For all i, and l,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq -\tau \ .$$

If $\tau \leq 0$, this implies that all participation constraints are satisfied.

■

A.2 Proof of Proposition 6

Part i) If only the most productive firm enters and there are no participation constraints, it follows from Proposition 4 that the optimal entry fee satisfies

$$\tau_A^* = \Pi_A(\beta^r) = E\left[\sum_{i=1}^{n} \theta_i q^{r*}(\theta) - k(q^{r*}(\theta))\right] \ ,$$

where we have used that $\beta^r = 1$. Note that τ_A^* equals the first-best level of surplus, conditional on $\beta = \beta^r$. If there are participation constraints, then consumers benefit from tax revenues, which, by Lemma 5, are given by

$$\tau_B^* = \Pi_B(\beta^r) = E\left[\sum_{i=1}^{n} (\theta_i - h(\theta_i)) q_i^{r**}(\theta) - k(q^{r**}(\theta))\right] \ ,$$

and from information rents which are equal to

$$S_B(\beta^r) := E\left[\sum_{i=1}^{n} h(\theta_i) q_i^{r**}(\theta)\right] \ .$$

Adding these two expressions yields a total expected consumer surplus of

$$E\left[\sum_{i=1}^{n}\theta_i q_i^{r**}(\theta)-k(q^{r**}(\theta))\right] .$$

This is only a second best level of surplus and hence less than τ_A^*. This implies that consumers are better off without participation constraints.

It remains to be shown that this is indeed the optimal outcome if f^r is sufficiently high.

Step 1. First, consider the optimal entry fee if there are no participation constraints. We seek to verify that, for all $l \neq r$, we have that

$$f^r\Pi_A(\beta^r)\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_A(\beta^l) ,$$

if f^r is sufficiently high. This follows because $\Pi_A(\beta^r) > \Pi_A(\beta^l)$, and f^r converges to $\left(\sum_{k=l}^{r}f^k\right)$ as f^r converges to 1.

Step 2. Now suppose that there are participation constraints. We need to show that for all $l \neq r$,

$$f^r(\Pi_B(\beta^r)+S_B(\beta^r))\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_A(\beta^l)+\sum_{k=l}^{r}f^kS_B(\beta^k) ,$$

or, equivalently that

$$f^r\Pi_B(\beta^r)\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_B(\beta^l)+\sum_{k=l}^{r-1}f^kS_B(\beta^k) .$$

As f^r goes to 1, the second term on the right hand side of this inequality goes to zero, and we have $f^r\Pi_B(\beta^r) > (\sum_{k=l}^{r}f^r)\,\Pi_B(\beta^l)$ for the same reason as in *Step 1.*

Part ii). Consider again the optimal entry fee if there are no participation constraints. We seek to verify that, for all $l \neq 1$, we have that

$$\Pi_A(\beta^1)\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_A(\beta^l) ,$$

if f^1 is sufficiently high. This follows since $\left(\sum_{k=l}^{r}f^k\right)$ goes to zero as f^1 goes to 1. Hence, for f^1 sufficiently large we have that that $\tau_A^*=\Pi_A(\beta^1)$.

Now consider the case with participation constraints. We claim that, for all $l \neq 1$,

$$\Pi_B(\beta^1)+\sum_{k=1}^{r}f^kS_B(\beta^k)\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_A(\beta^l)+\sum_{k=l}^{r}f^kS_B(\beta^k) ,$$

or, equivalently,

$$\Pi_B(\beta^1)+\sum_{k=1}^{l-1}f^kS_B(\beta^k)\geq\left(\sum_{k=l}^{r}f^k\right)\Pi_A(\beta^l) ,$$

if f^1 is sufficiently high. Again, this follows since $\left(\sum_{k=l}^{r} f^k\right)$ goes to zero as f^1 goes to 1. Hence for f^1 sufficiently large we have that that $\tau_B^* = \Pi_B(\beta^1)$.

Therefore, if f^1 is large, to see whether participation constraints are desirable, it suffices to compare the expected consumer surplus without participation constraints, $\Pi_A(\beta^1)$, and the expected consumer surplus with participation constraints, $\Pi_B(\beta^1) + \sum_{k=1}^{r} f^k S_B(\beta^k)$: participation constraints are desirable if

$$\Pi_B(\beta^1) + \sum_{k=1}^{r} f^k S_B(\beta^k) \geq \Pi_A(\beta^1) ,$$

which proves part ii) of Proposition 6.

■

A.3 Proof of Proposition 7

Part i) If there are participation constraints, the mechanism that the firm proposes is characterized in Proposition 5. With a quadratic cost function this mechanism has the following properties: A consumer is admitted to consume the public good if and only if $\theta_i - h(\theta_i) > 0$. Consumers who are admitted, consume the second-best quantity

$$q^{k**}(\theta) = \frac{\sum_{i=1}^{n} \mathbf{1}(\theta_i \geq h(\theta_i))(\theta_i - h(\theta_i))}{\beta^k} =: \frac{\tilde{q}^{**}(\theta)}{\beta^k} ,$$

where $\mathbf{1}$ is the indicator function, and β^k is the firm's cost parameter. The firm's expected profits can be written as

$$\Pi_B(\beta^k) = \frac{\pi_B}{\beta^k} , \quad \text{where} \quad \pi_B := \tfrac{1}{2} E\left[(\tilde{q}^{**}(\theta))^2\right] . \tag{30}$$

The expected consumer surplus equals

$$S_B(\beta^k) = \frac{s_B}{\beta^k} , \quad \text{where} \quad s_B := E\left[\sum_{i=1}^{n} \mathbf{1}(\theta_i \geq h(\theta_i)) h(\theta_i) \tilde{q}^{**}(\theta)\right] . \tag{31}$$

Under the assumption that f is a uniform distribution, the optimal tax is such that even the firm with the worst technology, $\beta = \beta^1 = r$, enters, i.e.,

$$\tau_B^* = \frac{\pi_B}{r} . \tag{32}$$

To see this, suppose that τ_B is such that all firms with a cost parameter larger or equal than $\beta^k = r - k + 1$ enter. The resulting expected consumer surplus is hence given by

$$\begin{aligned} \left(\sum_{l=k}^{r} f^r\right) \frac{\pi_B}{\beta^k} + \sum_{l=k}^{r} f^l \frac{s_B}{\beta^k} &= \frac{r-k+1}{r} \frac{\pi_B}{r-k+1} + \frac{s_B}{r} \sum_{l=k}^{r} \frac{1}{\beta^k} \\ &= \frac{\pi_B}{r} + \frac{S_B}{r} \sum_{l=1}^{r-k+1} \frac{1}{l} . \end{aligned} \tag{33}$$

It is now easily verified that this expression is decreasing in k; i.e., lowering k unambiguously increases total expected consumer surplus. The optimal level of expected consumer surplus with participation constraints is therefore given by

$$\Sigma_B := \frac{\pi_B}{r} + \frac{s_B}{r} \sum_{l=1}^{r} \frac{1}{l} \; . \tag{34}$$

Part ii) If there are no participation constraints, the mechanism that the firm proposes is characterized in Proposition 4. If the cost function is quadratic, this implies that there is no exclusion and every consumer consumes the first-best quantity

$$q^{k*}(\theta) = \frac{\sum_{i=1}^{n} \theta_i}{\beta^k} =: \frac{\tilde{q}^*(\theta)}{\beta^k} \; ,$$

where β^k is the firm's cost parameter. The firm's expected profits are equal to

$$\Pi_A(\beta^k) = \frac{\pi_A}{\beta^k} \; , \quad \text{where} \quad \pi_A := \tfrac{1}{2} E\left[(\tilde{q}^*(\theta))^2\right] \; . \tag{35}$$

Under the assumption that f is a uniform distribution, optimal taxation leads an optimal level of total consumer surplus which equals

$$\tau_B^* = \frac{\pi_A}{r} \; ; \tag{36}$$

the optimal tax rate is, however, indeterminate. To see this, suppose first that the entry fee is such that only the most productive firm with $\beta^r = 1$ enters. This leads to expected tax revenues of

$$f^r \Pi_A(\beta^r) = f^r \pi_A = \frac{\pi_A}{r} \; .$$

Now suppose the entry fee is set such that the firms with $\beta^r = 1$ and $\beta^{r-1} = 2$ enter. This yields $(f^{r-1} + f^r)\Pi_A(\beta^{r-1}) = \frac{2}{r}\frac{\pi_A}{2} = \frac{\pi_A}{r}$. Likewise, if we consider entry be firms with $\beta \in \{\beta^r, \beta^{r-1}, \beta^{r-2}\}$ we once more get expected tax revenues of $\frac{\pi_A}{r}$, etc. The optimal level of expected consumer surplus without participation constraints is therefore given by

$$\Sigma_A := \frac{\pi_A}{r} \; . \tag{37}$$

Part iii) Given the expressions in (34) and (37), the imposition of participation constraints is desirable provided that $\Sigma_B > \Sigma_A$, or, equivalently, that

$$s_B \sum_{l=1}^{r} \frac{1}{l} > \pi_A - \pi_B \; .$$

The right hand side of this inequality is positive, simply because first-best profits exceed second-best profits. s_B is a positive number because second-best profit maximization leaves information rents to the consumers. Finally, the sum $\sum_{l=1}^{r} \frac{1}{l}$ goes to infinity, as r goes out of bounds. Hence, if r is large, participation constraints are desirable. ■

B Further Proofs

B.1 Preliminaries

In this part of the supplementary material, we study various auxiliary optimization problems. The proofs of the Propositions below repeatedly draw on these results.

B.1.1 Revenue maximization subject to the consumers' constraints

Revenue maximization without participation constraints.

We first study the following optimization problem: Given a provision rule $(q_i)_{i=1}^n$ that satisfies the monotonicity constraint $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l, and given expected payments $(T_i(\theta^0))_{i=1}^n$ for individuals with a θ^0-preference, we seek to maximize expected revenue $E[\sum_{i=1}^n t_i(\theta, \beta)]$ subject to the individuals' incentive compatibility constraints,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^k) - T_i(\theta^k), \tag{38}$$

for all i, l, and k. For brevity we refer to this problem as problem AUX_1. The following Proposition characterizes the solution to this problem.

Proposition 8 *A solution $(t_i)_{i=1}^n$ to problem AUX_1 has the following properties:*

i) For all i and l, the local downward incentive constraint,

$$\theta^{l+1} Q_i \theta^{l+1} - T_i(\theta^{l+1}) \geq \theta^{l+1} Q_i \theta^l - T_i(\theta^l) \ ,$$

is binding, and all other incentive compatibility constraints are not binding.

ii) The expected revenue equals

$$\sum_{i=1}^n T_i(\theta^0) + E\left[\sum_{i=1}^n (\theta_i - h(\theta_i)) q_i(\theta, \beta)\right] \ . \tag{39}$$

iii) The expected consumer surplus equals

$$E\left[\sum_{i=1}^n h(\theta_i) q_i(\theta, \beta)\right] - \sum_{i=1}^n T_i(\theta^0) \ . \tag{40}$$

The proof follows from Lemmas 1 and 2.

Lemma 1 *The revenue that can be achieved at a solution to problem AUX_1 is bounded from above by the expression in (39). Reaching this upper bound requires that all local downward incentive constraints are binding. Moreover, if the upper bound is reached, the expected consumer surplus is given by the expression in (40).*

Proof *Step 1.* We first establish the following: Let a provision rule $(q_i^n)_{i=1}^n$ for the public good be given and let the expected payments $(T_i^n(\theta^0))_{i=1}^n$ of individuals with $\theta_i = \theta^0$ be given. If all local downward incentive constraints are binding, then the expected revenue is given by the expression in (39) and the expected consumer surplus is given by the expression in (40).

To see this, note that since all local downward incentive constraints are binding, we have that for any given i and any $k \geq 1$,

$$T_i(\theta^k) = T_i(\theta^0) + \sum_{l=1}^{k} \theta^l (Q_i(\theta^l) - Q_i(\theta^{l-1})) \ .$$

Rearranging terms and using that $\theta^0 = 0$, $\theta^1 = 1$, etc., shows that this can be equivalently written as

$$T_i(\theta^k) = T_i(\theta^0) + \theta^k Q_i(\theta^k) - \sum_{l=0}^{k} Q_i(\theta^l) \ .$$

By the law of iterated expectations,

$$E[t_i(\theta, \beta)] = T_i(\theta^0) + \sum_{k=0}^{m} f(\theta^k)\theta^k Q_i(\theta^k) - \sum_{k=0}^{m} f(\theta^k) \sum_{l=0}^{k} Q_i(\theta^l) \ .$$

Straightforward manipulations show that

$$\sum_{k=0}^{m} f(\theta^k) \sum_{l=0}^{k} Q_i(\theta^l) = \sum_{k=0}^{m} f(\theta^k) \frac{1 - F(\theta^k)}{f(\theta^k)} Q_i(\theta^k) \ .$$

Using the law of iterated expectations once more implies that $E[t_i^n(\theta)]$ can be written as

$$E[t_i(\theta, \beta)] = T_i(\theta^0) + E\left[\left(\theta_i - \frac{1 - F(\theta)}{f(\theta)}\right) q_i(\theta)\right] = T_i(\theta^0) + E\left[(\theta_i - h(\theta_i))\, q_i(\theta)\right] \ .$$

This also implies that

$$E[\theta_i q_i(\theta, \beta) - t_i(\theta, \beta)] = E[h(\theta_i) q_i(\theta, \beta)] - T_i(\theta^0)$$

Step 2. To complete the proof of the Lemma, take an arbitrary decision rule $(q_i^n)_{i=1}^n$ as given and consider the problem of maximizing aggregate revenues subject to the local downward incentive compatibility constraints of all individuals, and with given expected payments, $(T_i(\theta^0))_{i=1}^n$, for individuals with a θ^0-preference. Obviously, at a solution to the relaxed problem all constraints have to be binding. Otherwise, for some types of some individuals, expected payments could be increased, without violating any one of the constraints. By the reasoning in *Step 1.*, this implies that the maximal aggregate revenue is given by (39). Since this problem takes only a subset of all incentive compatibility and participation constraints into account this expression is an upper bound on the revenues that can be generated if decision rule $(q_i^n)_{i=1}^n$ is to be implemented and all incentive constraints are taken into account.

$\blacksquare$

Lemma 2 *Consider a mechanism such that all local downward incentive compatibility constraints are binding, and a provision rule so that, for all i, and all k the following monotonicity constraints are satisfied, $Q_i^n(\theta^k) \leq Q_i^n(\theta^{k+1})$. Then, this mechanism satisfies all incentive compatibility constraints.*

Proof We first show that if the monotonicity constraints hold, and all local downward incentive constraints are satisfied, then all downward incentive constraints (i.e., constraints of the form $\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^l) - T_i(\theta^l)$, for some $l \leq k$) are satisfied. To see this, note that the two local constraints,

$$\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^{k-1}) - T_i(\theta^{k-1}) ,$$

and

$$\theta^{k-1} Q_i(\theta^{k-1}) - T_i(\theta^{k-1}) \geq \theta^{k-1} Q_i(\theta^{k-2}) - T_i(\theta^{k-2}) ,$$

imply that

$$\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^{k-2}) - T_i(\theta^{k-2}) + (\theta^k - \theta^{k-1})(Q_i(\theta^k) - Q_i(\theta^{k-1})) .$$

The monotonicity constraint implies that the right hand side of this inequality is larger than $\theta^k Q_i(\theta^{k-2}) - T_i(\theta^{k-2})$. Hence, the downward incentive constraint,

$$\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^{k-2}) - T_i(\theta^{k-2}) ,$$

is satisfied. Iterating this argument further shows that all downward incentive constraints hold.

In a similar way, one can show that, if the monotonicity constraints holds, then local upward incentive compatibility – $\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^{k+1}) - T_i(\theta^{k+1})$ for all i and k – is sufficient to ensure that all upward incentive constraints – $\theta^k Q_i(\theta^k) - T_i(\theta^k) \geq \theta^k Q_i(\theta^l) - T_i(\theta^l)$ for all i, k and $l > k$ – are satisfied.

To complete the proof, we show that if all local downward incentive compatibility constraints are binding, and the monotonicity constraint holds, then all local upward incentive compatibility constraints are satisfied. To see this, suppose that

$$\theta^k Q_i(\theta^k) - T_i(\theta^k) = \theta^k Q_i(\theta^{k-1}) - T_i(\theta^{k-1}) ,$$

or, equivalently,

$$T_i(\theta^k) - T_i(\theta^{k-1}) = \theta^k \left(Q_i(\theta^k) - Q_i(\theta^{k-1}) \right) . \tag{41}$$

The local upward incentive compatibility constraint,

$$\theta^{k-1} Q_i(\theta^{k-1}) - T_i(\theta^{k-1}) \geq \theta^{k-1} Q_i(\theta^k) - T_i(\theta^k) ,$$

can be equivalently written as

$$T_i(\theta^k) - T_i(\theta^{k-1}) \geq \theta^{k-1} \left(Q_i(\theta^k) - Q_i(\theta^{k-1}) \right) . \tag{42}$$

If $Q_i(\theta^k) \geq Q_i(\theta^{k-1})$ holds, then (41) implies (42). ■

Revenue maximization with participation constraints.

We define problem AUX_2 as follows: Given a provision rule $(q_i)_{i=1}^n$ that satisfies the monotonicity constraint $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l, we seek to maximize expected revenue $E[\sum_{i=1}^n t_i(\theta, \beta)]$ subject to the individuals' incentive compatibility constraints in (38) and the participation constraints

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq 0 , \tag{43}$$

for all i, and l.

Proposition 9 *A solution $(t_i)_{i=1}^n$ to problem AUX_2 has the following properties:*

i) It has all the properties stated in Proposition 8.

ii) The participation constraint

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) = -T_i(\theta^0) \geq 0 ,$$

is binding for every individual i, whereas all other participation constraints are not binding.

Note that if we modify problem AUX_1 so that the payments $(T_i(\theta^0))_{i=1}^n$ can be freely chosen subject to the participation constraint for θ^0-types, $T_i(\theta^0) \leq 0$, for all i, then the solution will be such that $T_i(\theta^0) = 0$, for all i. To complete the proof it therefore suffices to establish the following Lemma.

Lemma 3 *Consider a mechanism that satisfies local downward incentive compatibility. Suppose that for all i, $\theta^0 Q_i(\theta^0) - T_i(\theta^0) \geq 0$. Then, this mechanism satisfies all participation constraints.*

Proof By assumption the mechanism satisfies the participation constraints for θ^0-individuals. The local downward incentive compatibility constraint for a θ^l-individual implies that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) \geq \theta^{l-1} Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) \; .$$

Consequently, if the participation constrained is satisfied for a θ^{l-1}-individual, then it is also satisfied for a θ^l individual. ■

B.1.2 Revenue minimization subject to the firm's constraints

We define problem AUX_3 as follows: Given a provision rule $(q_i)_{i=1}^n$ that satisfies the monotonicity constraint $K(\beta^l) \leq K(\beta^{l+1})$, for all l, we seek to minimize expected revenue $E[\sum_{i=1}^n t_i(\theta, \beta)]$ subject to the firm's incentive compatibility constraints in

$$R(\beta^l) - \beta^l K(\beta^l) \geq R(\beta^k) - \beta^l K(\beta^k), \tag{44}$$

for all l, and k, and the non-negative profit conditions,

$$R(\beta^l) - \beta^l K(\beta^l) \geq 0 \; , \tag{45}$$

for all l.

Proposition 10 *A solution $(t_i)_{i=1}^n$ to problem AUX_3 has the following properties:*

i) For all k, the local downward incentive constraint,

$$R(\beta^{k+1}) - \beta^{k+1} K(\beta^{k+1}) \geq R(\beta^k) - \beta^{k+1} K(\beta^k) \; ,$$

is binding and all other incentive compatibility constraints are not binding.

ii) The non-negative profit constraint

$$R(\beta^1) - \beta^1 K(\beta^1) \geq 0 \; ,$$

is binding, whereas all other non-negative profit constraints are not binding.

iii) The expected revenue equals

$$E[(\beta + g(\beta)) k(q(\theta, \beta))] \; ,$$

where $q(\theta, \beta) := \max_{i \in I} q_i(\theta, \beta)$.

Proof *Step 1.* We show that, for any mechanism satisfying the the budget balance conditions in (45) and the firm's incentive compatibility conditions in (44), $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] \geq E\left[(\beta + g(\beta))\, k(q(\theta,\beta))\right]$.

Let $(q_i)_{i=1}^{n}$ be an arbitrary given provision rule and consider the relaxed problem of minimizing $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$ subject to the budget balance condition for $\beta = \beta^1$ and the local downward incentive compatibility conditions for the firm, $R(\beta^l) - \beta^l K(\beta^l) \geq R(\beta^{l-1}) - \beta^l K(\beta^{l-1})$, for all l. Since this minimization problem takes only a subset of all budget balance conditions and all incentive compatibility conditions into account, the solution of this minimization problem will be a lower bound to the minimal value of $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$ that can be obtained if all budget and incentive constraints are taken into account.

At a solution to the relaxed problem all constraints have to be binding. Otherwise it was possible to reduce the expected revenues for some type of firm without violating any of the constraints of the relaxed problem, thereby attaining a lower value of $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$. This makes it possible to verify that $R(\beta^l) = \beta^l K(\beta^l) + \sum_{j=1}^{l-1} K(\beta^j)$, for $l \in \{2,\ldots,r\}$, and that $R(\beta^1) = \beta^1 K(\beta^1)$. Using the law of iterated expectations, we obtain

$$\begin{aligned}
E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] &= \sum_{l=1}^{r} f^l R(\beta^l) = E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=2}^{r} f^l \sum_{j=1}^{l-1} K(\beta^j) \\
&= E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=1}^{r} (1 - F(\beta^l)) K(\beta^l) \\
&= E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=1}^{r} f^l \frac{1 - F(\beta^l)}{f(\beta^l)} K(\beta^l) \\
&= E\left[(\beta + g(\beta))\, k(q(\theta,\beta))\right] .
\end{aligned}$$

Step 2. Suppose public goods provision is such that the following monotonicity constraint holds: For all l, $K(\beta^l) \geq K(\beta^{l-1})$. We show that, under this assumption, there is a mechanism such that $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] = E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right]$, satisfying all the budget balance conditions in (45) and all incentive compatibility conditions in (44).

Using arguments that are analogous to those in the proof of Lemma 3, we find that if the firm's non-negative profit condition holds for $\beta = \beta^1$, then it also holds for all $\beta \neq \beta^1$.

The fact that all local downward incentive compatibility constraints are binding and that the monotonicity constraint $K(\beta^l) \leq K(\beta^{l+1})$ holds for all l, implies that all firm incentive compatibility conditions are satisfied. This follows from similar arguments as in Lemma 2.

■

B.2 Proof of Proposition 1

We first consider a relaxed problem of maximizing expected consumer surplus taking the only the firm's non-negative profit conditions in (45) and the firm's incentive compatibility conditions in (44) into account. We refer to this problem in the following as auxiliary problem AUX_4.

We will then argue, in a second step, that there is a payoff equivalent mechanism which satisfies also the consumers' incentive constraints in (38)

Lemma 4 *The mechanism which solves the auxiliary problem AUX_4 has all the properties stated in Proposition 1.*

Proof Let us assume, for a moment, that the mechanism which maximizes expected consumer subject to (45) and (44) satisfies the monotonicity condition $K(\beta^l) \leq K(\beta^{l+1})$, for all l. This property will be verified below.

A necessary condition for the maximization of consumer surplus is that the payments of individuals are minimized. Hence, by Proposition 10,

$$E\left[\sum_{i=1}^{n} t_i(\theta, \beta)\right] = E[(\beta + g(\beta))k(q(\theta, \beta))] \tag{46}$$

where $q(\theta, \beta) := \max_{i \in I} q_i(\theta, \beta)$. Therefore we can write the problem of consumer surplus maximization as follows: Choose a provision rule $(q_i)_{i=1}^n$ in order to maximize

$$S = E\left[\sum_{i=1}^{n} \theta_i q_i(\theta, \beta)\right] - E[(\beta + g(\beta))k(q(\theta, \beta))] \,.$$

Now suppose that the solution to this problem involves exclusion: for some (θ, β) there is i such that $q_i(\theta, \beta) < q(\theta, \beta)$. Then increasing $q_i(\theta, \beta)$ involves no cost, i.e., $E[(\beta + g(\beta))k(q(\theta, \beta))]$ remains unaffected, but increases consumer welfare since $E[\sum_{i=1}^n \theta_i q_i(\theta, \beta)]$ goes up. This is a contradiction to the assumption that the optimum involves exclusion. Hence, we need have all (θ, β), and all i that $q_i(\theta, \beta) = q(\theta, \beta)$.

We can therefore once more rewrite the problem of choosing an optimal provision rule: Choose $q : (\theta, \beta) \mapsto q(\theta, \beta)$ in order to maximize

$$S = E\left[\left(\sum_{i=1}^{n} \theta_i\right) q(\theta, \beta)\right] - E[(\beta + g(\beta))k(q(\theta, \beta))] \,.$$

The solution q^* to this problem is such that, for every (θ, β), the following first order condition is satisfied:

$$\sum_{i=1}^{n} \theta_i = (\beta + g(\beta))k'(q^*(\theta, \beta)) \ . \tag{47}$$

Note that, by assumption, $\beta + g(\beta)$ is an increasing function of β, or equivalently, $k \geq l$ implies that $\beta^k + g(\beta^k) \geq \beta^l + g(\beta^l)$. Consequently, the first order conditions imply that, for every θ, $q^*(\theta, \beta^k) \leq q^*(\theta, \beta^l)$. This implies that the monotonicity condition $K(\beta^l) \leq K(\beta^{l+1})$, for all l, is satisfied. ■

Lemma 5 *There is a mechanism which solves problem AUX_4 and satisfies also the consumers's incentive compatibility constraints in (38).*

Proof We first note that the first order conditions in (47) imply that, for every i, every l, every θ_{-i}, and every given β, $q_i(\theta^l, \theta_{-i}, \beta) \leq q_i(\theta^{l+1}, \theta_{-i}, \beta)$. This implies that the solution to problem AUX_4 satisfies the monotonicity constraints $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l.

Now construct expected payments of individuals such that all local downward incentive compatibility constraints are binding and choose $(T_i(\theta^0))_{i=1}^n$ such that

$$\sum_{i=1}^{n} T_i(\theta^0) = E\left[\left(\sum_{i=1}^{n} \theta_i - h(\theta_i)\right) q^*(\theta, \beta)\right] - E[(\beta + g(\beta))k(q^*(\theta, \beta))] \ . \tag{48}$$

It follows from Lemma 2 that all of the consumers' incentive constraints are satisfied. Also it follows from Proposition 8, equation (39), that the expected revenues are given by

$$E\left[\sum_{i=1}^{n} t_i(\theta, \beta)\right] = E[(\beta + g(\beta))k(q^*(\theta, \beta))] \ .$$

■

B.3 Proof of Proposition 2

Lemma 6 *A mechanism $(q_i^*, t_i^*)_{i=1}^n$ that maximizes S subject to the constraints in (38), (44), and (45) satisfies the constraints in (43) if and only if*

$$E\left[\sum_{i=1}^{n} (\theta_i - h(\theta_i)) q^*(\theta, \beta)\right] \geq E[(\beta + g(\beta))k(q^*(\theta, \beta))] \ . \tag{49}$$

Proof As has been shown in the proof of Lemma 5, the provision rule $(q_i^*)_{i=1}^n$ is such that the monotonicity constraints $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l, are satisfied. It follows from Proposition 9, that the maximal revenue that can be extracted from individuals is therefore equal to $E\left[\sum_{i=1}^{n} (\theta_i + h(\theta_i)) q^*(\theta, \beta)\right]$.

As has been shown in the proof of Lemma 4, the provision rule $(q_i^*)_{i=1}^n$ is such that the monotonicity condition $K(\beta^l) \leq K(\beta^{l+1})$, for all l, is satisfied. It follows from Proposition 10 that the minimal revenue for the firm is equal to $E[(\beta + g(\beta))k(q^*(\theta, \beta))]$.

Consequently, a necessary condition for the implementability of $(q_i^*)_{i=1}^n$ is that

$$E\left[\sum_{i=1}^{n}(\theta_i + h(\theta_i))q^*(\theta,\beta)\right] \geq E[(\beta + g(\beta))k(q^*(\theta,\beta))] \; .$$

Sufficiency of this condition can be shown by using, once more, the construction in the proof of Lemma 5. If condition (49) holds and we let, for all i,

$$T_i(\theta^0) = -\left(E\left[\left(\sum_{i=1}^{n}\theta_i - h(\theta_i)\right)q^*(\theta,\beta)\right] - E\left[(\beta + g(\beta))k(q^*(\theta,\beta))\right]\right) \geq 0 \; ,$$

we obtain a mechanism that achieves $(q_i^*)_{i=1}^n$, satisfies all relevant constraints, and has the properties stated in Proposition 1. ■

Consider the following problem, referred to henceforth as problem AUX_5: Choose $(q_i, t_i)_{i=1}^n$ in order to maximize S subject to the constraints that

$$E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] \geq E[(\beta + g(\beta))k(q(\theta,\beta))] \; . \tag{50}$$

and

$$E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q_i(\theta,\beta)\right] \geq E[(\beta + g(\beta))k(q(\theta,\beta)) \; . \tag{51}$$

Lemma 7 *Suppose that condition (49) is violated. Then a solution to problem AUX_5 has properties i), ii) and iii) in Proposition 2.*

Proof We first note that (50) has to hold as an equality. Otherwise, we could increase S by lowering $E[\sum_{i=1}^n t_i(\theta,\beta)]$ without upsetting (51). We can hence rewrite the objective S as follows

$$S = E\left[\sum_{i=1}^{n}\theta_i q_i(\theta,\beta)\right] - E[(\beta + g(\beta))k(q(\theta,\beta))] \; .$$

If condition (49) is violated, then the inequality in (51) is binding, and the optimal provision rule maximizes the following Lagrangean

$$\begin{aligned}\mathcal{L} &= E\left[\textstyle\sum_{i=1}^n \theta_i q_i(\theta,\beta)\right] - E[(\beta + g(\beta))k(q(\theta,\beta))] \\ &\quad +\lambda\left(E\left[\textstyle\sum_{i=1}^n(\theta_i - h(\theta_i))q_i(\theta,\beta)\right] - E[(\beta + g(\beta))k(q(\theta,\beta))\right) \\ &= (1+\lambda)E\left[\textstyle\sum_{i=1}^n(\theta_i - \frac{\lambda}{1+\lambda}h(\theta_i))q_i(\theta,\beta) - (\beta + g(\beta))k(q(\theta,\beta))\right] \; ,\end{aligned}$$

where λ is the Langrangean multiplier which, at a solution to this maximization problem, has to be strictly positive, $\lambda > 0$.

The Lagrangean $\mathcal{L}$ is increasing in $q_i(\theta, \beta)$ if $\theta_i - \frac{\lambda}{1+\lambda} h(\theta_i) \geq 0$ and is decreasing otherwise. The assumption that h is a decreasing function and the observation that $\lambda > 0$ imply that there there is a cutoff value $\theta^k > \theta^0$, so that $\theta_i - \frac{\lambda}{1+\lambda} h(\theta_i) \geq 0$ if and only if $\theta_i \geq \theta^k$.

Maximization of the Lagrangean requires that $q_i(\theta, \beta) = q(\theta, \beta) := \max_i q_i(\theta, \beta)$ if $\theta_i \geq \theta^k$, and $q_i(\theta, \beta) = 0$, otherwise. This establishes property i) in Proposition 2.

Given this observation, we can rewrite the Lagrangean as

$$\mathcal{L} = (1+\lambda) E\left[\textstyle\sum_{\{i|\theta_i \geq \theta^k\}} (\theta_i - \frac{\lambda}{1+\lambda} h(\theta_i)) q(\theta, \beta) - (\beta + g(\beta)) k(q(\theta, \beta))\right] .$$

The optimal level of $q(\theta, \beta)$ therefore satisfies the first order condition,

$$\sum_{\{i|\theta_i \geq \theta^k\}} \left(\theta_i - \frac{\lambda}{1+\lambda} h(\theta_i)\right) = (\beta + g(\beta)) k'(q(\theta, \beta)) ,$$

for all θ, and β. This proves property iii) in Proposition 2.

■

Lemma 8 *Suppose that condition (49) is violated. The level of S generated by solution to problem AUX_5 is an upper bound on the level of S is the constraints in (38), (44),(45) and (43) have to be taken into account. Moreover, the solution to problem AUX_5 can be implemented by means of a mechanism that satisfies these constraints.*

Proof It follows from Propositions 9 and 10 that the consumer surplus S is bounded from above by the surplus that is generated by a mechanism that solves the following problem AUX_6: maximize S subject to the constraints

$$E\left[\sum_{i=1}^{n} t_i(\theta, \beta)\right] \geq E[(\beta + g(\beta)) k(q(\theta, \beta))] . \tag{52}$$

and

$$E\left[\sum_{i=1}^{n} (\theta_i - h(\theta_i)) q_i(\theta, \beta)\right] \geq E\left[\sum_{i=1}^{n} t_i(\theta, \beta)\right] . \tag{53}$$

and that this upper bound can be reached if the monotonicity constraints $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l; and $K(\beta^l) \leq K(\beta^{l+1})$, for all l, are satisfied.

Obviously at a solution to this problem the constraint (52) has to be binding, because otherwise it would be possible to increase S by lowering $E[\sum_{i=1}^{n} t_i(\theta, \beta)]$. This implies that the constraint in (53) can be written as

$$E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q_i(\theta,\beta)\right] \geq E[(\beta + g(\beta))k(q(\theta,\beta))] \ . \tag{54}$$

If condition (49) is violated, this inequality constraint is binding, which implies that the solution to problem AUX_6 coincides with the solution to problem AUX_5.

To complete the proof it remains to be shown that a solution to problem AUX_5 satisfies the monotonicity constraints.

To see that $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$, for all i and l holds, note that the monotone hazard rate assumption implies that $\theta_i - h(\theta_i)$ is an increasing function. Consequently, the solution to problem AUX_5 is such that

$$q_i(\theta^l, \theta_{-i}, \beta) \leq q_i(\theta^{l+1}, \theta_{-i}, \beta) \ ,$$

for all i, l, θ_{-i} and β.

To see that $K(\beta^l) \leq K(\beta^{l+1})$, for all l, note that $\beta + g(\beta)$ also is an increasing function. This implies that, for all θ, and l, $q(\theta, \beta^l) \leq (\theta, \beta^{l+1})$. Consequently,

$$k(q(\theta,\beta^l)) \leq k(q(\theta,\beta^{l+1})) \ .$$

■

B.4 Proof of Proposition 4

We first consider a relaxed problem of maximizing expected profits

$$E\left[\sum_{i=1}^{n} t_i(\theta) - \ \beta k(q(\theta))\right]$$

taking only the constraint that the expected consumer surplus must be non-negative,

$$E\left[\sum_{i=1}^{n}(\theta_i q_i(\theta) - t_i(\theta)\right] \geq 0 \ , \tag{55}$$

into account. We show in *Step 1* that the solution to this problem has all the properties stated in Proposition 4. We will then show in *Step 2* that this outcome can also be obtained in such a way that the consumers' incentive compatibility constraints in (38) are satisfied.

Step 1. Obviously, at a solution to the relaxed problem the constraint in (55) has to be binding. Otherwise, expected payments of individuals and therefore the firm's expected profit could be increased. This establishes property *ii*) in Proposition 4. Expected profits can hence be rewritten as

$$E\left[\sum_{i=1}^{n}\theta_i q_i(\theta) - \ \beta k(q(\theta))\right] \ , \tag{56}$$

and the optimal provision rule $(q_i)_{i=1}^n$ maximizes this expression. The solution has no exclusion (property i) in Proposition 4). Otherwise it would be possible to increase $\sum_{i=1}^n \theta_i q_i(\theta)$, for some θ without having to increase $\beta k(q(\theta))$. This would lead to a higher value of the objective function, so that a situation with exclusion cannot be optimal.

This implies that the objective in (56) can be rewritten once more as

$$E\left[\left(\sum_{i=1}^{n}\theta_i\right) q(\theta) - \ \beta k(q(\theta))\right] \ , \tag{57}$$

which is the expression for profits in part iv) of Proposition 4. Maximization of this expression yields to the Samuelson rule, property iii) in Proposition 4.

Step 2. It is easily verified that public goods provision according to the Samuelson rule implies that for all i, and l, the monotonicity constraint $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$ is satisfied. Consequently, if the expected payments for all individuals are chosen in such a way that the local downward incentive compatibility constraints are binding, then, by Lemma 2, all incentive constraints are satisfied. If moreover, we choose $(T_i(\theta^0))_{i=1}^n$ such that,

$$E\left[\sum_{i=1}^{n}h(\theta_i)q_i(\theta,\beta)\right]\sum_{i=1}^{n}T_i(\theta^0) \ . \tag{58}$$

then by part iii) of Proposition 8, the expected consumer surplus is equal to 0. ■

B.5 Proof of Proposition 5

It follows from Propositions 8 and 9 that, if we limit attention to provision rules satisfying the monotonicity constraint $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all i and l, then revenue maximization yields

$$E\left[\sum_{i=1}^{n}t_i(\theta)\right] = E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q_i(\theta)\right] \ .$$

Hence, the profit maximizing provision rule is the one that maximizes

$$E\left[\sum_{i=1}^{n}t_i(\theta) - \ \beta k(q(\theta))\right] = E\left[\sum_{i=1}^{n}(\theta_i - h(\theta_i))q_i(\theta) - \ \beta k(q(\theta))\right] \tag{59}$$

and, by iii) of Proposition 8, yields an expected consumer surplus of

$$E\left[\sum_{i=1}^{n}h(\theta_i)q_i(\theta)\right]$$

We proceed in two steps. We first show that the provision rule $(q_i)_{i=1}^n$ that maximizes the right hand side of (59) satisfies properties *i)* and *iii)* in Proposition 5, *Step 1.* We then show that this provision rule satisfies the monotonicity constraint, *Step 2.*

Step 1. Expected profits $E\left[\sum_{i=1}^n(\theta_i - h(\theta_i))q_i(\theta) - \ \beta k(q(\theta))\right]$ are decreasing in $q_i(\theta)$ if $\theta_i - h(\theta_i) < 0$ and non-decreasing otherwise. Hence, it is optimal to have $q_i(\theta) = 0$ in the first place and $q_i(\theta) = q(\theta)$ in the latter. This proves property *i)* in Proposition 5.

The objective function can therefore be rewritten as

$$E\left[\sum_{i=1}^n h(\theta_i)q_i(\theta)\right] = E\left[\left(\sum_{\{i|\theta_i - h(\theta_i)\geq 0\}} \theta_i - h(\theta_i)\right) q(\theta) \ - \beta k(q(\theta))\right] .$$

Choosing $q : \theta \mapsto q(\theta)$ so as to maximize this expression yields property *iii)* in Proposition 5.

Step 2. The assumption that h is a decreasing function implies that $\theta_i - h(\theta_i)$ is increasing in θ_i. Hence if property *i)* in Proposition 5 holds, then we have that for all i and θ_{-i}, $q_i(\theta^l, \theta_{-i}) \geq q_i(\theta^{l-1}, \theta_{-i})$. This implies, in particular, that the monotonicity constraint $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all i and l, is satisfied.

■