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On the legitimacy of coercion for the financing of public goods

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Abstract

The literature on public goods has shown that efficient outcomes are impossible if participation constraints have to be respected. This paper addresses the question whether they should be imposed. It asks under what conditions efficiency considerations justify that individuals are forced to pay for public goods that they do not value. It is shown that participation constraints are desirable if public goods are provided by a malevolent Leviathan. By contrast, with a Pigouvian planner, efficiency can be achieved. Finally, the paper studies the delegation of public goods provision to a profit-maximizing firm. This also makes participation constraints desirable.

Keywords: Public goods, Mechanism Design, Incomplete Contracts, Regulation

JEL: D02, D82, H41, L51

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1 Introduction

Should public goods be financed solely out of voluntary contributions or is there a role for taxes as a source of public goods finance? The disadvantage of a system based on voluntary contributions is that public goods are underprovided because voluntary contributions tend to neglect the social benefits of increased public goods provision. By contrast, a system based on taxes which are raised independently of an individual's valuation of public goods can provide sufficient funds for efficient public goods provision. However, if individuals can be forced to pay for public goods that they do not value, this coercive power may also be abused. If politicians, bureaucrats or managers can use funds from the public budget to finance projects, their choices may be biased towards their private interests.

This paper formalizes a tradeoff between the efficiency of public goods provision, on the one hand, and protection against an abuse of coercive power, on the other. It asks the question under what circumstances coercion is legitimate in the sense that it makes individuals, by and large, better off even though occasionally they will be forced to pay for a public good that they do not value. Broadly speaking, the main result is that if public goods are provided by a benevolent Pigouvian planner, then individuals are happy to equip this institution with coercive power. Being benevolent, it will use the instruments at its disposal in the individuals' best interests, which implies that public goods are provided efficiently. By contrast, if public goods are provided by a malevolent planner who seeks to maximize its own payoff at the expense of individuals, individuals prefer public goods provision based on voluntary contributions.

The result shows that at the heart of the question whether or not the use of coercion is legitimate is a distributive conflict. Both a malevolent Leviathan and a benevolent Pigouvian planner will provide a surplus-maximizing amount of public goods if coercion is possible. However, the malevolent Leviathan will keep the entire surplus for himself so that individuals will not benefit from public goods provision at all. Voluntary public goods finance, by contrast, leaves them at least a positive share of the surplus, albeit a smaller one.

This paper contributes to the literature on public goods provision under conditions of incomplete information about public goods preferences. This literature has arrived at two major results. On the one hand, there is a *possibility result*: It is possible to reach an efficient allocation of public goods, even if individuals have private information on their preferences.[1] On the other hand, there is an *impossibility result*: Efficient outcomes are out of reach if participation constraints have to be respected, so that each individual has to be better off relative to a status

[1] This result is due to d'Aspremont and Gérard-Varet (1979) and Arrow (1979). For a more recent generalization, see d'Aspremont et al. (2004).

quo allocation without public goods provision.[2]

This literature confronts us with a choice between outcomes that are efficient and second-best outcomes that avoid the use of coercion. The contribution of the present paper is to answer the question how this choice should be made; i.e., we clarify under which conditions the objective to reach efficient outcomes justifies the use of coercion.

Our answer to this question is based on a constitutional choice perspective; that is, we make the following thought experiment: Suppose there is an ex ante stage at which individuals have not yet discovered what their preferences are. More specifically, an individual's objective is to maximize expected utility, with expectations taken about her future preferences. At this ex ante stage, individuals decide about the rules according to which public goods are provided. In particular, they face a choice between a strong and a weak formulation of participation constraints. The strong formulation requires that, at the interim stage, where individuals have discovered their preferences, each individual benefits from public goods provision. The weak version requires only that individuals benefit from public goods provision at the ex ante stage.

To illustrate this by means of an example, think of the construction of a bridge, and suppose that there are individuals who cross the bridge frequently and others who do so only rarely. If we impose participation constraints in the weak, ex ante sense, this allows us to force the non-frequent users to contribute to the financing of the bridge, provided that their utility loss is compensated for by the utility gain of the frequent users. By contrast, if we impose participation constraints in the strong, ex interim sense, we lose this opportunity. In this case, the less frequent users must also be made better off by the construction of the bridge, which implies that they cannot be forced to pay for a bridge that they hardly ever use.

We say that coercion is legitimate if, at the constitutional stage, individuals opt for participation constraints in the weak sense.

We approach the question whether coercion and hence efficient public goods provision is legitimate from two different angles.

First, we take a mechanism design perspective. With this approach, we arrive at the conclusion that strong participation constraints, which protect individuals from having to pay for a public good that they do not value, should be imposed if and only if there is a pronounced agency conflict between individuals and the mechanism designer.[3] The logic is as follows: While strong participation constraints have detrimental consequences from an efficiency perspective,

[2] See Güth and Hellwig (1986), Rob (1989), Mailath and Postlewaite (1990), Hellwig (2003), or Neeman (2004).

[3] Hellwig (2003) studies public goods provision by a benevolent mechanism designer who faces participation constraints in the strong sense. Our result shows that such a mechanism design problem cannot occur if the relevant constraints are endogenized by means of a constitutional decision at the ex ante stage.

they also imply that individuals can at least realize an information rent. If the mechanism designer's objective is to extract the surplus from public goods provision (and hence to minimize the expected payoff of individuals), then the imposition of strong participation constraints is desirable because individuals prefer getting an information rent over not getting anything.

Based on this answer, we turn to the more fundamental question where such an agency conflict should come from. To motivate this question, suppose for a moment that at the constitutional stage individuals could not only choose between strong and weak participation constraints, but that they could also specify the mechanism designer's objective function. Obviously, they would opt for a benevolent mechanism designer who maximizes the expected payoff of individuals. As a consequence, individuals would be happy to remove any constraint from the mechanism designer's problem. In particular, they would be willing to accept that they occasionally have to pay for a public good that they do not value. Hence, the use of coercion would be legitimate and efficiency could be achieved.

While this reasoning may seem somewhat contrived, it raises the question whether we can provide a more plausible "microfoundation" for agency conflicts that make strong participation constraints desirable. In the second part of the paper, we show that there is a positive answer to this question, and moreover, that giving such a positive answer requires us to leave the mechanism design framework and to adopt instead an incomplete contracts perspective.

More specifically, the second part of the paper is based on an extended model in which there are not only individuals with private information about their preferences but also a firm which produces the public good and has private information about its technology. In this environment, a mechanism design approach leads to complete contingent planning, i.e., to a specification of public goods production and individual payments as a function of the individuals' public goods preferences and the firm's technology. Again, we obtain the result that if the mechanism designer is sufficiently benevolent, then the use of coercion is justified so that efficiency can be reached.

We contrast this with the following incomplete contracts model: There is a regulator who delegates public goods provision to the firm, i.e., he sells the right to produce the public good and to collect payments from individuals to the firm. Afterwards, the firm becomes residual claimant; i.e., the firm communicates with individuals about their preferences and organizes public goods supply so that its profits are maximized. From the regulator's perspective, this arrangement is incomplete in the sense that he remains ignorant with respect to public goods preferences. His interaction with the firm can at most be contingent on the firms's technology.

We show that this approach may indeed leave so much discretion to the firm that it is able to extract the entire surplus from public goods provision. Consequently, individuals prefer the imposition of strong participation constraints, i.e., in the incomplete contracts model, the use of coercion is not legitimate.

To sum up, the analysis shows that, unless there are pronounced conflicts of interests between the consumers of a public good and the institution which is organizing its supply, coercion is legitimate and an efficient supply of public goods is possible. By contrast, if there are such conflicts, strong participation constraints should protect individuals against an abuse of coercive power. Finally, it is shown that the delegation of public goods provision to a profit-maximizing firm generates a case in which individuals should be protected: a profit-maximizing firm should not be given the possibility to charge individuals in excess of their willingness to pay, even if this comes at the cost of an inefficient public goods supply.

The remainder of the paper is organized as follows. The next section gives a more detailed literature review. Section 3 introduces the model. Section 4 establishes the result that the legitimacy of coercion depends on the mechanism designer's degree of benevolence. An extended model in which public goods provision is delegated to a firm with private information about its technology is analyzed in Section 5. The last section contains concluding remarks.

2 Related Literature

The paper contributes to the literature on public goods provision under the assumption that individuals have private information on their preferences. For a model with independent private values, d'Aspremont and Gérard-Varet (1979) and Arrow (1979) have shown that an efficient allocation of public goods can be implemented as a Bayes-Nash equilibrium. However, this outcome is out of reach if, in addition, ex interim participation constraints are imposed. This has been established by Güth and Hellwig (1986) and Mailath and Postlewaite (1990).[4] This impossibility result has led various authors to study second-best problems where public goods provision is subject to ex interim participation constraints. In particular, Güth and Hellwig (1986) and Schmitz (1997) study public goods provision by a profit-maximizing monopolist, and Hellwig (2003) and Norman (2004) study public goods provision by a benevolent utilitarian planner.

This paper contributes to this literature in various respects. First, it provides an answer to the question whether, from a normative perspective, strong participation constraints should be imposed, even if this implies that efficient outcomes cannot be reached. This question has not been asked before.

Second, the paper makes a technical contribution to the study of second-best problems. Most of the existing literature assumes that an individual's public goods preferences are the realiza-

[4]These papers show that a version of the Myerson and Satterthwaite (1983) theorem on the impossibility of efficient trade holds in an economy with public goods.

tion of a continuous random variable. This paper, by contrast, works under the assumption of a discrete number of types (where the number of possible types may be arbitrarily large). While this has no bearing on the results of the analysis, the model with a discrete set of types has two major advantages. There is no need to impose differentiability and continuity assumptions to make the mechanism design problem tractable. Also, it becomes more explicit which incentive compatibility constraints are binding and which ones are slack, which leads to a better understanding of the tradeoffs which shape the optimal mechanism.

Finally, it uses ideas from the literature on incomplete contracts to show that the delegation of public goods provision to a profit-maximizing firm yields a situation in which the imposition of strong participation constraints is desirable.[5] It thereby bridges two different literatures that are both relevant for public goods provision, and which are typically treated separately; namely the literature on the revelation of public goods preferences (which is a major topic in theoretical public economics) and the literature on the regulation of firms who are in charge of public production (which is a major topic in industrial organization).[6]

This work is also related to two recent papers in which participation constraints play a significant role.

Grüner (2008) asks the question whether an efficient allocation of public goods can be achieved with a different set of participation constraints. He requires that each individual is made better off relative to a situation with majority voting about public goods provision. In this model, efficient public goods provision is possible. This result is given a positive interpretation; i.e., it explains why despite the impossibility results in the existing literature, public goods are provided in the real world and why these outcomes may even be efficient: In the real world the status quo outcome is shaped by democratic institutions. The present paper offers an alternative positive explanation. Individuals may be willing to accept that, occasionally, they have to pay for public goods that they do not value if, on average, they benefit from the provision of public goods. This leads to a weaker notion of participation constraints, so that efficient outcomes can be achieved.

Acemoglu et al. (2008) compare public and private provision of insurance contracts. They formalize the following tradeoff: private provision suffers from inefficiencies due to participation constraints. These problems may be overcome by state provision of insurance, given that the state has coercive power. The disadvantage of state provision, however, is that the coercive power may be abused by selfish politicians. The tradeoff “markets versus governments” therefore

[5] For an overview of the incomplete contracts approach, see Hellwig (1996) and Tirole (1999).

[6] The seminal article in the latter branch of the literature is Baron and Myerson (1982). For a textbook treatment and a literature survey, see Laffont and Tirole (1993).

becomes a tradeoff between distortions due to participation constraints and distortions due to agency problems between citizens and politicians. While this tradeoff seems empirically plausible, the present paper shows that, from a normative perspective, agency problems are what justifies the imposition of participation constraints. Hence, why should the state be given coercive power if politicians are not acting in the citizen's interest? The normative analysis in this paper suggests that the state should have coercive power only if the agency problems between citizens and politicians are less significant than the agency problems between private providers of insurance and their customers.

3 The environment

There is a finite set of individuals, $I = \{1, \dots, n\}$. The preferences of individual i are given by the utility function

$$u_i = \theta_i q - t_i ,$$

where $q \in \mathbb{R}_+$ is the provision level of a public good, t_i is individual i's contribution to the cost of public good provision and θ_i is a taste parameter that affects individual i's valuation of the public good. For each i, θ_i belongs to a finite ordered set $\Theta = \{\theta^0, \theta^1, \dots, \theta^m\}$, with $\theta^0 = 0$. We assume that $\theta^l - \theta^{l-1} = 1$, for all l. We denote a vector of all individual taste parameters by $\theta = (\theta_1, \dots, \theta_n)$.

From an ex ante perspective, the taste parameters of individuals are independent and identically distributed random variables that take values in Θ. For any i, we denote the probability that $\theta_i = \theta^l$ by p^l. The following notation will prove helpful. For every i, let $p(\theta_i)$ be a random variable that takes the value p^l if θ_i takes the value θ^l and $P(\theta_i)$ be a random variable that takes the value $\sum_{k=0}^{l} p^k$ if θ_i takes the value θ^l. Define $h^l = \frac{1-P(\theta^l)}{p(\theta^l)}$. In the literature this fraction is known as the hazard rate. We assume that the hazard rate is decreasing, $h^l < h^{l-1}$, for all $l \geq 1$. This assumption is imposed in the following without further mention.

We study public goods provision from an interim perspective, i.e., after individuals have learned what their preferences are. With an appeal to the Revelation Principle we limit attention to direct mechanisms and to truthful Bayes-Nash equilibria. A direct mechanism consists of a provision rule for the public good and, for each individual i, a payment rule. The provision rule is a function $q : \Theta^n \rightarrow \mathbb{R}_+$ that specifies a public good provision level as a function of the preferences that individuals communicate to the mechanism designer. Analogously, the payment rule for individual i is a function $t_i : \Theta^n \rightarrow \mathbb{R}$.

A mechanism has to satisfy participation constraints, incentive compatibility constraints and a budget constraint. The budget constraint requires that expected payments of individuals are sufficient to cover the expected cost of public good provision,[7]

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right] \geq E[\beta k(q(\theta))] \ . \tag{1}$$

where k is a strictly increasing and strictly convex function with $k(0) = 0$, $\lim_{q\to 0} k'(q) = 0$, and $\lim_{q\to\infty} k'(q) = \infty$. β is a parameter that, for the moment, is treated as commonly known.[8] The expectations operator E applies to the vector θ of all individual taste parameters.

The incentive compatibility constraints ensure that that truth-telling is a Bayes-Nash equilibrium: given that all other individuals reveal their taste parameter, the best response of individual i is to reveal the own taste parameter as well. Formally, for each i, for each $\theta_i \in \Theta$, and for each $\hat{\theta}_i \in \Theta$,

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) \geq \theta_i Q_i(\hat{\theta}_i) - T_i(\hat{\theta}_i), \tag{2}$$

where

$$Q_i(\hat{\theta}_i) := E[q(\theta_{-i}, \hat{\theta}_i) \mid \hat{\theta}_i]$$

is the expected level of public goods provision from the perspective of individual i, given that all other individuals reveal their preferences to the mechanism designer and individual i announces $\hat{\theta}_i$. Likewise,

$$T_i(\hat{\theta}_i) = E[q(\theta_{-i}, \hat{\theta}_i) \mid \hat{\theta}_i]$$

is i's expected payment.

A mechanism also has to satisfy participation constraints which ensure that individuals benefit from the provision of the public good. We distinguish between participation constraints at the ex interim state or at the ex ante stage. The ex interim participation constraints are as follows: For all i, and all θ_i,

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) \geq 0. \tag{3}$$

These constraints ensure that, after all individuals have discovered what their public goods preferences are, no individual is worse off relative to a status quo situation in which the public good is not provided. An alternative interpretation is that individuals are given veto rights that protect them from having to pay for a public good that they do not value. Consequently, a deviation from the status quo requires a unanimous agreement to provide the public good.

[7]It has been shown by d'Aspremont et al. (2004) that to any incentive compatible mechanism that satisfies the budget constraint in expectation there exists a payoff equivalent incentive compatible mechanism that satisfies budget balance in an ex post sense, i.e., $\sum_{i=1}^{n} t_i(\theta) \geq \beta k(q(\theta))$, for every θ. Hence, working with budget balance in expectation is without loss of generality.

[8]In Section 5 we will assume that the supplier of the public good has private information on β.

The ex ante participation constraints require that, for all i,

$$E[\theta_i q(\theta) - t_i(\theta)] \geq 0 \ , \tag{4}$$

so that each individual benefits from public good provision at an ex ante stage, i.e., prior to learning what the own preferences are. These participation constraints are less restrictive than those in (3). To make this more explicit we can use the law of iterated expectations to write (4) as follows: for all i,

$$\sum_{l=0}^{m} p^l(\theta^l Q_i(\theta^l) - T_i(\theta^l)) \geq 0 \ .$$

Consequently, the ex ante participation constraints in (4) require that the ex interim participation constraints in (3) hold "on average", but not necessarily for each possible realization of individual i's preferences.

These constraints in (4) ensure that the provision of the public good is a Pareto-improvement if considered behind a "veil of ignorance" where individuals can form an expectation about how the public good is going to affect their well-being, but are not yet fully informed about their preferences. These constraints make it possible to rely on coercion when financing the provision of a public good. Individuals can be forced to pay for a public good that they do not value, provided that, behind the veil of ignorance, they benefit from public goods provision. The participation constraints in (3), by contrast, exclude coercion under each and every circumstance. We can therefore interpret them as providing a maximal protection of economic freedom: No one may interfere with an individual's decision to spend his money on the uses that are most attractive to him.

The analysis focusses on the question whether the use of coercion is beneficial for individuals. To this end we will compare mechanisms where the ex ante participation constraints have to be satisfied to mechanisms where the ex interim participation constraints are imposed. The standard of comparison is the ex ante expected utility of individuals. If this utility is larger with ex ante participation constraints, then we say that the use of coercion is legitimate in the sense that if, behind a veil of ignorance, individuals were confronted with a constitutional choice about the use of coercion, they would unanimously vote in favor of it.

3.1 The tradeoff between efficiency and voluntary participation

We will show in the following that there efficiency is compatible with participation constraints at the ex ante stage but not with participation constraints at the ex interim stage.

We say that a mechanism $(q, t_1, \ldots, t_n)$ is constrained efficient if it is incentive compatible and budgetary feasible, and there is no other incentive compatible and budgetary feasible mechanism

$(q', t'_1, \ldots, t'_n)$, such that for all i, $E[\theta_i q'(\theta) - t'_i(\theta)] \geq E[\theta_i q(\theta) - t_i(\theta)]$, with a strict inequality for some i.[9]

Proposition 1 *A mechanism is constrained efficient if and only if the budget condition (1) holds as an equality and the public goods provision rule is surplus-maximizing; i.e., for every θ, $q(\theta)$ is chosen so as to maximize $\left(\sum_{i=1}^n \theta_i\right) q(\theta) - \beta k(q(\theta))$.*

It is well known that, under conditions of complete information, surplus maximization in conjunction with budget balance is both necessary and sufficient for Pareto-efficiency if preferences are quasilinear in money. Proposition 1 shows that the same is true with private information on public goods preferences, i.e., private information on preferences does not alter the efficiency conditions.[10]

The main step in the proof of the Proposition, which can be found in the Appendix, is to show that to any mechanism that is efficient in the set of allocations that are budgetary feasible (but not necessarily incentive compatible) there exists a budgetary feasible and incentive compatible mechanism that leads, for all individuals, to the same level of ex ante expected utility. Intuitively, incentive compatibility is a condition that affects how expected utility increases ex interim with an individual's valuation of the public good.[11] However, this is no restriction on the "base utility",

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) = -T_i(\theta^0),$$

to which these increments are added. Hence, upon manipulating $T_i(\theta^0)$ one can generate any level of $E[\theta_i q(\theta) - t_i(\theta)]$ in an incentive compatible way.

In particular, the if-part of Proposition 1 implies that there exists a constrained efficient allocation such that the surplus from public goods provision is shared equally among individuals, i.e., such that the ex ante expected utility of each individual i is equal to

$$E[\theta_i q(\theta) - t_i(\theta)] = \frac{1}{n} E\left[\sum_{i=1}^n \theta_i q^*(\theta) - \beta k(q^*(\theta))\right] > 0\ ,$$

[9]It should be noted that this definition of efficiency is based on the ex ante expected payoff of individuals. This efficiency criterion, which implies the desirability of surplus maximization, is predominant in the literature. However, one could also define an ex interim notion of Pareto efficiency, see Holmstrom and Myerson (1983), and Ledyard and Palfrey (1999) for an application to public goods provision.

[10]It has been shown by d'Aspremont and Gérard-Varet (1979) and Arrow (1979) that surplus maximization can be achieved, given the requirement of incentive compatibility. Proposition 1 is more general. It shows that surplus maximization is both necessary and sufficient for constrained efficiency.

[11] To see that interim utility is increasing note that the incentive compatibility conditions imply that, for each i, and each l, $\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) > \theta^{l-1} Q_i(\theta^{l-1}) - T_i(\theta^{l-1})$.

where q^* is the surplus-maximizing provision rule. In the following we will refer to this mechanism as the *symmetric constrained efficient mechanism.*

Obviously, since the expected surplus from public good provision is strictly positive, under the symmetric constrained efficient mechanism all ex ante participation constraints hold as a strict inequality. This shows that there is no conflict between efficiency on the one hand and the imposition of ex ante participation constraints on the other. If the expected benefits from public goods provision are evenly distributed, then every individual is made better off by public good provision.[12] We summarize these observations in the following Corollary.

Corollary 1 *There exists a constrained efficient mechanism that satisfies the ex ante participation constraints.*

By contrast, efficiency may be out of reach with ex interim participation constraints.

Proposition 2 *There exists a constrained efficient mechanism that satisfies the interim participation constraints if and only if*

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q^*(\theta)\right] \geq E[\beta k(q^*(\theta))] \ . \tag{5}$$

The inequality in (5) is violated if the number of individuals is sufficiently large. For a proof of this claim, see part B of the Appendix. The intuition is that, as the number of individual's increases, each single individual's influence on the public goods provision level becomes smaller and smaller, so that it becomes more and more attractive to articulate a low taste parameter in order to mitigate the own contribution to the cost of public good provision. Hence, with more individuals it is more difficult to raise enough money for efficient public goods provision.[13]

3.2 Sketch of the Proof of Proposition 2

A formal proof of Proposition 2 can be found in the Appendix. In the following the argument is sketched. The proof is based on a characterization of revenue maximizing mechanisms, i.e., of mechanisms that maximize $E[\sum_{i=1}^{n} t_i(\theta)]$, taking the public goods provision rule q as given, and uses arguments that are familiar from the analysis of optimal non-linear pricing mechanisms.[14]

[12] There exist constrained-efficient mechanisms that violate these constraints. Given that utility is perfectly transferrable between individuals, efficiency can also be achieved if one individual is made very badly-off and receives a negative expected payoff.

[13] For a model so that θ_i is distributed according to an atomless probability distribution, this impossibility result holds irrespective of the number of individuals, see Hellwig (2003).

[14] See Mussa and Rosen (1978), Matthews and Moore (1987), or Bolton and Dewatripont (2005).

In particular, a “relaxed revenue maximization problem” is studied which takes only a subset of all constraints into account, namely the ex interim participation constraints and the local downward incentive compatibility constraints,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) \ ,$$

for all i and l. The observation, that ex interim expected utility is increasing in θ_i (see footnote 11) then implies that only the participation constraints for types $\theta_i = \theta^0$ need to be taken explicitly into account. If these are satisfied, then those for higher types are automatically satisfied as well. A further observation is that the participation constraints for $\theta_i = \theta^0$ and all downward incentive compatibility constraints have to be binding. Otherwise, taking the provision rule for the public good as fixed, it would be possible to increase the expected payments of some types of some individuals without violating any one of the constraints of the relaxed problem. Given this pattern of binding constraints, it is possible to solve for the expected payments of individuals as a function of public goods provision rule. This derivation yields

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right] = E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] \ .$$

Consequently, $E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right]$ is an upper bound on the revenues that can be raised if public goods are provided according to q^*. Constrained efficiency is therefore possible only if these maximal revenues exceed the expected cost from public good provision, $E[\beta k(q^*(\theta))]$.

The if-part of the Proposition establishes that this upper bound can actually be reached if all incentive compatibility constraints; i.e., not only the local downward ones, are taken into account. By well-known arguments from the analysis of incentive constraints, this is the case if the public goods provision rule satisfies the following monotonicity constraint: For all i and l, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$. Obviously, this monotonicity property is satisfied by the surplus-maximizing provision rule q^*, which is strictly increasing in every individual's taste parameter.

The expression for maximal revenues is interpreted as follows: Individuals have to be granted rents because they have private information about their preferences. This implies, in particular, that a revenue-maximizing mechanism is unable to extract the whole expected surplus from public goods provision which equals $E\left[\sum_{i=1}^{n} \theta_i q(\theta)\right]$. As we show in part B of the Appendix, the expected payoff of individual i under a revenue maximizing mechanism is equal to $E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q(\theta)\right]$. An inspection of (5) reveals that the sum of these informational rents reduces the revenues that can be extracted from individuals.

3.3 Benevolent Second-best Mechanisms

The observation that ex interim participation constraints may imply that efficiency cannot be reached has led various authors to study second best mechanisms. Typically, the objective is to maximize expected utilitarian welfare, $E\left[\sum_{i=1}^{n}(\theta_i q(\theta) - t_i(\theta))\right]$, subject to the budget constraint (1), the incentive compatibility constraints (2), and the ex interim participation constraints (3). For brevity, we refer to this problem in the following as the *benevolent second best problem.*

Proposition 3 *If condition (5) holds, then the symmetric constrained efficient mechanism solves the benevolent second best problem. Otherwise, each individual is, in terms of ex ante expected utility, strictly worse off.*

The Proposition, which is proven in the Appendix, shows that from a normative perspective, the imposition of ex interim participation constraints is undesirable. These constraints are never beneficial but sometimes harmful, depending on whether or not the inequality in (5) holds. Hence, if individuals were given a choice between the imposition of participation constraints at the ex interim or the ex ante stage they would unanimously opt for the latter. Put differently, at the ex ante stage, individuals are happy to accept that they occasionally will have to pay for public goods that they do not value if they are assured that their share of the expected surplus from public goods provision is sufficiently high. However, these results rely on the assumption that the mechanism designer is benevolent. In the following section we relax this assumption. As will become clear, this may give rise to a role for the imposition of participation constraints at the ex interim stage.

4 Conflicts of interest

In the following, we allow for the possibility that the mechanism designer does not only care about the expected payoffs of individuals, but also derives utility from resources that he extracts for himself. In particular, we will formulate a model in which we can vary the mechanism designer's degree of benevolence. With this model we will ultimately show that coercion is legitimate if and only if the mechanism designer is sufficiently benevolent.

Formally, we assume that the public good is provided by a mechanism designer whose objective is to maximize expected profits,

$$\Pi := E\left[\sum_{i=1}^{n} t_i(\theta)\right] - E[\beta k(q(\theta))] \ . \tag{6}$$

However, there is a fraction a fraction τ of profits that are redistributed to individuals in a lump sum fashion. We treat τ as a given parameter which measures the mechanism designer's degree of benevolence. As will become clear, low values of τ indicate that the mechanism designer extracts a large fraction of the surplus from public goods provision, whereas a high value of τ indicates that any surplus extracted will be transferred back to individuals. For ease of exposition, all individuals are entitled to the same fraction of profits. The per capita share of profits is therefore equal to $\frac{\tau}{n}$.

The interaction between the mechanism and the individuals who consume the public good proceeds as follows. First, prior to the operation of the mechanism, the mechanism designer makes an unconditional lump sum payment of $\frac{\tau}{n}\Pi^*$ to each individual, where Π^* are the expected profits that result from the profit-maximization problem. After this payment is made, a mechanism is chosen in order to maximize Π subject to the incentive compatibility constraints in (2), and participation constraints. Again, the participation constraints are either imposed at the ex ante stage as in (4), or at the ex interim stage as in (3).

Remark 1 *Imposing a sequential structure where expected profits are distributed prior to the operation of the mechanism has the following convenient implication: Profits do not enter the incentive compatibility constraints because the upfront transfer is not conditional on the behavior of individuals under the mechanism. In part B of the Appendix, we discuss an alternative version of the model in which profits are not redistributed ex ante (before the operation of the mechanism), but ex post; that is, after θ has been observed, each individual receives a transfer*

$$\frac{\tau}{n}\left(\sum_{i=1}^{n} t_i(\theta) - \beta k(q(\theta))\right) .$$

In part B of the Appendix, it is shown that the above sequential structure can be imposed without loss of generality: The outcome of the model with a redistribution of profits after the operation of the mechanism can be replicated by the model with an upfront transfer of expected profits prior to the operation of the mechanism.

Remark 2 *The individuals' share of expected profits does not enter the participation constraints. This may be questioned on the following grounds. The participation constraints serve to ensure that individuals are not worse off as compared to a status quo situation with no public goods provision. If their share of monopoly profits provides them with utility that they would not be able to realize in the status quo, then this should be included in the utility that they derive from public goods provision. Accordingly, the appropriate version of, say, the ex interim participation constraints would be as follows; for all i, and all θ_i,*

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) + \frac{\tau}{n}\Pi^* \geq 0. \tag{7}$$

Again, it turns out that the specification with participation constraints as in (7) can be interpreted *as a special case of the model with the participation constraints that do not include monopoly profits. This is also clarified in part B of the Appendix.*

The following Proposition compares, for an arbitrary individual i, ex ante expected utility with ex interim participation constraints, V_i^{int}, and ex ante participation constraints, V_i^{ant}. If V_i^{ant} is larger than V_i^{int} then the use of coercion is legitimate in the sense that the individual in question is, in an ex ante sense, made better off if coercion is possible.

Proposition 4 *Public goods provision is surplus-maximizing with ex ante participation constraints and distorted downwards with ex interim participation constraints. Moreover, there exists $\hat{\tau} \in (0,1)$ such that $V_i^{ant}(\tau) > V_i^{int}(\tau)$ if and only if $\tau > \hat{\tau}$.*

The Proposition shows that the legitimacy of coercion depends on the mechanism designer's degree of benevolence. If it is high, then expected payoffs with coercion are close to the symmetric constrained efficient mechanism. In this case, the imposition of ex interim participation constraints is harmful for individuals because it results in a lower level of aggregate surplus.

By contrast, for a low degree of benevolence, individuals prefer the imposition of ex interim participation constraints, even though this implies that public goods provision is inefficient. Given that the mechanism designer retains almost the whole surplus, they cannot benefit from surplus-maximizing public goods provision. The only remaining source of payoffs is therefore the information rent that individuals can reap provided that ex interim participation constraints are imposed. Hence, they prefer a larger fraction of a smaller, second-best surplus over a smaller fraction of the maximal, first best surplus.

This result can be summarized as follows. If public goods are provided in a benevolent way, the use of coercion is legitimate. A benevolent mechanism designer acts in the interests of individuals and should hence face as few constraints as possible. By contrast, if a malevolent institution is in charge of public goods provision then the use of coercion is not legitimate. A malevolent mechanism designer maximizes its own well-being at the expense of individuals. Hence, it is in the interest of individuals that he faces as many constraints as possible.

4.1 Sketch of the Proof of Proposition 4

The proof follows from the characterization of the optimal mechanism with ex interim and ex ante participation constraints, respectively, which we sketch below. A complete proof is in the Appendix.

Ex interim participation constraints. Consider first the mechanism designer's problem with ex interim participation constraints. Once upfront payments to individuals are made, the mechanism designer aims at surplus extraction. Whatever his provision rule is, he will therefore choose the payments of individuals so that expected revenues $E\left[\sum_{i=1}^n t_i(\theta)\right]$ are maximized. This problem of revenue maximization, see the discussion following Proposition 2, yields the following expression for maximal revenues as a function of the public goods provision rule q,

$$E\left[\sum_{i=1}^n t_i(\theta)\right] = E\left[\sum_{i=1}^n \left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] .$$

The mechanism designer therefore chooses the provision rule which maximizes second best profits,

$$E\left[\sum_{i=1}^n \left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta) - \beta k(q(\theta))\right] .$$

We denote this provision rule in the following by q_Π^{**}. With this second best provision rule, public goods provision is, for every θ, distorted downwards relative to the surplus-maximizing level. This follows since q_Π^{**} is characterized implicitly by the first order condition,

$$\sum_{i=1}^n \left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) = \beta k'(q_\Pi^{**}(\theta)) ,$$

whereas the surplus-maximizing provision level is given by $\sum_{i=1}^n \theta_i = \beta k'(q^*(\theta))$.

The expected payoffs of individuals, from the ex ante perspective are derived as follows: First, with ex interim participation constraints, they get an information rent which equals $E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q_\Pi^{**}(\theta)\right]$; second, they get a fraction $\frac{\tau}{n}$ of the second-best profit. After algebraic manipulations which make use of the assumption that the individuals' taste parameters are *iid* random variables, we can therefore derive the following expression for individual i's expected payoff,

$$V_i^{int}(\tau) = \tau \frac{1}{n} E\left[\sum_{i=1}^n \theta_i q_\Pi^{**}(\theta) \ - \ \beta k(q_\Pi^{**}(\theta))\right] + (1-\tau) E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q_\Pi^{**}(\theta)\right] . \tag{8}$$

Hence, an individual's expected payoff is a convex combination of the expected profit that is transferred to individuals and the information rent. In particular, the smaller τ, i.e., the smaller the fraction of profits that is transferred to individuals, the larger is the contribution of the information rent to the individuals' expected payoff.

Ex ante participation constraints. The mechanism designer now aims at surplus maximization subject to ex ante participation constraints and incentive compatibility constraints.

However, to characterize the outcome of this mechanism design problem, we may ignore the incentive constraints. As was we already pointed out in the discussion of Proposition 1, incentive compatibility constraints have no bearing on the ex ante expected utility levels. Consequently, if there is some mechanism that satisfies the budget constraint and guarantees individuals some non-negative expected utility level, then there is also another mechanism that generates these expected utility levels and is, in addition, incentive compatible.

Obviously, at a solution to the mechanism design problem, all ex ante participation constraints are binding. This implies that, for each individual i, $E[t_i(\theta)] = E[\theta_i q(\theta)]$. Upon substituting these expected payments in the mechanism designer's objective function, we find that he chooses q in order to maximize

$$E\left[\sum_{i=1}^{n} t_i(\theta) - \beta k(q(\theta))\right] = E\left[\sum_{i=1}^{n} \theta_i q(\theta) - \beta k(q(\theta))\right] ,$$

i.e., he chooses to provide public goods in a surplus-maximizing way.

Given that, with ex ante participation constraints, individuals are unable to reap an information rent, their expected payoff from the ex ante perspective consists entirely of their share of the first surplus which is given by

$$V_i^{ant}(\tau) = \tau \frac{1}{n} E\left[\sum_{i=1}^{n} \theta_i q^*(\theta) \ - \ \beta k(q^*(\theta))\right] . \tag{9}$$

Note that, whatever τ, the mechanism designer chooses to provide public goods according to q^*, i.e., public goods provision is surplus-maximizing. Hence, the parameter τ only affects the distribution of the surplus between the mechanism designer and the individuals, but has no bearing on the public goods provision rule. In particular, if τ is close to zero, the expected payoff of individuals is close to zero.

5 Endogenous Conflicts of Interest

In the previous section we did not treat the mechanism designer's degree of benevolence τ as a choice variable, but rather as a parameter that makes it possible to define formally what benevolence and malevolence mean. Obviously, if there was a choice of τ, then a complete redistribution of profits would be the ideal outcome. There is nothing better than the use of coercion in conjunction with a benevolent mechanism designer. But this raises the question whether we can find a more convincing case for participation constraints. The analysis so far has only shown that the imposition of participation constraint is justified if, for some reason, the ideal outcome is out of reach.

This issue is taken up in the following. The aim is to develop a more microfounded model of public goods provision that is empirically plausible and, moreover, endogenously creates a distributive conflict that justifies the imposition of strong participation constraints.

More specifically, we study an extended model in which the public good is produced by a firm with private information about its cost function. As a benchmark, we first study the implications of this extension for a model of mechanism design. We will see that if there is a mechanism designer who engages in complete contingent planning of public goods production and individual payments, then we obtain the same results as in the benchmark model with a commonly known technology: Coercion is legitimate if and only if the mechanism designer is sufficiently benevolent.

We will then contrast this result with the following incomplete contracts model: a regulator delegates public goods production to a profit-maximizing firm, possibly in exchange for an upfront payment that the firm has to make and which is then transferred to individuals. This interaction is incomplete, relative to the mechanism design benchmark, because the regulator remains ignorant with respect to the individuals' preferences. The fine-tuning of public goods supply and of individual payments to the state of demand lies in the hands of a profit-maximizing firm. We show that the firm may be able to extract the whole surplus from public goods provision, and that this may imply that strong participation constraints should be imposed.

5.1 Mechanism Design

We assume that the firm producing the public good has private information on the parameter β in the cost function. From an ex ante perspective, the cost parameter β is a random variable that takes values in a finite ordered set $\{\beta^1, \ldots, \beta^r\}$, with $\beta^1 = r$, $\beta^2 = r-1$, ..., $\beta^r = 1$. Hence, a firm with cost parameter β^1 has the worst technology. The probability that β equals β^j is in the following denoted by $f(\beta^j) = f^j$. We assume that the random variable β is stochastically independent of the vector of taste parameters θ. We impose another monotone hazard rate assumption: for any $j \geq 1$, $g^{j+1} \leq g^j$, where, for any j, $g^j = \frac{1-F(\beta^j)}{f(\beta^j)}$.

A mechanism in this extended model consists of a provision rule for the public good $q : (\theta, \beta) \mapsto q(\theta, \beta)$, which specifies how much of the public good is provided as a function of the vector of taste parameters θ and the firm's cost parameter β, and a payment rule which specifies for each individual i, a contribution to the cost of public good provision $t_i : (\theta, \beta) \mapsto t_i(\theta, \beta)$.

The revelation principle implies that it entails no loss of generality to assume that individuals send messages about their taste parameters and that the firm sends a message about its cost parameter, and that truth-telling constitutes a Bayes-Nash equilibrium. The incentive compatibility conditions for individual i, are still given by the inequalities in (2), except that $T_i(\hat{\theta}_i)$ and $Q_i(\hat{\theta}_i)$ now also involve expectations about the firm's cost parameter; i.e., $T_i(\hat{\theta}_i) := E[t_i(\theta_{-i}, \hat{\theta}_i, \beta) \mid \hat{\theta}_i]$, and $Q_i(\hat{\theta}_i) := E[q(\theta_{-i}, \hat{\theta}_i, \beta) \mid \hat{\theta}_i]$. The incentive compatibility constraints for the firm are as follows: For all β, and all $\hat{\beta}$,

$$R(\beta) - \beta K(\beta) \geq R(\hat{\beta}) - \beta K(\hat{\beta}) \ , \tag{10}$$

where $R(\hat{\beta}) := E\left[\sum_{i=1}^{n} t_i(\theta, \hat{\beta}) \mid \hat{\beta}\right]$ is the firm's expected revenue conditional on announcing a cost parameter $\hat{\beta}$, and $K(\hat{\beta}) := E[k(q(\theta, \hat{\beta})) \mid \hat{\beta}]$. These constraints are based on the assumption that the firm is able to manipulate the production costs that are observable to outsiders. If it claims a cost parameter $\hat{\beta}$ and has a true cost parameter β, then its expected production costs are equal to $\beta K(\hat{\beta})$. These production costs differ from those of a firm whose true cost parameter equals $\hat{\beta}$. This difference, however, is assumed to be unobservable to anyone who is an outsider to the firm.

We require that budget balance holds for each type of firm. Hence, for all β,

$$R(\beta) - \beta K(\beta) \geq 0 \ . \tag{11}$$

To sum up, in the extended model with private information on preferences and production costs, any mechanism has to satisfy the budget balance conditions in (11), the individual incentive compatibility conditions in (2), and the firm's incentive compatibility conditions in (11). In addition to these constraints, we may impose participation constraints for individuals in the ex ante or the ex interim sense.

Proposition 5 *A mechanism is constrained efficient if and only if*

$$E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] = E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right] \ ,$$

and the public goods provision rule maximizes

$$E\left[\left(\sum_{i=1}^{n} \theta_i\right) q(\theta,\beta) \ - \ \left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right] \ .$$

This Proposition shows that private information on production costs affects the cost function on which the analysis of constrained efficient mechanisms is based.[15] In the basic model with a commonly known technology β, the expected costs of public good provision are given by $E[\beta k(q(\theta))]$. In the extended model, conditional on β, there is an additional cost which equals $E\left[\frac{1-F(\beta)}{f(\beta)}\ k(q(\theta,\beta)) \mid \beta\right]$. This additional costs are due to the information rents that the firm is able to realize.

The proof of Proposition 5, which is in the Appendix, follows from similar arguments as the proof of Proposition 2. A necessary condition for constrained efficiency is that, for a given public goods provision rule, the expected payments of individuals are chosen such that $E[\sum_{i=1}^{n} t_i(\theta,\beta)]$ is minimized subject to the firm's budget balance conditions in (11) and the firm's incentive compatibility conditions in (10). Moreover, by analyzing a relaxed problem which minimizes

[15]For the extended model, we use the same definition of constrained efficiency as in the basic model in which β was assumed to be known, i.e., we follow Baron and Myerson (1982) in that only the expected payoffs of individuals are of relevance and there is no weight given to rents that the firm may extract.

$E[\sum_{i=1}^{n} t_i(\theta, \beta)]$ taking only the budget balance condition for the least efficient firms, i.e., those with $\beta = \beta^1$, and the local downward incentive compatibility conditions,

$$R(\beta^l) - \beta^l K(\beta^l) \geq R(\beta^{l-1}) - \beta^l K(\beta^{l-1}) \ .$$

into account, we find that $E[\sum_{i=1}^{n} t_i(\theta, \beta)]$ is bounded from below by $E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) \ k(q(\theta, \beta))\right]$. Finally, we show that this lower bound can actually be obtained because the provision rule which solves the relaxed problem satisfies the monotonicity constraint $K(\beta^l) \geq K(\beta^{l-1})$, for all l.

Upon replacing the cost function $E[\beta k(q(\theta))]$ of the basic model by the "virtual" cost function $E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) \ k(q(\theta, \beta))\right]$ which takes account of the firm's information rent we can not only reproduce Proposition 1, but all further results of Sections 3 and 4 in the extended model. In particular, we obtain, once more the conclusions that (i) ex interim participation constraints can only do harm to individuals if the mechanism designer is benevolent, and (ii) upon reducing the mechanism designer's degree of benevolence, we eventually obtain a situation in which the imposition of strong participation constraints is desirable.

In the following, we will show that a case for ex interim participation constraints arises endogenously with an incomplete contracts perspective on public goods provision.

5.2 Incomplete Contracts

We now drop the assumption that there is a mechanism designer who specifies public goods production and individual payments as a function of the firm's technology and the individuals' preferences. Instead we assume that there is a regulator who delegates the task of adjusting the final allocation to the preference intensities of individuals to a profit-maximizing firm, in exchange for an upfront payment that the firm has to deliver. We assume that the regulator is benevolent and redistributes this payment to individuals.

The regulator differs from the mechanism designer in that he remains ignorant with respect to the preferences of individuals and also with respect to the firm's operating profits; i.e., after the firm has made its upfront payment it becomes the residual claimant and is no longer monitored by the regulator. This arrangement is incomplete in the sense that the interaction between the regulator and the firm is not made contingent on the public goods preferences of individuals.

In the basic model with a commonly known technology, this regulatory approach would make it possible to reach efficient public goods provision. Given that the firm's expected profits are known, the regulator can just require an upfront payment that equals the expected surplus from efficient public goods provision. This would imply that individuals realize the same expected payoff as under the symmetric constrained efficient mechanism. Hence, they would legitimize coercion at the constitutional ex ante stage. We will see, in the following, that this reasoning breaks down if the firm has private information about its technology.

The delegation of public goods provision to a self-interested firm is also of interest because it is a common practice in reality. Public transport is an example for which the question whether the use of coercion is legitimate or not can be framed as follows: Should public transportation be organized in such a way that the revenue from the tickets that are sold to customers cover all of the costs, or is there a role for lump sum payments, i.e., for payments that individuals have to deliver irrespective of whether or not they make use of public transportation. In the latter case, there might be people who pay for public transportation, even though they never use it, and who would hence be better off if there was no public transportation system at all.

Formally, we consider the following sequential structure. First, the firm learns its technology and individuals learn their preferences. Then, the regulator asks the firm for an upfront payment that may possibly depend on the firm's technology. Finally, the firm interacts with individuals according to a profit-maximizing mechanism that has to satisfy incentive compatibility and participation constraints. Once more, we focus on the question whether, from an ex ante perspective, individuals prefer the imposition of strong, ex interim participation constraints over weak, ex ante participation constraints.

We assume, without loss of generality that the interaction between the firm and regulator is based on a direct revelation mechanism such that the firm reports its cost parameter to the regulator, and has to deliver a payment $S^c(\hat{\beta})$ that is made contingent on its report. The index $c \in \{ant, int\}$ refers to the kind of participation constraint that has to be respected. We denote the firm's profits by $\Pi^c(\beta)$.

An implication of the assumption that the regulator does not observe the firm's profits, production costs, etc. is that the firm's upfront payment can not be made contingent on the firm's cost parameter. To see this, consider two types of firms β and β' who are both supposed to produce the public good. Incentive compatibility requires that

$$\Pi^c(\beta) - S^c(\beta) \geq \Pi^c(\beta) - S^c(\beta') \quad \text{and} \quad \Pi^c(\beta') - S^c(\beta') \geq \Pi^c(\beta') - S^c(\beta) \; .$$

These inequalities imply that $S^c(\beta) = S^c(\beta')$. In words, if there is a firm of who is supposed to produce the public good and the firm knows that, if it was less productive, it would still become the producer of the public good, then it will reveal its high productivity level only if its payment is not higher than the the one it had to pay if it was less productive. In the following we can therefore drop the dependence of the upfront payment on the firm's type, and interpret S^c as an entry fee that each firm who wants to become the producer of the public good has to pay.

We assume in the following that the upfront payment is set in such a way that each type of firm participates, i.e., S^c is such that even the firm with the worst technology and therefore the lowest profits is willing to participate,

$$\Pi^c(\beta^1) - S^c \geq 0 \; .$$

In principle, the regulator could require an upfront payment that exceeds $\Pi^c(\beta^1)$. This would imply that a firm with a bad technology would refrain from becoming the provider of the public good, so that there would be no public goods provision. From an ex post perspective, this would be an inefficient outcome because even if β is very high, the marginal cost of public goods provision converges to zero as the provision level converges to zero. Hence, even with a bad technology, it would be desirable to have a strictly positive public goods supply. However, the advantage of choosing S^c strictly larger than $\Pi^c(\beta^1)$ is that it makes it possible to extract a larger fraction of the profits of firms with a better technology. In general, this tradeoff between efficiency considerations on the one hand and the desirability of rent extraction on the other, need not be such that it is optimal to have a positive supply of public goods with each type of firm. However, if the probability f^1 of facing a firm with a bad technology is sufficiently high, then the scope for rent extraction is limited anyway and the regulator will choose the upfront payment such that $S^c \leq \Pi^c(\beta^1)$.

Remark 3 *An alternative foundation for this assumption would be that the regulator faces a commitment problem. The intuition is as follows: Suppose that the regulator sets an upfront payment that is so high that not all types of the firm are willing to enter. Also, suppose that the regulator finds himself in the situation that indeed entry has not occurred. Then he knows that if he lowers the entry fee, eventually the firm will be ready to enter and that this will create a strictly positive surplus. Lack of commitment means that the regulator is unable to resist this temptation. Anticipating this behavior, the firm will be willing to produce the public good only if the payment has been reduced to a level such that each type of firm would be willing to participate. It is beyond the scope of this paper to provide a rigorous game-theoretic analysis of the relationship between the regulator and the firm in the absence of commitment. In the context of a buyer-seller relationship, these considerations have been formalized by Hart and Tirole (1988); see also Bolton and Dewatripont (2005).*

For the purposes of this paper, the assumption $S^c \leq \Pi^c(\beta^1)$, is made for ease of exposition. As will become clear, it is a sufficient condition that makes it possible to show that, with an incomplete contracts perspective, we may find a role for strong participation constraints.

Proposition 6 *For sufficiently large β^1, the use of coercion is not legitimate, and public goods provision should be distorted downwards.*

If β^1 is large this implies that the least productive firm makes hardly any profit. Given that the profits of the least productive firm provide an upper bound on the entry fee that the regulator charges, this implies that the entry fee converges to zero as β^1 goes out of bounds. Consequently, all other firms can keep the whole profit from public goods provision for themselves. Given this observation, the arguments from the previous section imply that coercion is not legitimate and that the firm should face strong, ex interim participation constraints.

Hence, the analysis of the incomplete contracts model can be summarized as follows: If public goods provision is delegated to a profit-maximizing firm, and moreover, the firm may credibly claim that its maximal profit is close to zero, then the use of coercion for public goods finance is not legitimate; i.e., the firm must not be given access to external funds in order to cover its production costs.

5.3 Proof of Proposition 6

Ex interim participation constraints. We first consider expected payoffs of the firm and of individuals if ex interim participation constraints have to be respected. A straightforward extension of the analysis in Section 4 implies that the expected net profits (taking the upfront payment into account) of a firm of type β^l are given by

$$\Pi^{*int}(\beta^l) = \left(\max_{q \in \mathbf{Q}} E\left[\sum_{i=1}^{n} \left(\theta_i - \frac{1 - P(\theta_i)}{p(\theta_i)} \right) q(\theta) - \beta k(q(\theta)) \mid \beta = \beta^l \right] \right) - S^{int} \,, \tag{12}$$

where $\mathbf{Q}$ is the set of functions from Θ^n to $\mathbb{R}_+$. An individual's expected utility at the ex ante stage can then be written as $U_i^{int} := \frac{1}{n} S^{int} + E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q^{**}_{\Pi e}(\theta, \beta) \right]$, where $q^{**}_{\Pi e}(\theta, \beta)$ is the level of public goods provision that results from the profit maximization problem in (12), if the vector of taste parameters equals θ and the cost parameter equals β.

As in Section 4, the imposition of ex interim participation constraints implies that individuals get an information rent. This information rent reduces the revenues that a profit-maximizing firm can realize. As a consequence, the firm chooses a second-best public goods provision level which falls short of the surplus-maximizing first best level. The information rent also enters the expected payoff of individuals. In addition, each individual gets an equal share of the upfront payment that the firm has to deliver.

Ex ante participation constraints. We can also extend the analysis of Section 4 to determine expected profits and payoffs if ex ante participation constraints are imposed. A type β^l firm then realized expected payoffs of

$$\Pi^{*ant}(\beta^l) = \left(\max_{q \in \mathbf{Q}} E\left[\left(\sum_{i=1}^{n} \theta_i \right) q(\theta) - \beta k(q(\theta)) \mid \beta = \beta^l \right] \right) - S^{ant} \,, \tag{13}$$

where $S^{ant} \leq \Pi^{*ant}(\beta^1)$ is the upfront payment in a model with ex ante participation constraints. The expected utility of individual i is equal to $U_i^{ant} := \frac{1}{n} S^{ant}$.

With the weaker ex ante participation constraints, the firm is able to extract higher payments from individuals. Individuals no longer get an information rent, which implies that the firm chooses a surplus-maximizing public goods provision level $q_e^*(\theta,\beta)$. The expected payoff of individuals is entirely due to the firm's upfront payment.

Legitimacy of coercion. The use of coercion for public goods finance is legitimate if the expected payoff of individuals is higher with participation constraints at the ex ante stage, or equivalently, if

$$\frac{1}{n}S^{ant} > \frac{1}{n}S^{int} + E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q^{**}_{\Pi e}(\theta,\beta)\right] . \tag{14}$$

We seek to show that this condition is violated for sufficiently large β^1. This follows from the observation that both $\Pi^{*ant}(\beta^1)$ and $\Pi^{*int}(\beta^1)$ converge to 0 as β^1 goes to ∞. To see this, note that the first order conditions of the profit maximization problems (12) and (13), respectively, are

$$\frac{\sum_{i=1}^n \theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}}{\beta} = k'(q^{**}_{\Pi e}(\theta,\beta)) \quad \text{and} \quad \frac{\sum_{i=1}^n \theta_i}{\beta} = k'(q_e^*(\theta,\beta)) \; .$$

Hence, as β goes out of bounds, optimal quantities and expected profits go to zero. This implies that for large β^1, condition (14) reduces to

$$0 > E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q^{**}_{\Pi e}(\theta,\beta)\right] ,$$

a contradiction. ■

6 Concluding Remarks

The analysis has provided an answer to the question whether the financing of public goods should be subject to participation constraints. The advantage of a system based on participation constraints is that all individuals benefit from public goods provision. The disadvantage, however, is that public goods finance is generally insufficient to induce efficient outcomes. Which of these two forces is dominating depends on whether or not there are pronounced agency problems between individuals and the institution in charge of public goods provision. If the latter acts in the individuals' best interests, then the imposition on participation constraint is not attractive. By contrast, if it seeks to maximize his own payoff at the expense of individuals, then participation constraints should be imposed. Finally, we have shown that, under certain assumptions, if the production of public goods is delegated to a monopolistic firm with private information about its production costs, the latter case applies; that is, a regulated monopolist should not be given access to public funds.

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A Appendix

A.1 Proof of Proposition 1

Only if - part. We show that for every constrained efficient mechanism the budget constraint is binding and the provision rule is surplus-maximizing.

Without loss of generality of we can characterize a Parto-efficient mechanism as the solution of the following optimization problem: Choose a mechanism in order to maximize $E[\theta_1 q(\theta) - t_1(\theta)]$ subject to the incentive compatibility constraints in (2), the budget constraint in (1) and the following set of reservation utility constraints: For each $i \neq 1$,

$$E[\theta_i q(\theta) - t_i(\theta)] \geq \bar{u}_i \ , \tag{15}$$

for some given vector of reservation utility levels $(\bar{u}_2, \ldots, \bar{u}_n)$.

Consider a relaxed problem which does not include the incentive compatibility constraints. As is well-known in the literature the solution to this relaxed problem is such that the budget constraint in (1) and the constraints in (15) are binding. Moreover, public goods provision has to be surplus-maximizing. In the following we show that this solution can also be achieved subject to incentive compatibility constraints.

The surplus-maximizing provision rule satisfies $q(\theta_{-i}, \theta^l) < q(\theta_{-i}, \theta^{l+1})$, for all i and l. This implies that for all i, and all l, the monotonicity constrained $Q_i(\theta^{l+1}) > Q_i(\theta^l)$, is satisfied.

Given this provision rule, if we choose the expected payments of each individual i in such a way that all local downward incentive compatibility constraints are binding,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) = \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}),$$

then Lemma 2 in part B of the Appendix implies that the resulting allocation is incentive compatible, and Lemma 3 implies that the expected utility of individual i is given by

$$E[\theta_i q(\theta) - t_i(\theta)] = E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] - T_i(\theta^0) \; .$$

Consequently, if we choose for each $i \neq 1$, $T_i(\theta^0)$ such that

$$E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] - T_i(\theta^0) = \bar{u}_i$$

and for all $l > 0$,

$$T_i(\theta^l) = \theta^l(Q_i(\theta^l) - Q_i(\theta^{l-1})) + T_i(\theta^{l-1}) \; ,$$

then the resulting allocation is incentive compatible and yields for each individual $i \neq 1$, the same expected utility as the solution of the relaxed problem. We proceed in a similar way with individual 1, except that the utility level for individual 1, u_1^*, follows from the solution of the relaxed problem and is given by

$$u_1^* = E\left[\sum_{i=1}^{n} \theta_i q^*(\theta) - \beta k(q^*(\theta))\right] - \sum_{i=2}^{n} \bar{u}_i \; . \tag{16}$$

It remains to be shown that the payments are such that the budget constraint holds as an equality. By construction, $E[t_i(\theta)] = E[\theta_i q^*(\theta)] - \bar{u}_i$, for $i \neq 1$, and $E[t_1(\theta)] = E[\theta_1 q^*(\theta)] - u_1^*$. This implies that

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right] = E\left[\sum_{i=1}^{n} \theta_i q^*(\theta)\right] - \left(u_1^* + \sum_{i=2}^{n} \bar{u}_i\right) \stackrel{(16)}{=} E\left[\beta k(q^*(\theta))\right] \; .$$

If-Part. We now show that every incentive compatible mechanism such that the provision rule is surplus-maximizing and the budget constraint holds as an equality is constrained efficient.

The proof proceeds by contradiction. Suppose that $(q, t_1, \ldots, t_n)$ is an incentive compatible mechanism such that q is surplus-maximizing and the budget constraint holds as an equality. Suppose there exists an incentive compatible mechanism $(q', t_1', \ldots, t_n')$ such that, for all i,

$$E[\theta_i q'(\theta) - t_i'(\theta)] \geq E[\theta_i q(\theta) - t_i(\theta)] \; ,$$

with a strict inequality for some i. This implies that

$$E\left[\sum_{i=1}^{n}(\theta_i q'(\theta) - t_i'(\theta))\right] > E\left[\sum_{i=1}^{n}(\theta_i q(\theta) - t_i(\theta))\right] \; ,$$

Using that $E\left[\sum_{i=1}^{n} t_i(\theta))\right] = E[\beta k(q(\theta))]$, and that $E\left[\sum_{i=1}^{n} t_i'(\theta))\right] \geq E[\beta k(q'(\theta))]$, this implies that

$$E\left[\sum_{i=1}^{n} \theta_i q'(\theta) - \beta k(q'(\theta))\right] > E\left[\sum_{i=1}^{n} \theta_i q(\theta) - \beta k(q(\theta))\right] \; ,$$

hence a contradiction to the assumption that q is a surplus-maximizing provision rule. ■

A.2 Proof of Proposition 2

Only if - part. By Proposition 1, under every constrained efficient mechanism the provision rule is equal to q^*. Given this provision rule, Lemma 7 in part B of the Appendix implies that the maximal revenue that is possible in the presence of ex interim participation constraints equals

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right] .$$

If this is smaller than $E[\beta k(q^*(\theta))]$, budget balance can not be achieved. Hence, constrained efficiency can not be achieved.

If - part. We need to show that if

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right] \geq E[\beta k(q^*(\theta))] .$$

then, given the surplus-maximizing provision rule q^*, $(t_1, \ldots, t_n)$ can be chosen such that the budget constraint binds and that for all i, the incentive compatibility constraints and the ex interim participation constraints are satisfied.

By the arguments in the proof of Lemma 7, the participation constraints of individual i are satisfied if and only if $T_i(\theta^0) \leq 0$. Since the provision rule q^* implies that the monotonicity constraints $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$ are satisfied for all i and l, Lemmas 2 and 3 in part B of the Appendix imply that incentive compatibility holds if expected payments are chosen such that all local downward incentive compatibility constraints are binding and that the expected payments of individual i are in this case equal to

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + T_i(\theta^0) .$$

Now choose

$$T_i(\theta^0) = \frac{1}{n}\left(E[\beta k(q^*(\theta))] - E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right]\right) ,$$

for all i. By assumption this is smaller or equal to zero, so that the ex interim participation constraints are satisfied, for all i. It remains to be shown that budget balance holds. This follows since, by construction,

$$\begin{aligned} E\left[\sum_{i=1}^{n} t_i(\theta)\right] &= E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right] \\ &\quad + \left(E[\beta k(q^*(\theta))] - E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right]\right) . \end{aligned}$$

■

A.3 Proof of Proposition 3

Step 1. At a solution to the second best problem, the budget constraint has to be binding. Otherwise it would be possible to reduce the expected payments of individuals without violating any of the incentive compatibility or participation constraints, and without violating the budget constraint. Hence, at a solution to the second best problem

$$E\left[\sum_{i=1}(\theta_i q(\theta) - t_i(\theta))\right] = E\left[\left(\sum_{i=1}\theta_i\right) q(\theta) - \beta k(q(\theta))\right] .$$

Step 2. The expected revenue $E[\sum_{i=1}^{n} t_i(\theta)]$ at a solution of the second best problem satisfies

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] \geq E[\sum_{i=1}^{n} t_i(\theta)] \ .$$

This follows from the arguments in the proof of Lemma 7 in Appendix B , which imply, that if, for a given provision rule q, $E[\sum_{i=1}^{n} t_i(\theta)]$ is maximized taking only a subset of the constraints of the second best problem – namely the ex interim participation constraints and the local downward incentive compatibility constraints – into account, then the maximal revenue equals $E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right]$. This expression is therefore an upper bound on the expected payments of individuals. Combining this observation and the budget constraint yields

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] \geq E[\beta k(q(\theta))] \ . \tag{17}$$

Step 3. Steps 1 and 2 imply that the surplus that is generated at a solution of the auxiliary problem to maximize $E\left[\left(\sum_{i=1} \theta_i\right) q(\theta) - \beta k(q(\theta))\right]$ subject to the constraint in (17) is an upper bound on the second best surplus. Moreover, it is straightforward to verify that (17) is binding if and only if

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q^*(\theta)\right] < E[\beta k(q^*(\theta))] \ .$$

Step 4. Suppose that (17) is not binding. Then provision rule q^* and the payments in the mechanism in the proof of the if-part of Proposition 2 solve the auxiliary problem. Moreover, with this mechanism the surplus of public goods provision is shared equally among individuals, i.e., for all i,

$$E[\theta_i q(\theta) - t_i(\theta)] = \frac{1}{n} \, E\left[\sum_{i=1}^{n} \theta_i q^*(\theta) - \beta k(q^*(\theta))\right] \ .$$

This proves the first statement in Proposition 3.

Step 5. Now suppose that (17) is binding. The provision rule that solves the auxiliary problem satisfies the monotonicity constraint, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all i, and l. This follows because the optimal level of $q(\theta)$ is either equal to zero or given by the first order condition,

$$\beta k'(q(\theta)) = \sum_{i=1}^{n} \theta_i - \frac{\lambda}{1+\lambda} \sum_{i=1}^{n} \frac{1-P(\theta^i)}{p(\theta_i)}$$

where λ is the multiplier on the constraint. The assumption that the hazard rate is decreasing implies that whenever, for one individual the taste parameter θ^l is replaced by the taste parameter θ^{l+1}, the right hand side goes up, which implies that $q(\theta_{-i}, \theta^l) \leq q(\theta_{-i}, \theta^{l+1})$, for all i, θ_{-i}, and l. This implies, in particular, that $Q_i(\theta^l) \leq Q_i(\theta^{l-1})$, for all i, and l. This follows from Lemma 7 in Appendix B, this implies that the the surplus that is generated by the auxiliary problem can be achieved by a second best mechanism. Moreover, the arguments in the proof of this Lemma show that this requires that for all i, the ex interim Participation constraint $T_i(\theta^0) \leq 0$ and all local downward incentive compatibility constraints are binding. Lemma 3 in Appendix B implies that, at a solution to the second best problem, for all i, ex ante expected utility is equal to

$$E[\theta_i q(\theta) - t_i(\theta)] = E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q^{**}(\theta)\right] \tag{18}$$

where $q^{**}(\theta)$ is the provision rule that solves the auxiliary problem. Given that the constraint of the auxiliary problem is binding we have

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q^{**}(\theta)\right] = E[\beta k(q^{**}(\theta))] \ ,$$

or, equivalently,

$$E\left[\sum_{i=1}^{n}\frac{1-P(\theta_i)}{p(\theta_i)}q^{**}(\theta)\right] = E\left[\left(\sum_{i=1}^{n}\theta_i\right)q^{**}(\theta) - \beta k(q^{**}(\theta))\right]$$

Using that the random variables $(\theta_i)_{i=1}^n$ are independently and identically distributed, this implies that

$$E\left[\frac{1-P(\theta_i)}{p(\theta_i)}q^{**}(\theta)\right] = \frac{1}{n}\ E\left[\left(\sum_{i=1}^{n}\theta_i\right)q^{**}(\theta) - \beta k(q^{**}(\theta))\right] \tag{19}$$

Equations (18) and (19) imply that at a solution to the second best problem,

$$E[\theta_i q(\theta) - t_i(\theta)] \quad = \quad \frac{1}{n}\ E\left[\left(\sum_{i=1}^{n}\theta_i\right)q^{**}(\theta) - \beta k(q^{**}(\theta))\right] \ .$$

By definition of the surplus-maximizing provision rule q^*, $q^{**} \neq q^*$ implies that this is less than the ex ante expected payoff under the symmetric constraint efficient mechanism. ■

A.4 Proof of Proposition 4

The proof uses arguments from part B of the Appendix, in particular Lemmas 1 - 7, which provide a characterization of incentive compatible mechanisms and of revenue maximizing mechanisms.

The optimal mechanism with ex interim participation constraints. It follows from Lemma 5, that, at a solution to the mechanism design problem, for all i, the participation constraints in (3) are binding for $\theta_i = \theta^0$ and is slack otherwise. Otherwise it would be possible to increase the monopolist's revenue while holding the monopolist's provision rule fixed.

Consider the relaxed problem of choosing $(q, t_1, \ldots, t_n)$ in order to maximize Π subject to the downward incentive compatibility constraints in (24) and the ex interim participation constraints in (3). It follows from Lemma 6 that at a solution to this problem, all downward incentive compatibility constraints are binding, and the participation constraint in (3) is binding for $\theta_i = \theta^0$ and is slack otherwise. Otherwise it would be possible to increase the monopolist's revenue while holding the monopolist's provision rule fixed. But then the arguments in the proof of Lemma 7 imply that the expected revenue of the monopolist, at a solution to the relaxed problem, is equal to $E\left[\sum_{i=1}^n t_i(\theta)\right] = E\left[\sum_{i=1}^n \left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta)\right]$. Hence, the provision rule which is part of the solution of the relaxed problem maximizes

$$\Pi = E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta) - \beta k(q(\theta))\right] \ .$$

In the following this provision rule is denoted by q_{Π}^{**}.

The relaxed problem takes only a subset of all incentive compatibility constraints into account. Hence, the expected profits that are generated by the mechanism which solves the relaxed problem are an upper bound on the expected profits that are generated by the mechanism that solves the "full" problem of maximizing Π subject to all participation and incentive compatibility constraints.

It follows from Lemma 2 that if the provision rule q^{**}_{Π} is such that the monotonicity constraints $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$ are satisfied for all i and l, then the solution to the relaxed problem satisfies all incentive compatibility constraints and is hence also a solution to the full problem.

In the remainder we verify that under q^{**}_{Π} the monotonicity constraints are indeed satisfied. For every given θ, $q^{**}_{\Pi}(\theta)$ is either given by the first order condition,

$$\sum_{i=1}^{n} \left(\theta_i - \frac{1 - P(\theta_i)}{p(\theta_i)} \right) = \beta k'(q^{**}_{\Pi}(\theta)) \, , \tag{20}$$

or, if this equation has only a negative solution, equal to 0. Given that $h^l < h^{l-1}$, for all l, for each i, the left-hand side of the first order condition is strictly increasing in θ_i. Given that k is increasing and convex, this implies that, for each i and each l, $q^{**}_{\Pi}(\theta_{-i}, \theta^l) < q^{**}_{\Pi}(\theta_{-i}, \theta^{l+1})$, and, as a consequence, the monotonicity constraint $Q_i(\theta^{l+1}) > Q_i(\theta^l)$ holds for all i, and all l.

Given that all local downward incentive compatibility constraints and the participation constraints for $\theta_i = \theta^0$ are binding for all i (so that $T_i(\theta^0) = 0$), it follows from Lemma 3 that $E[\theta_i q(\theta) - t_i(\theta)] + \frac{\tau}{n}\Pi^*$ is equal to

$$E\left[\frac{1 - P(\theta_i)}{p(\theta_i)} q^{**}(\theta) \right] + \frac{\tau}{n} E\left[\sum_{i=1}^{n} \left(\theta_i - \frac{1 - P(\theta_i)}{p(\theta_i)} \right) q^{**}(\theta) - \beta k(q^{**}(\theta)) \right]$$

Exploiting that the random variables $(\theta_i)_{i=1}^n$ are *iid* and rearranging term yields the expression for expected payoffs of individuals in equation (8).

The optimal mechanism with ex ante participation constraints We first consider a relaxed problem of maximizing Π taking only the ex ante participation constraints in (4) into account. Obviously, this implies that, for each i, the participation constraint has to be binding, for each i, $E[t_i(\theta)] = E[\theta_i q(\theta)]$. Using this expression to substitute for $E[t_i(\theta)]$ in the definition of Π yields $\Pi = E\left[\left(\sum_{i=1}^n \theta_i \right) q(\theta) - \beta k(q(\theta)) \right]$. Hence, q^* is the profit-maximizing provision rule.

In the following we show that the there is a mechanism which is payoff equivalent to the solution of this relaxed problem and satisfies all incentive compatibility constraints.

The surplus-maximizing provision rule q^* is such that for each i, $Q_i(\theta^l) \leq Q_i(\theta^{l+1})$. If all local downward incentive compatibility constraints hold as an equality, then all incentive compatibility constraints are satisfied. This follows from Lemma 2. To complete the proof it suffices show that, given public goods provision according to q^*, there is a payoff equivalent mechanism which is such that all local downward incentive compatibility constraints hold as an equality.

If all local downward incentive constraints are binding, then the arguments in the proof of Lemma 2 imply that $T_i(\theta^l) = \theta^l Q_i(\theta^l) - \sum_{k=0}^{l-1} Q_i(\theta^k) + T_i(\theta^0)$, for all l, and hence

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1 - P(\theta_i)}{p(\theta_i)} \right) q^*(\theta) \right] + T_i(\theta^0) \, . \tag{21}$$

Now if we let, in addition, $T_i(\theta^0) = E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q^*(\theta) \right]$, then also the ex ante participation constraints of all individuals are binding. This also implies that for each individual $E[\theta_i q^*(\theta) - t_i(\theta)] = 0$ and expected profits are equal to $\Pi^* = E\left[\left(\sum_{i=1}^n \theta_i \right) q^*(\theta) - \beta k(q^*(\theta)) \right]$. The ex ante expected payoff of individuals consists entirely of the fraction of profits that they receive. This observation yields the expression for expected utility in equation (9).

Comparison of expected utilities. Using equations (9) and (8) we find that $V_i^{ant}(\tau) - V_i^{int}(\tau)$ is equal to

$$\tau \frac{1}{n}\left(E\left[\left(\sum_{i=1}^{n}\theta_i\right) q^*(\theta) - \beta k(q^*(\theta))\right] - E\left[\left(\sum_{i=1}^{n}\theta_i\right) q_{\Pi}^{**}(\theta) - \beta k(q_{\Pi}^{**}(\theta))\right]\right)$$
$$-(1-\tau)E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta)\right]$$

Since q^* maximizes the surplus from public goods provision, this expression is increasing in τ. Moreover, it is negative for τ close to zero and positive for τ close to 1. ■

A.5 Proof of Proposition 5

Step 1. We show that, for any mechanism satisfying the the budget balance conditions in (11) and the firm's incentive compatibility conditions in (10), $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] \geq E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right]$.

Let q be an arbitrary given provision rule and consider the relaxed problem of minimizing $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$ subject to the budget balance condition for $\beta = \beta^1$ and the local downward incentive compatibility conditions for the firm, $R(\beta^l) - \beta^l K(\beta^l) \geq R(\beta^{l-1}) - \beta^l K(\beta^{l-1})$, for all l. Since this minimization problem takes only a subset of all budget balance conditions and all firm incentive compatibility conditions into account, the solution of this minimization problem will be a lower bound to the minimal value of $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$ that can be obtained if all budget and incentive constraints are taken into account.

At a solution to the relaxed problem all constraints have to be binding. Otherwise it was possible to reduce the expected revenues for some type of firm without violating any of the constraints of the relaxed problem, thereby attaining a lower value of $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right]$. This makes it possible to verify that $R(\beta^l) = \beta^l K(\beta^l) + \sum_{j=1}^{l-1} K(\beta^j)$, for $l \in \{1,\ldots,r-1\}$, and that $R(\beta^1) = \beta^1 K(\beta^1)$. Using the law of iterated expectations, we obtain

$$\begin{aligned}
E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] &= \sum_{l=1}^{r} f^l R(\beta^l) = E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=2}^{r} f^l \sum_{j=1}^{l-1} K(\beta^j) \\
&= E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=1}^{r}(1-F(\beta^l))K(\beta^l) \\
&= E\left[\beta k(q(\theta,\beta))\right] + \sum_{l=1}^{r} f^l \frac{1-F(\beta^l)}{f(\beta^l)} K(\beta^l) \\
&= E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right].
\end{aligned}$$

Step 2. Suppose public goods provision is such that the following monotonicity constraint holds: For all l, $K(\beta^l) \geq K(\beta^{l-1})$. We show that, under this assumption, there is a mechanism such that $E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] = E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right]$, satisfying the budget balance conditions in (11) and the firm's incentive compatibility conditions in (10),

Using arguments that are analogous to those in the proof of Lemma 4 in part B of the Appendix we find that if the firm's budget balance condition holds for $\beta = \beta^1$, then it also holds for all $\beta \neq \beta^1$. The fact that all local downward incentive compatibility constraints are binding and that the monotonicity constraint $K(\beta^l) \geq K(\beta^{l-1})$ holds for all l, implies that all firm incentive compatibility conditions are satisfied. This follows from similar arguments as Lemmas 1 and 2 in part B of the Appendix.

Given Steps 1 and 2, The arguments that are needed to complete the proof of Proposition 5 are a straightforward modification of the arguments that were used to in the proof of Proposition 1. Hence, we only sketch the arguments.

The first observation is that if in Proposition 1 we replace the budget condition (1) by the budget condition

$$E\left[\sum_{i=1}^{n} t_i(\theta,\beta)\right] \geq E\left[\left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right] \tag{22}$$

we obtain the following result: A mechanism is Pareto-efficient among those that satisfy the individuals' incentive compatibility conditions as well as the budget constraint (22) if and only if the constraint (22) binds, and the public goods provision rule maximizes the "'virtual surplus"

$$E\left[\left(\sum_{i=1}^{n} \theta_i\right) q(\theta,\beta) \;-\; \left(\beta + \frac{1-F(\beta)}{f(\beta)}\right) k(q(\theta,\beta))\right] .$$

Denote this provision rule in the following by q_e^*.

To obtain this result we need to show that the lower bound on payments identified in Step 1 can be reached. By Step 2, this is the case if q_e^* is such that for all l, $K(\beta^l) \geq K(\beta^{l-1})$. To verify this property, note that, for any (θ,β), $q_e^*(\theta,\beta)$ is characterized by the following first order condition,

$$\frac{\sum_{i=1}^{n} \theta_i}{\beta + \frac{1-F(\beta)}{f(\beta)}} = k'(q_e^*(\theta,\beta)) .$$

By the monotone hazard rate assumption the left hand side is decreasing in β. Hence, conditional on θ, a larger value of β implies that less of the public good is provided. Given our assumption that $\beta^l < \beta^{l-1}$, this means that, for every θ, $q_e^*(\theta,\beta^l) > q_e^*(\theta,\beta^{l-1})$. This implies, in particular that $K(\beta^l) \geq K(\beta^{l-1})$.

B Further Results

B.1 Characterization of Incentive Compatible Mechanisms

Lemma 1 *For all i, the incentive constraints in (2) hold if the following local incentive constraints are satisfied: For any $l < m$,*

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) \ , \tag{23}$$

and, for any $l > 0$,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) \ . \tag{24}$$

Moreover, the local incentive constraints (23) and (24) imply that, for all i, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all $l > 1$.

Proof We first show that for each i and for each l, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$. This follows from adding (23) for $\theta_i = \theta^l$ (as stated in the Lemma) and (24) for $\theta_i = \theta^{l+1}$,

$$\theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) \geq \theta^{l+1} Q_i(\theta^l) - T_i(\theta^l) \ .$$

We now show that (23) implies that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l+2}) - T_i(\theta^{l+2}) \ , \tag{25}$$

To see this, rewrite (23) as

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) - (\theta^{l+1} - \theta^l) Q_i(\theta^{l+1}) \ ,$$

Since $Q_i(\theta^{l+2}) \geq Q_i(\theta^{l+1})$ we also have

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) - (\theta^{l+1} - \theta^l) Q_i(\theta^{l+2}) \ ,$$

Moreover, condition (23) for $\theta_i = \theta^{l+1}$ is

$$\theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) \geq \theta^{l+1} Q_i(\theta^{l+2}) - T_i(\theta^{l+2}) \ ,$$

Adding the last two inequalities yields

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l+2}) - T_i(\theta^{l+2})$$

Hence, an individual with preference parameter θ^l does not benefit from announcing θ^{l+2}. Iterating this argument one more establishes that this individual does neither benefit from announcing θ^{l+3}, etc.
The proof that an individual with preference parameter θ^l does not benefit from announcing θ^{l-j} for any $j \geq 1$ is analogous and left to the reader. ■

Lemma 2 *Suppose that, for some individual i, all local downward incentive compatibility constraints are binding and that $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all $l > 1$. Then all incentive compatibility constraints of individual i are satisfied.*

Proof If all local downward incentive constraints are binding for individual i, this implies that, for all $l \geq 1$,

$$T_i(\theta^l) = \sum_{k=1}^{l} \theta^k (Q_i(\theta^k) - Q_i(\theta^{k-1})) + T_i(\theta^0) \; . \tag{26}$$

Using that $\theta^0 = 0$ and that $\theta^{l+1} - \theta^l = 1$, for all $l > 0$, equation (26) can be equivalently written as

$$T_i(\theta^l) = \theta^l Q_i(\theta^l) - \sum_{k=0}^{l-1} Q_i(\theta^k) + T_i(\theta^0) \; . \tag{27}$$

To establish incentive compatibility, Lemma 1 implies that it suffices to show that all local upward incentive compatibility constraints are satisfied, i.e., for all l,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^l Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) \; .$$

or, equivalently,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) - Q_i(\theta^{l+1}) \; .$$

By equation (27), this inequality can also be written as

$$\sum_{k=0}^{l-1} Q_i(\theta^k) \geq \sum_{k=0}^{l} Q_i(\theta^k) - Q_i(\theta^{l+1}) \; ,$$

or

$$Q_i(\theta^{l+1}) \geq Q_i(\theta^l) \; .$$

These monotonicity constraints are satisfied by assumption. ∎

Lemma 3 *If for individual i, all local downward incentive compatibility constraints are binding, then the expected utility of individual i from the ex ante perspective is given by*

$$E[\theta_i q(\theta) - t_i(\theta)] = E\left[\left(\frac{1 - P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] - T_i(\theta^0)$$

Proof Equation (27) in the proof of Lemma 2 and the law of iterated expectations imply that,

$$\begin{aligned} E[t_i(\theta)] &= \sum_{j=0}^{m} p^j T_i(\theta^j) \\ &= \sum_{j=0}^{m} p^j \theta^j Q_i(\theta^j) - \sum_{j=1}^{m} p^j \sum_{k=0}^{j-1} Q_i(\theta^k) + T_i(\theta^0) \end{aligned}$$

$$
\begin{aligned}
&= E[\theta_i q(\theta)] - \sum_{j=1}^{m}\left(1-\sum_{k=0}^{j} p^j\right) Q_i(\theta^j) + T_i(\theta^0) \\
&= E[\theta_i q(\theta)] - \sum_{j=1}^{m} p^j \frac{1-\sum_{k=0}^{j} p^j}{p^j} Q_i(\theta^j) + T_i(\theta^0) \\
&= E[\theta_i q(\theta)] - E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + T_i(\theta^0) \ .
\end{aligned}
$$

■

B.2 Characterization of Revenue Maximizing Mechanisms for a given provision rule

Lemma 4 *For all i, if the participation constraint in (3) is satisfied for $\theta_i = \theta^0$ then it is also satisfied for all $\theta_i \neq \theta^0$.*

Proof Let $\theta_i \neq \theta^0$. Then, by the incentive compatibility constraints in (2),

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) \geq \theta_i Q_i(\theta^0) - T_i(\theta^0) \ .$$

Moreover, $\theta_i > \theta^0$ implies that the right-hand side of this inequality exceeds

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) \ ,$$

which is nonnegative by the participation constraint for $\theta_i = \theta^0$. This proves that (3) is not binding for $\theta_i \neq \theta^0$. ■

Lemma 5 *Let q be an arbitrary given provision rule. Consider the problem of choosing a mechanism $(t_1, \ldots, t_n)$ in order to maximize total revenue*

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right]$$

subject to subject to the incentive compatibility constraints in (2) and the ex interim participation constraints in (3). At a solution to this problem, the participation constraint in (3) is binding for $\theta_i = \theta^0$ and is slack otherwise.

Proof By Lemma 4 we only need to show that it is binding for $\theta_i = \theta^0$. We show that it is possible to increase the expected payments of individual i in an incentive compatible way if, for some i, the participation constraint for $\theta_i = \theta^0$ does not hold as an equality. It is instructive to rewrite the incentive compatibility constraints in (2) as follows: For each i, for each $\theta_i \in \Theta$, and for each $\hat{\theta}_i \in \Theta$,

$$\theta_i Q_i(\theta_i) - \theta_i Q_i(\hat{\theta}_i) \geq T_i(\theta_i) - T_i(\hat{\theta}_i), \tag{28}$$

Consider a new payment rule for individual i such that for each $\theta_i \in \Theta$, $T_i(\theta_i)$ increases by some $\epsilon > 0$, this implies that the right hand side of the incentive constraints in (28) remains constant, i.e., the increase of i's expected payments does not upset incentive compatibility. Since revenue increase in the expected payments of individual i, the revenue maximizing mechanism must be such that a binding participation constraint for $\theta_i = \theta^0$ prevents a further increase of individual i's payments. ■

Lemma 6 *Let q be an arbitrary given provision rule. Consider the "relaxed problem" of choosing a mechanism $(t_1, \ldots, t_n)$ in order to maximize total revenue*

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right]$$

subject to the downward incentive compatibility constraints in (24) and the ex interim participation constraints in (3). At a solution to this problem, all downward incentive compatibility constraints are binding, and the participation constraint in (3) is binding for $\theta_i = \theta^0$ and is slack otherwise.

Proof It is straightforward to verify that, for all i, all downward incentive compatibility constraints are binding. Otherwise the expected payments of some individual could be increased without violating any one of the constraints of the relaxed problem. It remains to be shown that, for all i, the participation constraint in (3) is binding for $\theta_i = \theta^0$ and is slack otherwise. By Lemma 4 we only need to show that, for all i, the participation constraint in (3) is binding for $\theta_i = \theta^0$. Suppose otherwise. Then it was possible to increase $T_i(\theta^0)$ without violating any constraint. ■

Lemma 7 *Let q be a given provision rule with the property that for all i, and all l, the monotonicity constraints $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$ are satisfied. Consider the problem of choosing $(t_1, \ldots, t_n)$ in order to maximize the total revenue*

$$E\left[\sum_{i=1}^{n} t_i(\theta)\right]$$

subject to the incentive compatibility constraints in (2) and the ex interim participation constraints in (3). The maximal revenue at a solution to this problem is equal to

$$E\left[\sum_{i=1}^{n} \left(\theta_i - \frac{1 - P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] .$$

Proof First, consider the "relaxed problem" of maximizing subject to the local downward incentive constraints (24) and the the ex interim participation constraints in (3). The arguments in the proofs of Lemmas 4 - 6 imply that, for all i, all local downward incentive constraints as well as the ex interim participation constraints are binding for $\theta_i = \theta^0$.[16]

Since the given provision rule q satisfies the monotonicity constraints $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$ for all i and l, Lemma 2 implies that all incentive compatibility constraints are satisfied at a solution to the relaxed problem. Hence, the solution to the relaxed problem is the revenue maximizing mechanism.

[16] In these Lemmas the provision rule for the public good is not taken as given. However, this does not affect the logic of the argument.

Given that all local downward incentive compatibility constraints are binding, Lemma 3 implies that, for all i,

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + T_i(\theta^0)$$

Since the participation constraints are binding, for all i, whenever $\theta_i = \theta^0$, we have $T_i(\theta^0) = 0$, for all i, and hence

$$E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] .$$

■

B.3 On the impossibility to reach constrained efficiency subject to interim participation constraints with many individuals

Lemma 8 *Let $(k_n)_{n=1}^{\infty}$ be a sequence of cost functions, with the understanding that k_n is the cost function that applies if the number of individuals is equal to n. Suppose that the sequence $(k_n)_{n=1}^{\infty}$ converges pointwise to a cost function k_∞. Let q_n^* be the surplus maximizing provision rule for an economy with n individuals. Suppose that $(k_n)_{n=1}^{\infty}$ converges pointwise to a cost function k_∞ and that*

$$\lim_{n\to\infty} \frac{1}{n} E[\beta k_n(q_n^*(\theta))] > 0 .$$

Then there exists n', such that for all $n > n'$,

$$\frac{1}{n} E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_n^*(\theta)\right] < \frac{1}{n} E[\beta k_n(q_n^*(\theta))] .$$

Proof We show in the following that

$$\lim_{n\to\infty} \frac{1}{n} E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_n^*(\theta)\right] = 0 .$$

By the strong law of large numbers $\frac{1}{n}\sum_{i=1}^{n}\theta_i$ converges in probability to $E[\theta_i]$, and $\frac{1}{n}\sum_{i=1}^{n}\frac{1-P(\theta_i)}{p(\theta_i)}$ converges in probability to $E\left[\frac{1-P(\theta_i)}{p(\theta_i)}\right]$. Moreover, $E\left[\frac{1-P(\theta_i)}{p(\theta_i)}\right] = E[\theta_i]$. To see this, note that

$$E\left[\frac{1-P(\theta_i)}{p(\theta_i)}\right] = \sum_{l=0}^{m} p^l \frac{1-P(\theta^l)}{p^l} = \sum_{l=0}^{m}\sum_{k=l+1}^{m} p_k = \sum_{l=0}^{m} l p^l = \sum_{l=0}^{m} \theta^l p^l .$$

Hence,

$$\lim_{n\to\infty} \frac{1}{n} E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)\right] = 0 .$$

To complete the proof, note that $q_n^*(\theta)$ converges in probability to a deterministic public goods provision level. This follows since (i) for every given n, $q_n^*(\theta)$ is characterized by the first order condition $\frac{1}{n}\sum_{i=1}^{n}\theta_i = \frac{1}{n}\beta k_n'(q^*(\theta))$, (ii) $\frac{1}{n}\sum_{i=1}^{n}\theta_i$ converges in probability to $E[\theta_i]$, and (iii) $(k_n)_{n=1}^{\infty}$ converges pointwise to k_∞. ■

By Proposition 2 there is no constrained efficient mechanism that satisfies the ex interim participation constraints whenever

$$\frac{1}{n}E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q_n^*(\theta)\right] < \frac{1}{n}E[\beta k_n(q_n^*(\theta))]\ .$$

In conjunction with Lemma 8 this implies that for sufficiently large n, constrained efficiency is incompatible with the ex interim participation constraints.

B.4 A model with profit-sharing ex post

The basic model is such that individuals receive a share $\frac{\tau}{n}$ of the expected monopoly profits which does not depend on their behavior under the mechanism that the monopolist proposes. In this section of the Appendix we investigate an alternative model which is such that individuals receive a share of the profits that the mechanism designer realizes ex post and which therefore depends on the vector of taste parameters θ. This implies that an individual's incentives under the mechanism are complicated by the dependence of profits on the own announcement. The incentive compatibility constraints are now as follows: for each i, for each $\theta_i \in \Theta$, and for each $\hat{\theta}_i \in \Theta$,

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) + \frac{\tau}{n}\Pi_i(\theta_i) \geq \theta_i Q_i(\hat{\theta}_i) - T_i(\hat{\theta}_i) + \frac{\tau}{n}\Pi_i(\hat{\theta}_i), \tag{29}$$

where

$$\Pi_i(\hat{\theta}_i) := E\left[\sum_{i=1}^{n} t_i(\theta_{-i},\hat{\theta}_i) - \beta k(q(\theta_{-i},\hat{\theta}_i)) \mid \hat{\theta}_i\right]$$

are the expected profits from the perspective of individual i.

We want to show in the following that (i) this model is well-defined only if profits are included in the participation constraints i.e., the participation constraints are such that, for all i and all θ_i

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) + \frac{\tau}{n}\Pi_i(\theta_i) \geq 0 \tag{30}$$

as opposed to

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) \geq 0\ , \tag{31}$$

and (ii) that if the model is well defined, then it gives the same results as the basic model under the assumption that $\tau = 0$.

Proposition 7 *The mechanism that maximizes* Π *subject to the incentive constraints in (29), and the interim participation constraints (30) has the following properties.*

i) For all i*, the participation constraint for* $\theta_i = \theta^0$ *is binding and the participation constraints for* $\theta_i \neq \theta^0$ *are not binding.*

ii) For all i*, all local downward incentive constraints are binding; i.e., for all* $l > 0$*,*

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) = \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) + \frac{\tau}{n}\Pi_i(\theta^{l-1})\ ,$$

and all other incentive compatibility constraints are not binding.

iii) The profit maximizing provision rule is given by q_{Π}^{**}.

iv) For all $\tau < 1$*, the expected after tax profits are given as*

$$(1-\tau)\Pi^* = E\left[\left(\sum_{i=1}^{n}\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta) - \beta k(q_{\Pi}^{**}(\theta))\right]$$

v) For all $\tau < 1$*, the expected payoff of individual* i *from the ex ante perspective, is given by*

$$E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta)\right]$$

B.4.1 Proof of Proposition 7

The proof follows from the series of Lemmas below.

Lemma 9 *For all* i*, the participation constraint in (30) is binding if* $\theta_i = \theta^0$ *and is slack otherwise.*

Proof Let $\theta_i \neq \theta^0$. Then, by the incentive compatibility constraints in (29),

$$\theta_i Q_i(\theta_i) - T_i(\theta_i) + \frac{\tau}{n}\Pi_i(\theta_i) \geq \theta_i Q_i(\theta^0) - T_i(\theta^0) + \frac{\tau}{n}\Pi_i(\theta^0) \, .$$

Moreover, $\theta_i > \theta^0$ implies that the right-hand side of this inequality exceeds

$$\theta^0 Q_i(\theta^0) - T_i(\theta^0) + \frac{\tau}{n}\Pi_i(\theta^0) \, ,$$

which is nonnegative by the participation constraint for $\theta_i = \theta^0$, and the fact that the maximal profit-level is non-negative because the monopolist can always ensure zero profits by choosing $q \equiv 0$. This proves that (30) is not binding for $\theta_i \neq \theta^0$.

Now suppose that $\theta_i = \theta^0$. We show that it is possible to increase the expected payments of individual i in an incentive compatible way if, for some i, the participation constraint for $\theta_i = \theta^0$ does not hold as an equality. It is instructive to rewrite the incentive compatibility constraints in (29) as follows: For each i, for each $\theta_i \in \Theta$, and for each $\hat{\theta}_i \in \Theta$,

$$\theta_i Q_i(\theta_i) - \left(1 - \frac{\tau}{n}\right) T_i(\theta_i) + \frac{\tau}{n}\Pi_i'(\theta_i) \geq \theta_i Q_i(\hat{\theta}_i) - \left(1 - \frac{\tau}{n}\right) T_i(\hat{\theta}_i) + \frac{\tau}{n}\Pi_i'(\hat{\theta}_i), \tag{32}$$

where

$$\Pi_i'(\hat{\theta}_i) = E\left[\sum_{j\neq i} t_j(\theta_{-i}, \hat{\theta}_i) - \beta k(q(\theta_{-i}, \hat{\theta}_i)) \mid \hat{\theta}_i\right]$$

does not include the payments of individual i. (32) can equivalently be written as: For each i, for each $\theta_i \in \Theta$, and for each $\hat{\theta}_i \in \Theta$,

$$\theta_i Q_i(\theta_i) + \frac{\tau}{n}\Pi_i'(\theta_i) - \theta_i Q_i(\hat{\theta}_i) - \frac{\tau}{n}\Pi_i'(\hat{\theta}_i) \geq \left(1 - \frac{\tau}{n}\right)(T_i(\theta_i) - T_i(\hat{\theta}_i)), \tag{33}$$

Now fix the provision rule for the public good and the payments of all individuals j, $j \neq i$. This implies that for all of the incentive constraints in (33), the left hand side remains constant. Now, if the monopolist chooses a new payment rule for individual i such that for each $\theta_i \in \Theta$, $T_i(\theta_i)$ increases by some $\epsilon > 0$, this implies that also the right hand side of the incentive constraints in (33) remains constant, i.e., the increase of i's expected payments does not upset incentive compatibility. Since monopoly profits increase in the expected payments of individual i, the profit-maximizing mechanism must be such that a binding participation constraint for $\theta_i = \theta^0$ prevents a further increase of individual i's payments. ■

Lemma 10 *For all i, the incentive constraints in (29) hold if the following local incentive constraints are satisfied: For any $l < m$,*

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^l Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) \ , \tag{34}$$

and, for any $l > 0$,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^l Q_i(\theta^{l-1}) - T_i(\theta^{l-1}) + \frac{\tau}{n}\Pi_i(\theta^{l-1}) \ . \tag{35}$$

Moreover, the local incentive constraints (34) and (35) imply that, for all i, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all $l > 1$.

Proof We first show that for each i and for each l, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$. This follows from adding (34) and the (35) constraint for $\theta_i = \theta^{l+1}$, which is given by

$$\theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) \geq \theta^{l+1} Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \ .$$

We now show that (34) implies that

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^l Q_i(\theta^{l+2}) - T_i(\theta^{l+2}) + \frac{\tau}{n}\Pi_i(\theta^{l+2}) \ , \tag{36}$$

To see this, rewrite (34) as

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) - (\theta^{l+1} - \theta^l) Q_i(\theta^{l+1}) \ ,$$

Since $Q_i(\theta^{l+2}) \geq Q_i(\theta^{l+1})$ we also have

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) - (\theta^{l+1} - \theta^l) Q_i(\theta^{l+2}) \ ,$$

Moreover, condition (34) for $\theta_i = \theta^{l+1}$ is

$$\theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) \geq \theta^{l+1} Q_i(\theta^{l+2}) - T_i(\theta^{l+2}) + \frac{\tau}{n}\Pi_i(\theta^{l+2}) \ ,$$

Adding the last two inequalities yields

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^l Q_i(\theta^{l+2}) - T_i(\theta^{l+2}) + \frac{\tau}{n}\Pi_i(\theta^{l+2})$$

Hence, an individual with preference parameter θ^l does not benefit from announcing θ^{l+2}. Iterating this argument one more establishes that this individual does neither benefit from announcing θ^{l+3}, etc.
The proof that an individual with preference parameter θ^l does not benefit from announcing θ^{l-j} for any $j \geq 1$ is analogous and left to the reader. ■

Lemma 11 *At a solution to the monopoly problem, all local downward incentive compatibility constraints are binding, all other incentive compatibility constraints are not binding, and the expected payments of individual i are given by*

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + \frac{\tau}{n}\Pi\,. \tag{37}$$

Proof *Step 1.* We first consider the "relaxed problem" of maximizing Π subject to the local downward incentive constraints (35), the binding participation constraints for individuals with $\theta_i = \theta^0$, and the following monotonicity constraints: for all i, $Q_i(\theta^l) \geq Q_i(\theta^{l-1})$, for all $l > 1$.

If one of the downward incentive constraints was not binding, then the expected payments of some individual could be increased without violating any one of the constraints of the relaxed problem. Hence, all of these constraints have to be binding. This implies that, for all i and all $l \geq 1$,

$$T_i(\theta^l) = \sum_{k=1}^{l} \theta^k (Q_i(\theta^k) - Q_i(\theta^{k-1})) + \frac{\tau}{n}\left(\Pi_i(\theta^l) - \Pi_i(\theta^0)\right) + T_i(\theta^0)\,. \tag{38}$$

From the fact that participation constraints are binding whenever $\theta_i = \theta^0$, it follows that $T_i(\theta^0) = \Pi_i(\theta^0)$. Using that $\theta^0 = 0$ and that $\theta^{l+1} - \theta^l = 1$, for all $l > 0$, equation (38) can be equivalently written as

$$T_i(\theta^l) = \theta^l Q_i(\theta^l) - \sum_{k=0}^{l-1} Q_i(\theta^k) + \frac{\tau}{n}\Pi_i(\theta^l)\,. \tag{39}$$

By the law of iterated expectations,

$$\begin{aligned}
E[t_i(\theta)] &= \sum_{j=0}^{m} p^j T_i(\theta^j) \\
&= \sum_{j=0}^{m} p^j \theta^j Q_i(\theta^j) - \sum_{j=1}^{m} p^j \sum_{k=0}^{j-1} Q_i(\theta^k) + \frac{\tau}{n}\sum_{j=1}^{m} p^j \Pi_i(\theta^j) \\
&= E[\theta_i q(\theta)] - \sum_{j=1}^{m}\left(1 - \sum_{k=0}^{j} p^j\right) Q_i(\theta^j) + \frac{\tau}{n}\Pi \\
&= E[\theta_i q(\theta)] - \sum_{j=1}^{m} p^j \frac{1-\sum_{k=0}^{j} p^j}{p^j} Q_i(\theta^j) + \frac{\tau}{n}\Pi \\
&= E[\theta_i q(\theta)] - E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + \frac{\tau}{n}\Pi\,.
\end{aligned}$$

Step 2. To complete the proof, Lemmas 9-10 imply that it suffices to show that the solution to the relaxed problem satisfies all local upward incentive compatibility constraints, i.e., for all i, and all l,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^l Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1})\,.$$

or, equivalently,

$$\theta^l Q_i(\theta^l) - T_i(\theta^l) + \frac{\tau}{n}\Pi_i(\theta^l) \geq \theta^{l+1} Q_i(\theta^{l+1}) - T_i(\theta^{l+1}) + \frac{\tau}{n}\Pi_i(\theta^{l+1}) - Q_i(\theta^{l+1})\,.$$

By equation (39), this inequality can also be written as

$$\sum_{k=0}^{l-1} Q_i(\theta^k) \geq \sum_{k=0}^{l} Q_i(\theta^k) - Q_i(\theta^{l+1}) \ ,$$

or

$$Q_i(\theta^{l+1}) \geq Q_i(\theta^l) \ .$$

These monotonicity constraints are satisfied at a solution of the relaxed problem. ■

Lemma 12 *At a solution to the monopoly problem, expected after tax profits are given as*

$$(1-\tau)\Pi = E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta) - \beta k(q(\theta))\right] \ ,$$

and the ex ante expected payoff of individual i equals

$$E\left[\left(\frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta)\right]$$

Proof Using Lemma 11 to substitute for $E\left[\sum_{i=1}^{n} t_i(\theta)\right]$ in the definition of Π, (see equation (6)) and collecting terms gives the result for after tax profits. Using 11 to substitute for $E\left[t_i(\theta)\right]$ in

$$E[\theta^i q(\theta) - t_i(\theta)] + \frac{\tau}{n}\Pi$$

gives the result for individual i's expected payoff. ■

B.4.2 Participation Constraints that do not include monopoly profits

We now discuss briefly how the analysis changes if instead of the participation constraints in (30) those in (31) that do not include profits, need to be satisfied. The analysis of this problem requires only minor adjustments, relative to the proof of Proposition 7. Adjusting the arguments in the proof of Lemma 9, implies that

$$T_i(\theta^0) = 0. \tag{40}$$

This yields the following modifications in Lemma 11. The expected payments of individual i ex interim, provided that $\theta_i \neq \theta^0$, are now given as

$$T_i(\theta^l) = \theta^l Q_i(\theta^l) - \sum_{k=0}^{l-1} Q_i(\theta^k) + \frac{\tau}{n}\left(\Pi_i(\theta^l) - \Pi_i(\theta^0)\right) \tag{41}$$

From the ex ante perspective these expected payments are equal to

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta)\right] + \frac{\tau}{n}\Pi - (1-P(\theta^0))\frac{\tau}{n}\Pi_i(\theta^0) \tag{42}$$

Substituting these expected payments into the definition of Π implies that

$$(1-\tau)\Pi = E\left[\sum_{i=1}^{n}\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right)q(\theta) - \beta k(q(\theta))\right] - (1-P(\theta^0))\frac{\tau}{n}\sum_{i=1}^{n}\Pi_i(\theta^0)$$

Now it is easy to verify that the mechanism designer may choose the mechanism such that $\Pi(\theta^0)$ is arbitrarily small and hence $E[t_i(\theta)]$ is arbitrarily large even if $T_i(\theta^0) = 0$ and all local downward incentive compatibility constraints are binding.[17] Consequently, after tax profits are unbounded and the profit-maximization problem is no longer well defined.

B.5 A model in which expected profits are included in the participation constraints

The following proposition clarifies how the analysis in Section 4 would be modified if instead of the ex interim participation constraints in (3) that do not include monopoly profits the ex interim participation constraints in (7) are imposed.

Proposition 8 *The mechanism that maximizes* Π *subject to the incentive constraints in (2), and the interim participation constraints (7) has the following properties.*

i) The pattern of binding participation and incentive compatibility constraints and the profit-maximizing provision rule are as in Proposition 7.

ii) The monopolists expected after tax profits, are for all τ*, given as*

$$(1-\tau)\Pi^* = E\left[\left(\sum_{i=1}^{n} \theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta) - \beta k(q_{\Pi}^{**}(\theta))\right] .$$

For all τ*, individual i's ex ante expected payoff is given by,*

$$E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q_{\Pi}^{**}(\theta)\right] .$$

The proposition establishes a neutrality result. The parameter τ neither has an influence on the optimal mechanism nor on the distribution of payoffs between the monopolist and the consumers of the public good. The reason is that, whatever τ, the mechanism designer will ensure that the lowest type's participation constraint is binding, so that

$$T_i(\theta^0) = \frac{\tau}{n}\Pi^*$$

for all i. Consequently, a higher τ, implies that the mechanism designer can rise the expected payment of individuals with the lowest possible valuation, so that their ex interim expected utility level is equal to 0. Given that the lowest types have an expected utility level of 0, all higher types can only get the information rent that is due to their private information on their preferences. Hence, whatever τ, their expected utility level is equal to $E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q_{\Pi}^{**}(\theta)\right]$.

An implication of these observations is that the model with monopoly profits in the participation constraints yields exactly the same results as the model without monopoly profits in the participation constraints under the additional assumption that, in the latter model, $\tau = 0$. Hence, without loss of

[17] Suppose that there are only two individuals, $n = 2$, and only two possible preference parameters, $m = 1$. The monopolist can choose $q(\theta) = 0$, for all θ and still choose the payments $t_1(\theta)$ and $t_2(\theta)$ such that he makes an unbounded profit, even though incentive and participation constraints have to be satisfied.

generality, we may focus on the model with participation constraints that do not include monopoly profits.[18]

B.5.1 Proof of Proposition 8

The proof of Proposition 8 is very similar to the characterization of the profit maximizing mechanism with ex interim participation constraints in the proof of Proposition 4. In the following, we only sketch the steps where the arguments from the proof of Proposition **??** require some modification. Adjusting the arguments in the proof of Lemma 6, implies that

$$T_i(\theta^0) = \frac{\tau}{n}\Pi^*. \tag{43}$$

A straightforward modification of Lemma 7 implies that individual i's expected payments from the ex ante perspective are equal to

$$E[t_i(\theta)] = E\left[\left(\theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta)\right] + \frac{\tau}{n}\Pi^*\,. \tag{44}$$

Substituting these expected payments into the definition of Π yields

$$\Pi = E\left[\left(\sum_{i=1}^{n} \theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q(\theta) - \beta k(q(\theta))\right] + \tau\Pi^*$$

After plugging the profit maximizing provision rule q_{Π}^{**} into this formula we obtain, by definition of Π^*,

$$\Pi^* = E\left[\left(\sum_{i=1}^{n} \theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta) - \beta k(q_{\Pi}^{**}(\theta))\right] + \tau\Pi^*\ ,$$

or, equivalently,

$$(1-\tau)\Pi^* = E\left[\left(\sum_{i=1}^{n} \theta_i - \frac{1-P(\theta_i)}{p(\theta_i)}\right) q_{\Pi}^{**}(\theta) - \beta k(q^{**}(\theta))\right]$$

Finally, to solve for the the ex ante expected payoff of individual i, rearrange equation (44) to obtain

$$E[\theta_i q(\theta) - t_i(\theta)] + \frac{\tau}{n}\Pi^* = E\left[\frac{1-P(\theta_i)}{p(\theta_i)} q(\theta)\right]$$

Upon substituting the profit-maximizing provision rule q_{Π}^{**} into this formula we obtain the statement in the Proposition. ■

[18] Proposition 8 can also be proven for the model with ex ante instead of ex interim participation constraints. The details are left to the reader.